

# Determination of the small Solar system bodies orbital elements from astrometric observations with new OMT-800 telescope

V. Troianskyi, A. Bazyey, V. Kashuba, V. Zhukov

*Astronomical observatory of  
Odessa National University  
Odessa, Ukraine*

Gaia-FUN-SSO-3 Paris Observatory, 24-26 November 2014



# Observational station Mayaki

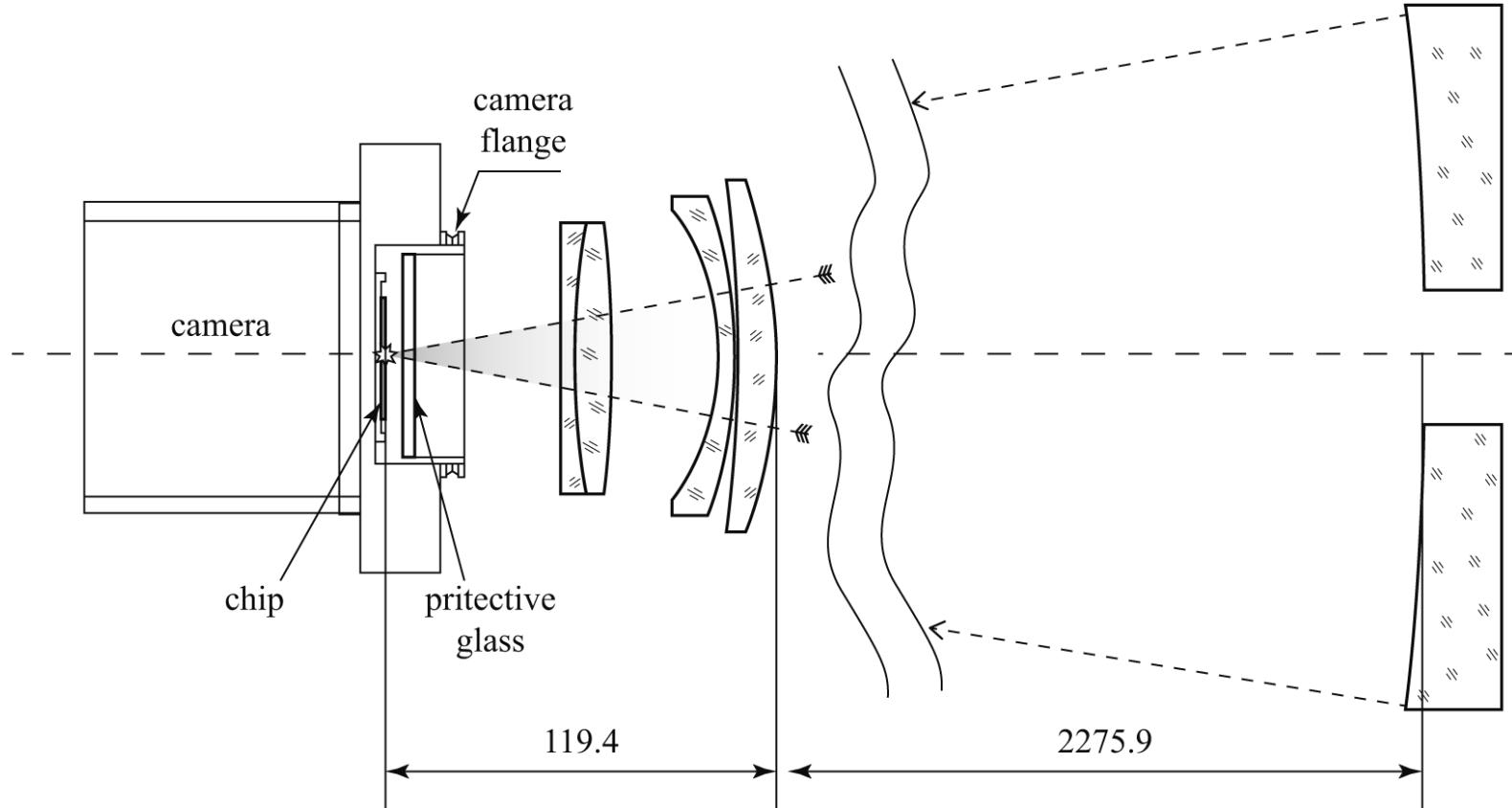


# New OMT-800 telescope in Mayaki

## 800 mm f/2.67 reflector + CCD

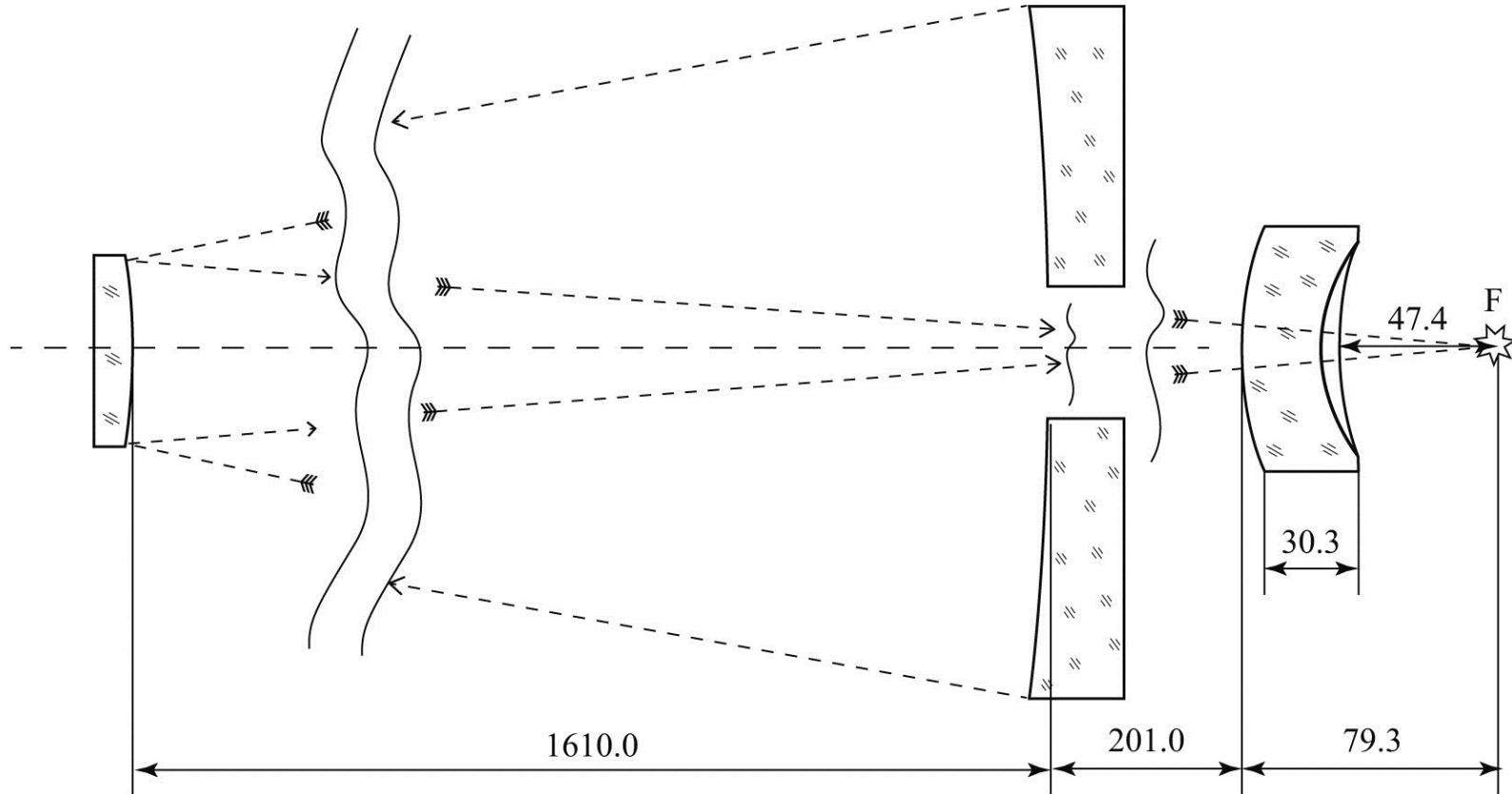


# The optical system of the telescope



Catadioprtic layout with modified Winn's corrector

# The optical system of the telescope



Alternative optical layout  
with two-mirror Ritchey–Chrétien scheme

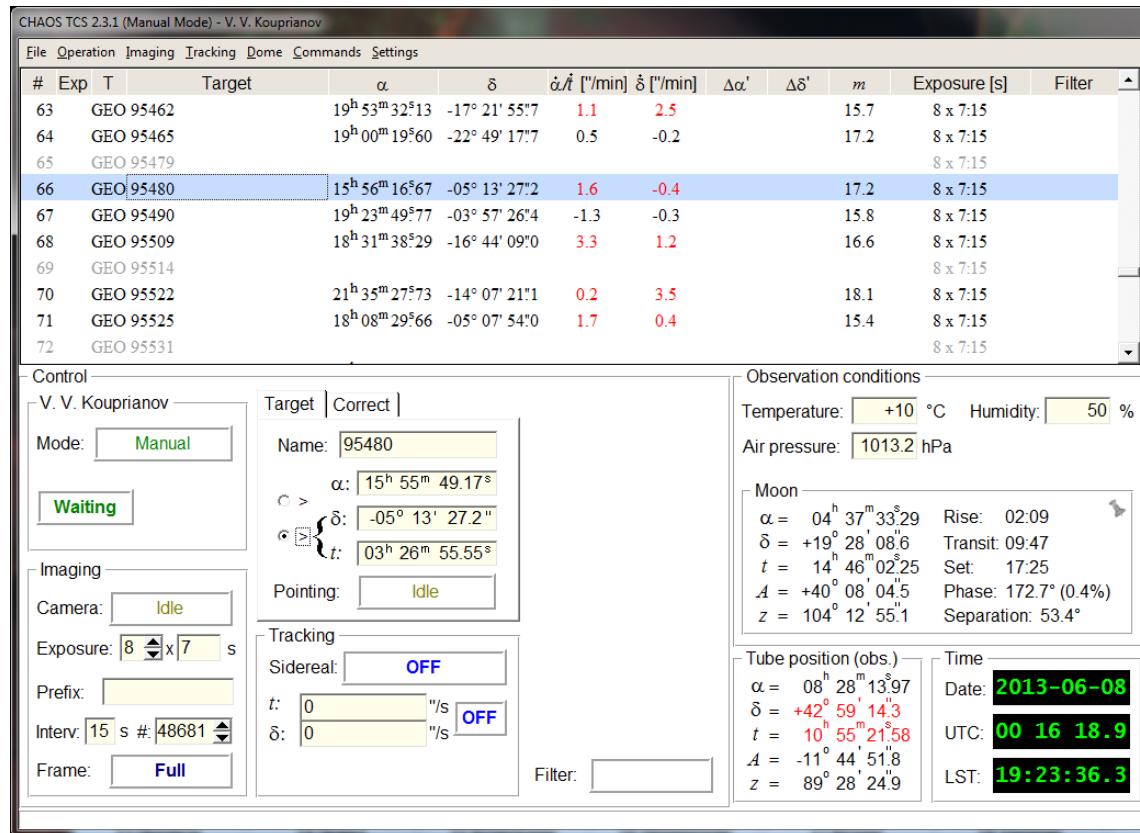
# The main characteristics of OMT-800



Main mirror diameter, <i>mm</i>	<b>800</b>
Telescope effective diameter, <i>mm</i>	<b>755</b>
Telescope total optical length, <i>mm</i>	<b>2395.3</b>
Telescope effective focal length, <i>mm</i>	<b>2134.3</b>
Focal ratio	<b>1:2.67</b>
Mass of main mirror, <i>kg</i>	<b>75</b>
Focal length of main mirror at apex, <i>mm</i>	<b>2398.2</b>
Main mirror eccentricity square	<b>1.1986</b>
Max deviation of main mirror from the ideal surface, <i>nm</i>	<b>60</b>
Linear diameter of the field, <i>mm</i>	<b>49.14</b>
Angular diameter of the field, <i>arc min</i>	<b>78</b>
Spectral range of achromatization, <i>nm</i>	<b>486-820</b>

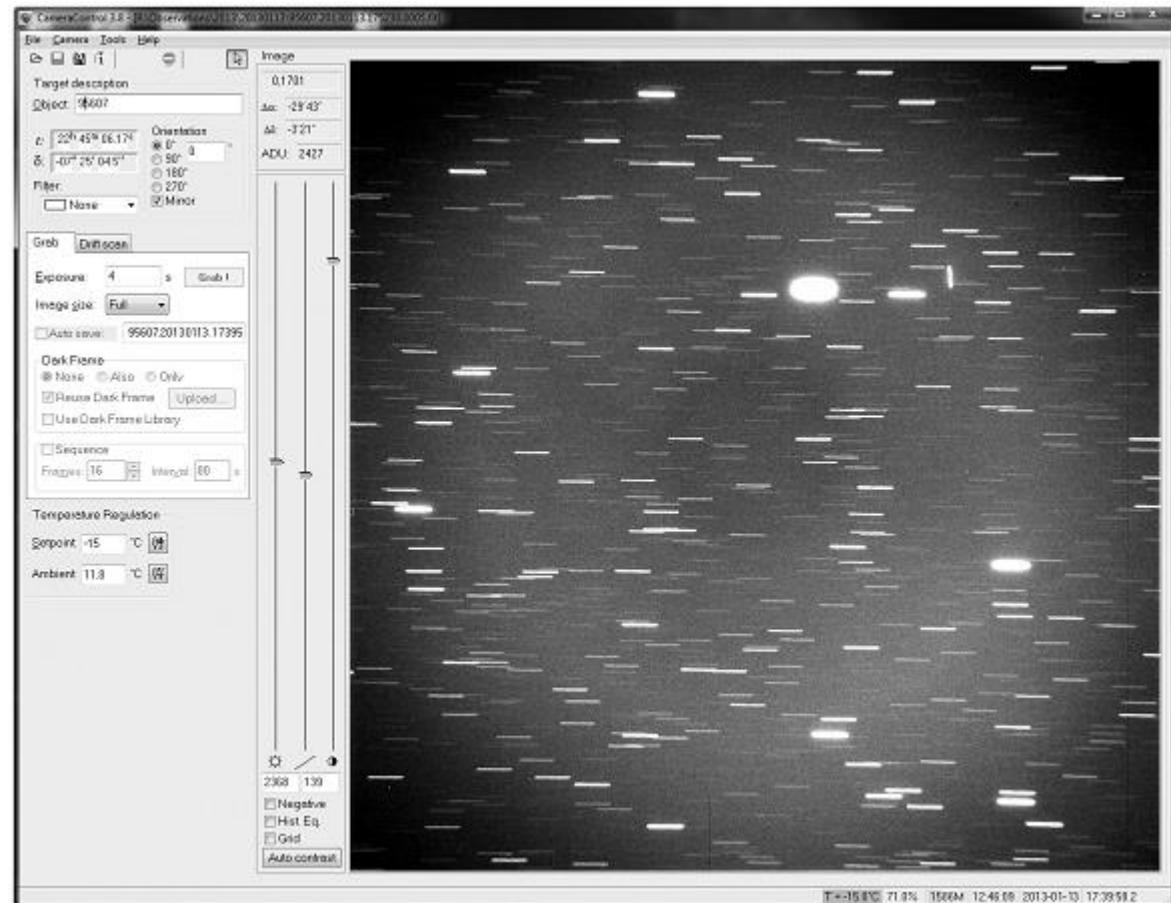
# CHAOS telescope control software on the OMT-800 control workstation

- Automatic observations mode with a previously prepared schedule
- Observation of different types of objects in one in a one mixed schedule
- Tracking of moving objects with known velocities ( $v_\alpha, v_\delta$  or  $v_t, v_\delta$ )
- Ability of using of tabular ephemeris, orbital elements of objects of the Solar system, orbital elements of major planets satellites relatively central planet.



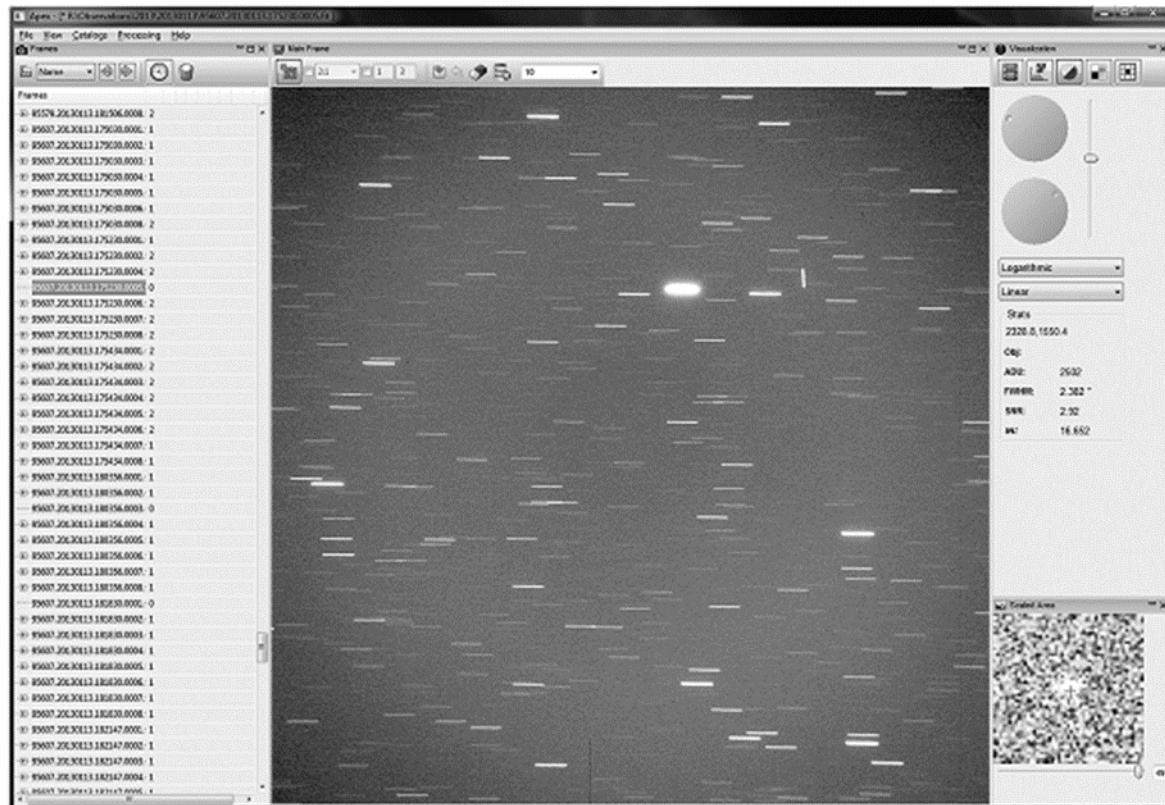
# Camera Control software on the OMT-800 control workstation

- Good tool for quick searching of faint objects
- Control of camera cooler
- Various types of image previews
- Recording of FITS headers



# APEX II astronomical images processing software

- Modular architecture
- Instrumental calibrations
- Source measurement for accurate determination of position and photometrical profile of objects.
- Differential astrometry with data of Tycho-2 and UCAC4 reference catalogues
- Differential photometry
- Post-processing of several images for searching of new objects
- Reporting

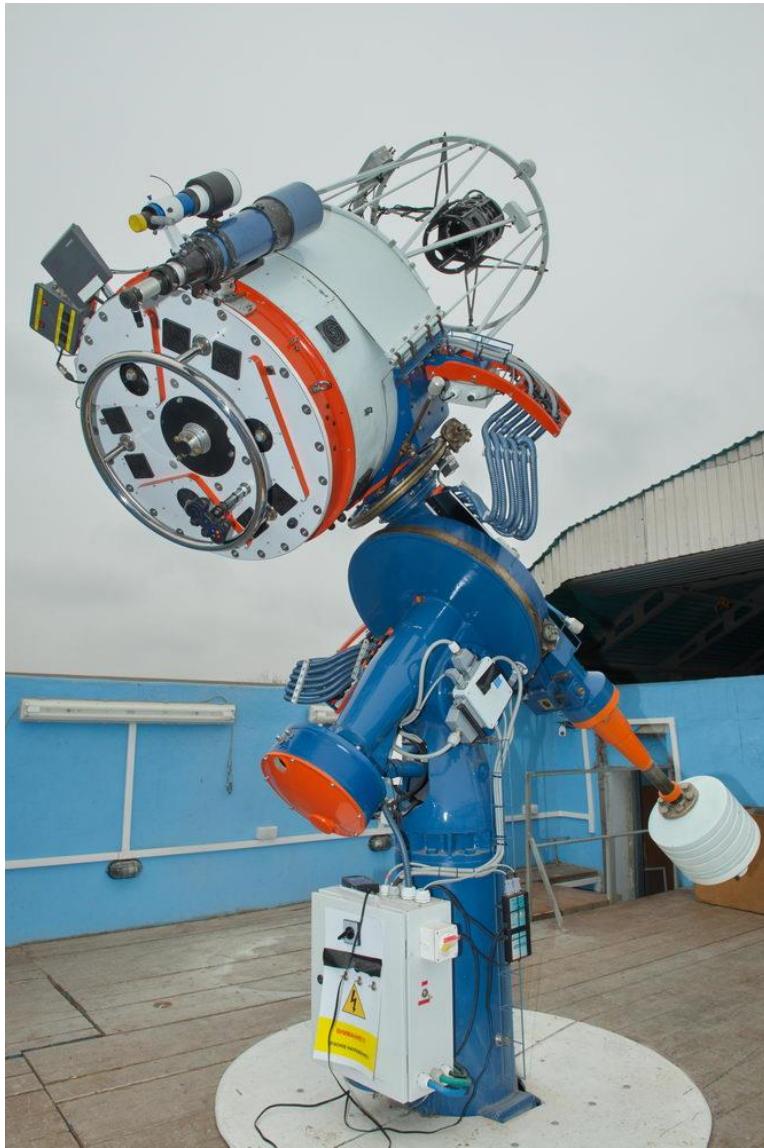


# Capabilities of our telescope

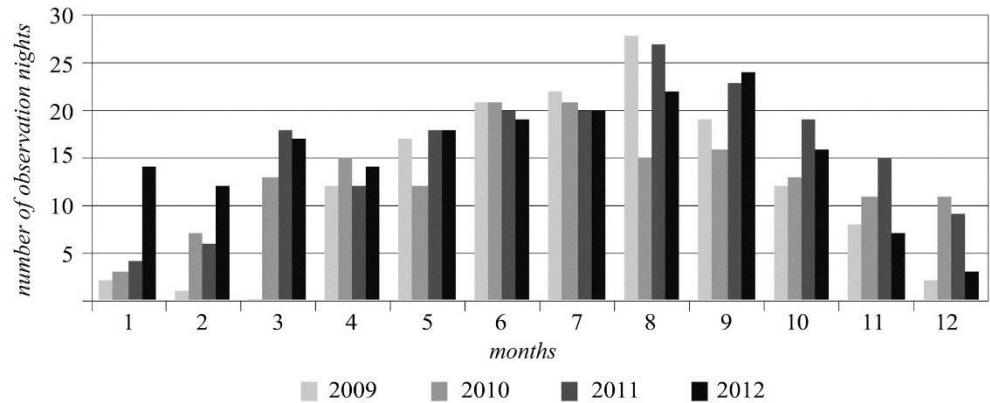
- Observations of artificial satellites in geostationary and geosynchronous orbits
- Observations of Near Earth Asteroids
- Observations of small Solar System bodies
- Differential photometrical observations of stars and stellar fields
- Observations of exoplanet transits



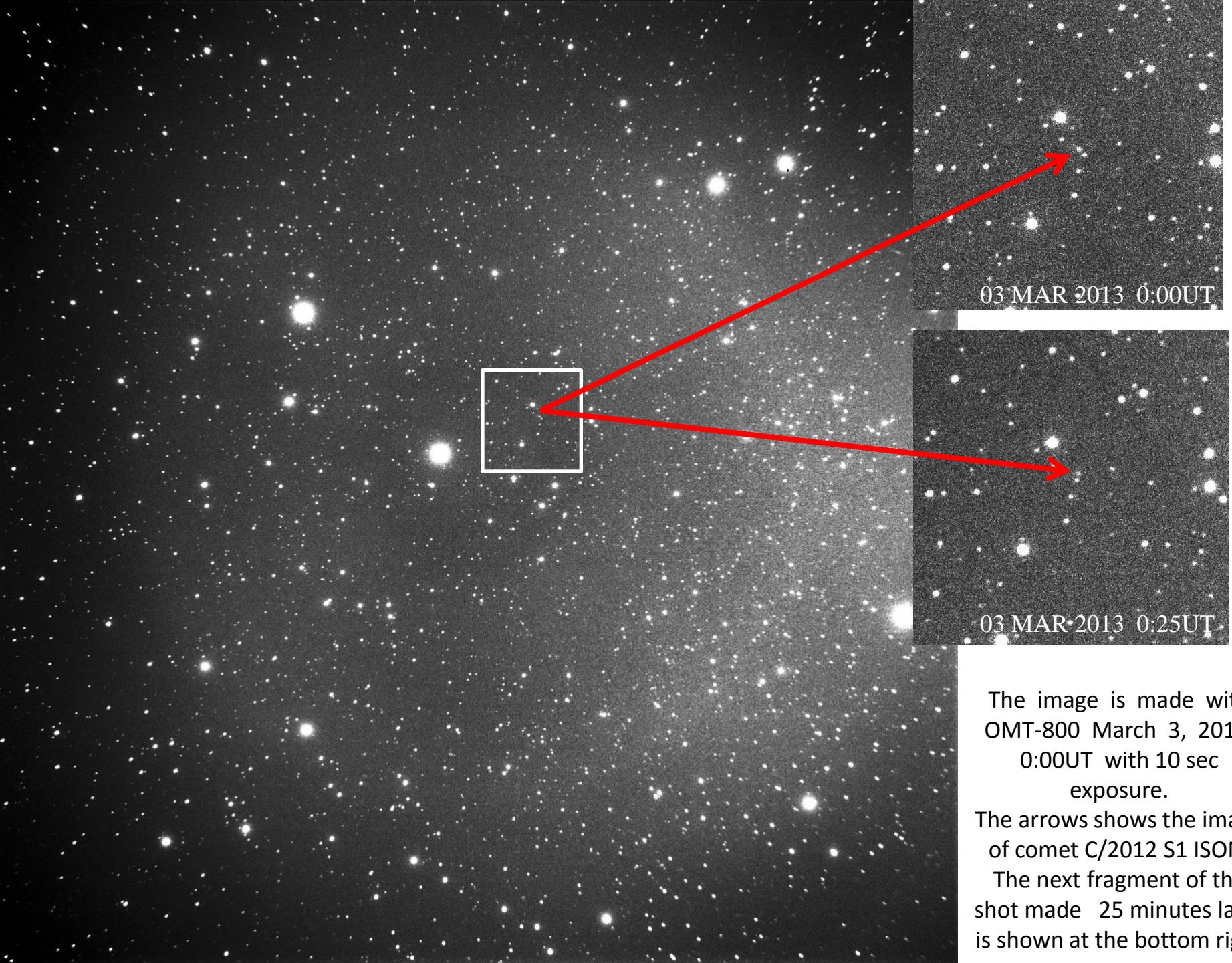
# Advantages of our telescope



Statistics on clear nights at Mayaki station  
for the last 4 years



- The possibility of observing the mixed programs
- High performance automatic observations
- Ability operational observing time-dependent processes (gamma-ray bursters, novae)
- Observations of objects with declination up to  $-35^{\circ}$  and magnitude up to  $21^m$ .



The image is made with OMT-800 March 3, 2013, 0:00UT with 10 sec exposure.

The arrows shows the image of comet C/2012 S1 ISON.

The next fragment of the shot made 25 minutes later is shown at the bottom right



M33 galaxy  
Sum of 3 exposure  
of 10 seconds.  
MaximDL processing



Great Orion Nebula  
M42  
10 seconds exposure  
MaximDL processing

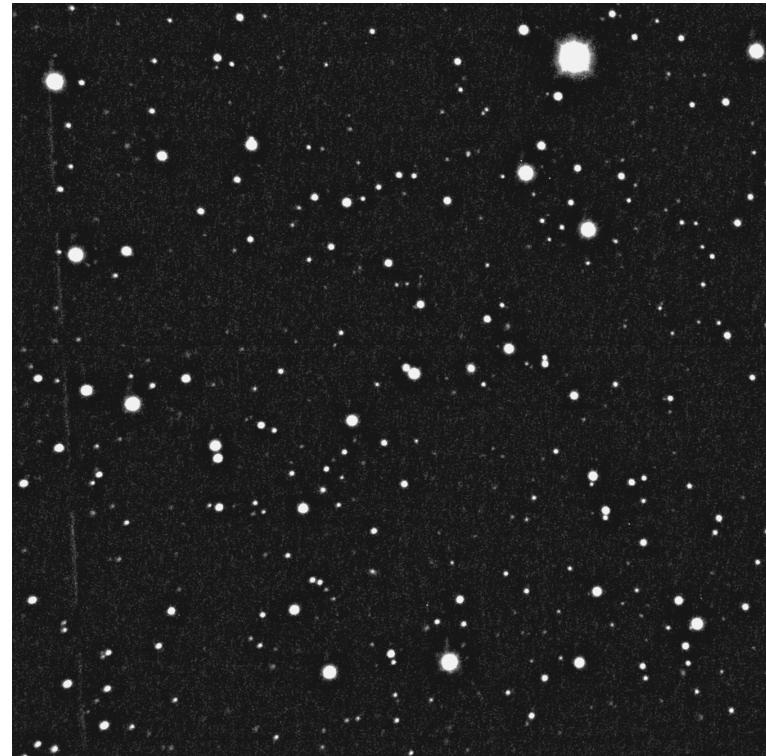


The Andromeda  
Galaxy M31  
Sum of 10 ten-second  
exposures.  
MaximDL processing

# CoLiTec software for automatical searching of asteroids on series of CCD frames



1 exposure  
of 10 seconds



Processing of 8 exposures  
of 10 seconds

# CoLiTec LookSky Software

LookSky - Работа с FITS файлами

Файл Изображение Сервис Справка

Брокер объектов

Астероиды

MPC ColiTec.MPC

Включить Подпись

Nº	Наименование	m	Vdpy
0001	0039P		00.10
0002	0145P		00.10
0003	01537	17.70	01.00
0004	09136	16.60	01.00
0005	11494	17.40	01.00
0006	17795	17.80	01.00
0007	22910	18.60	01.00
0008	23401	17.40	01.00
0009	28822	19.50	01.00
0010	35757	17.90	01.00
0011	36242	17.40	01.00
0012	37054	17.90	01.00
0013	37542	19.00	01.00
0014	38871	18.00	01.00
0015	41683	19.00	01.00
0016	44168	18.50	01.00

IAU Minor Planet Center

X = 01490 Y = 02156 GX = 01492 GY = 02159 I = 872.00000 RA = 14:04:51.35 DE = -14:19:17.10  
 001(001) :: 002(004) C:\Temp\Zones\2013.04.29.ZONE001\Step\_Sngl\_2.fit

# First report and results in Minor Planet Center for CoLiTec MPS 544817-545650

COD 583

CON V. Troianskyi [v.troianskyi@onu.edu.ua]

OBS V. Kashuba

TEL 0.80-m f/3 reflector + CCD

ACK AsteroidSurvey

NET UCAC4

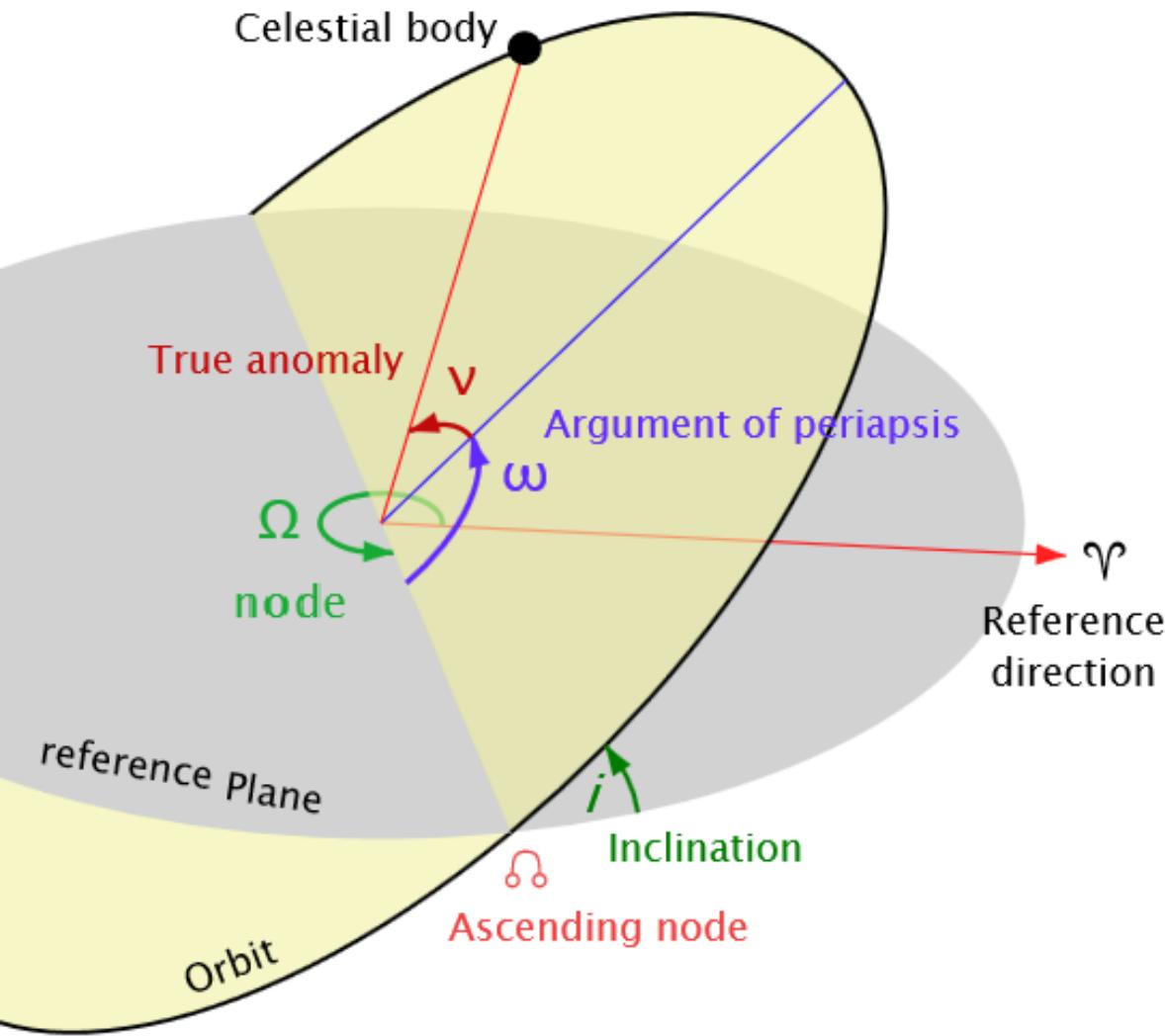
MEA V. Troianskyi

AC2 v.troianskyi@onu.edu.ua, v.kashuba@onu.edu.ua

AA01A02	C2014	10	05.08044	02	47	35.73	+15	35	08.6	13.9	V	583
AA01A02	C2014	10	05.09421	02	47	35.26	+15	35	07.2	14.0	V	583
AA01A02	C2014	10	05.10785	02	47	34.81	+15	35	06.0	14.0	V	583
AA01A02	C2014	10	05.12068	02	47	34.39	+15	35	04.7	14.6	V	583
AA03A02	C2014	10	05.08044	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	05.09421	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	05.10785	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	05.12068	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	05.13414	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	05.14765	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	05.16116	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	05.17467	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	05.18818	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	05.20169	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	05.21520	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	05.22871	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	05.24222	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	05.25573	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	05.26924	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	05.28275	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	05.29626	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	05.31977	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	05.33328	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	05.34679	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	05.36030	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	05.37381	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	05.38732	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	05.40083	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	05.41434	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	05.42785	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	05.44135	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	05.45486	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	05.46837	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	05.48188	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	05.49539	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	05.50890	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	05.52241	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	05.53572	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	05.54923	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	05.56274	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	05.57615	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	05.58966	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	05.60317	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	05.61668	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	05.63019	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	05.64360	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	05.65711	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	05.67062	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	05.68413	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	05.69764	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	05.71115	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	05.72466	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	05.73817	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	05.75168	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	05.76519	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	05.77870	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	05.79221	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	05.80572	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	05.81923	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	05.83274	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	05.84625	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	05.85976	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	05.87327	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	05.88678	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	05.89029	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	05.90370	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	05.91721	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	05.93072	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	05.94423	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	05.95774	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	05.97125	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	05.98476	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	05.99827	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	06.01178	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	06.02529	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	06.03880	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	06.05231	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	06.06572	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	06.07923	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	06.09274	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	06.10615	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	06.11966	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	06.13317	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	06.14668	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	06.16019	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	06.17370	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	06.18721	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	06.20072	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	06.21423	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	06.22774	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	06.24125	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	06.25476	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	06.26827	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	06.28178	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	06.29529	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	06.30880	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	06.32231	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	06.33572	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	06.34923	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	06.36274	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	06.37625	02	45	39.11	+16	15	48.5	16.1	V	583
AA03A02	C2014	10	06.39026	02	45	39.11	+16	15				

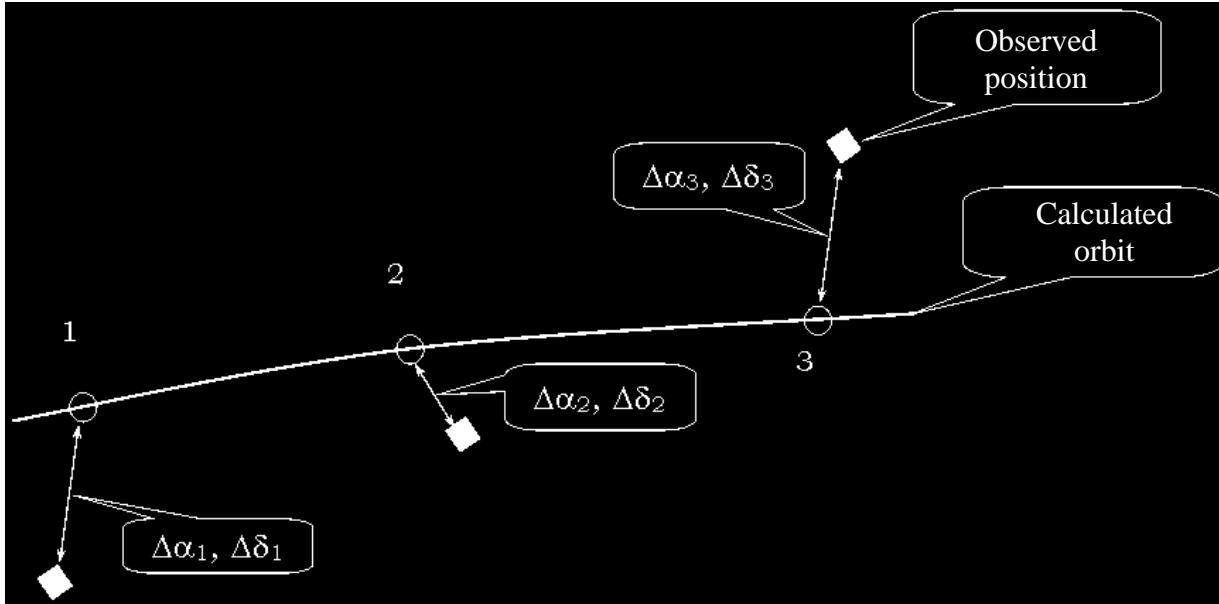
# Search parameters of motion of the object (Orbital elements, the position vector)

19



Preliminary determination of the position vector for the three positional measurements by Gauss and Laplace method

# Clarification prior position vector in all dimensions



$$\Delta\alpha = (\alpha_{t0})_i - (\alpha_{tc})_i$$

$$\Delta\delta = (\delta_{t0})_i - (\delta_{tc})_i$$

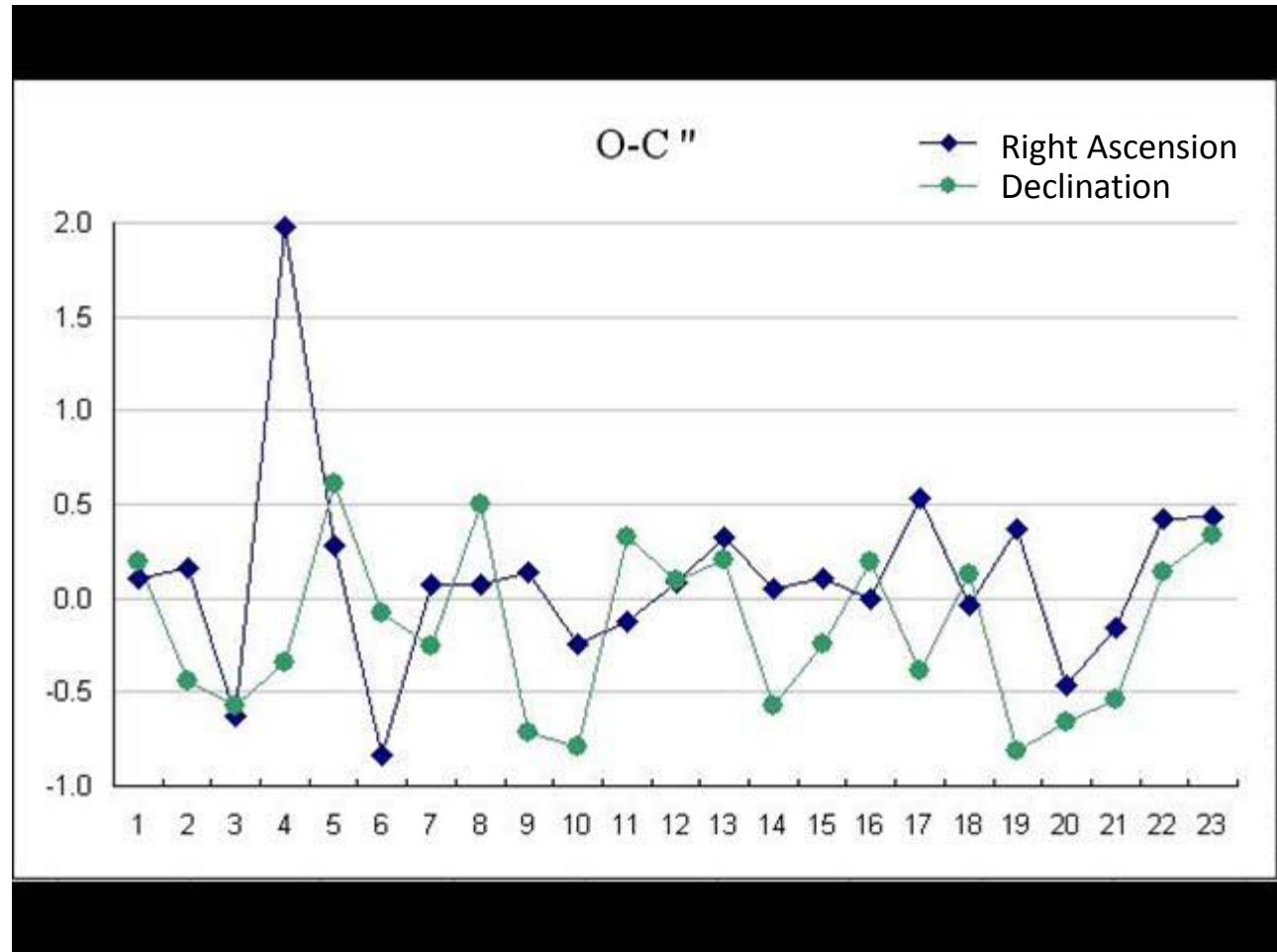
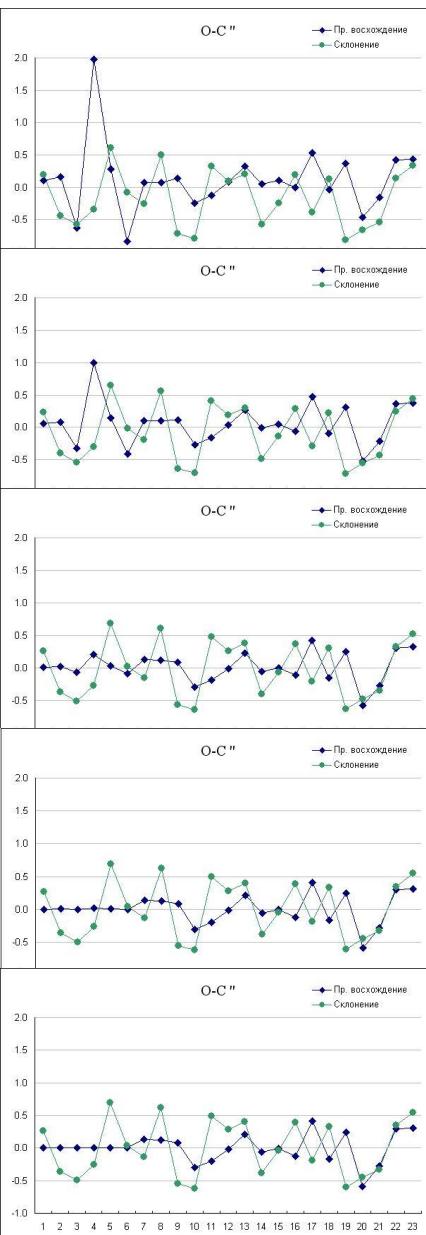
$$\alpha_t = \alpha_t(x_2, y_2, z_2, \dot{x}_2, \dot{y}_2, \dot{z}_2)$$

$$\delta_t = \delta_t(x_2, y_2, z_2, \dot{x}_2, \dot{y}_2, \dot{z}_2)$$

$$\Delta\alpha = \frac{\partial\alpha_i}{\partial x_2} \Delta x_2 + \frac{\partial\alpha_i}{\partial y_2} \Delta y_2 + \frac{\partial\alpha_i}{\partial z_2} \Delta z_2 + \frac{\partial\alpha_i}{\partial \dot{x}_2} \Delta \dot{x}_2 + \frac{\partial\alpha_i}{\partial \dot{y}_2} \Delta \dot{y}_2 + \frac{\partial\alpha_i}{\partial \dot{z}_2} \Delta \dot{z}_2$$

$$\Delta\delta = \frac{\partial\delta_i}{\partial x_2} \Delta x_2 + \frac{\partial\delta_i}{\partial y_2} \Delta y_2 + \frac{\partial\delta_i}{\partial z_2} \Delta z_2 + \frac{\partial\delta_i}{\partial \dot{x}_2} \Delta \dot{x}_2 + \frac{\partial\delta_i}{\partial \dot{y}_2} \Delta \dot{y}_2 + \frac{\partial\delta_i}{\partial \dot{z}_2} \Delta \dot{z}_2$$

# Clarification of the position vector - minimizing the residuals O-C



# The accuracy of calculating of the orbit in comparison with Jet Propulsion Laboratory or Minor Planet Center

$e = 0.559\textcolor{red}{3}2$

$0.55927 \text{ (MPC)}$

$W = 228.0\textcolor{red}{1}5^\circ$

$228.062^\circ \text{ (MPC)}$

$a = 2.234\textcolor{red}{9}1 \text{ a.u.}$

$2.23418 \text{ a.u. (MPC)}$

$i = 47.2\textcolor{red}{2}6^\circ$

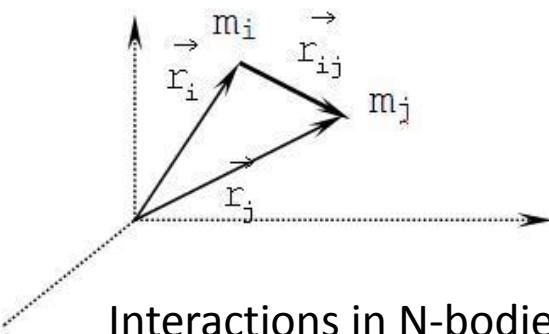
$w = 161.9\textcolor{red}{5}3^\circ$

$161.925^\circ \text{ (MPC)}$

$47.287^\circ \text{ (MPC)}$



# Equations of motion



Interactions in N-bodies problem

$$m_i \ddot{x}_i = -k^2 \sum_{j=1}^{n'} m_i m_j \frac{x_i - x_j}{r_{ij}^3},$$

$$m_i \ddot{y}_i = -k^2 \sum_{j=1}^{n'} m_i m_j \frac{y_i - y_j}{r_{ij}^3},$$

$$m_i \ddot{z}_i = -k^2 \sum_{j=1}^{n'} m_i m_j \frac{z_i - z_j}{r_{ij}^3}.$$

We have  $3N$  second order differential equations. That is, the problem is reduced to a system of  $6N$  order. To solve this system should have a  $6N$  initial conditions and, accordingly,  $6N$  integrals of motion.

# Integration of orbits with Everhart's method

Everhart E. // A New Method for Integrating Orbits // Bulletin of the American Astronomical Society. 1973.

А.А. Базей, И.В. Кара // Применение методов Эверхарта 15, 17, 19, 21 порядков для вычисления траектории движения небесных тел в околопланетном пространстве // Вісник Астрономічної школи. - 2009. - Т.6, № 2. - С.155-157.

# The coordinates of the major planets and the Moon, obtained from numerical theory Solar system DE431 (JPL, 2013)

IPN Progress Report 42-196 • February 15, 2014

## The Planetary and Lunar Ephemerides DE430 and DE431

William M. Folkner,\* James G. Williams,† Dale H. Boggs,†  
Ryan S. Park,\* and Petr Kuchynka\*

# Gravitational potential of Celestial body

$$U_x = \left\{ -\frac{\mu}{r^2} \sum_{\ell=0}^{\infty} (\ell+1) \left( \frac{a}{r} \right)^{\ell} \sum_{m=0}^{\ell} P_{\ell,m}(\sin \phi) [C_{\ell,m} \cos m\lambda + S_{\ell,m} \sin m\lambda] \right\}$$

$$U_y = \left\{ \frac{\mu}{r^2} \sum_{\ell=1}^{\infty} \left( \frac{a}{r} \right)^{\ell} \sum_{m=0}^{\ell} \frac{\partial P_{\ell,m}(\sin \phi)}{\partial \phi} [C_{\ell,m} \cos m\lambda + S_{\ell,m} \sin m\lambda] \right\}$$

$$U_z = \left\{ \frac{\mu}{r^2} \sum_{\ell=1}^{\infty} \left( \frac{a}{r} \right)^{\ell} \sum_{m=1}^{\ell} m \frac{P_{\ell,m}(\sin \phi)}{\cos \phi} [-C_{\ell,m} \sin m\lambda + S_{\ell,m} \cos m\lambda] \right\}$$

$\mu$  – gravitational constant ( $GM$ )

$a$  – equatorial radius of the celestial body

$C_{l,m}, S_{l,m}$  – the coefficients of the gravitational field

$r, \varphi, \lambda$  – spherical coordinates of the asteroid

$P_{l,m}$  – associated Legendre functions

# Accounting 343 most massive Small bodies selected JPL

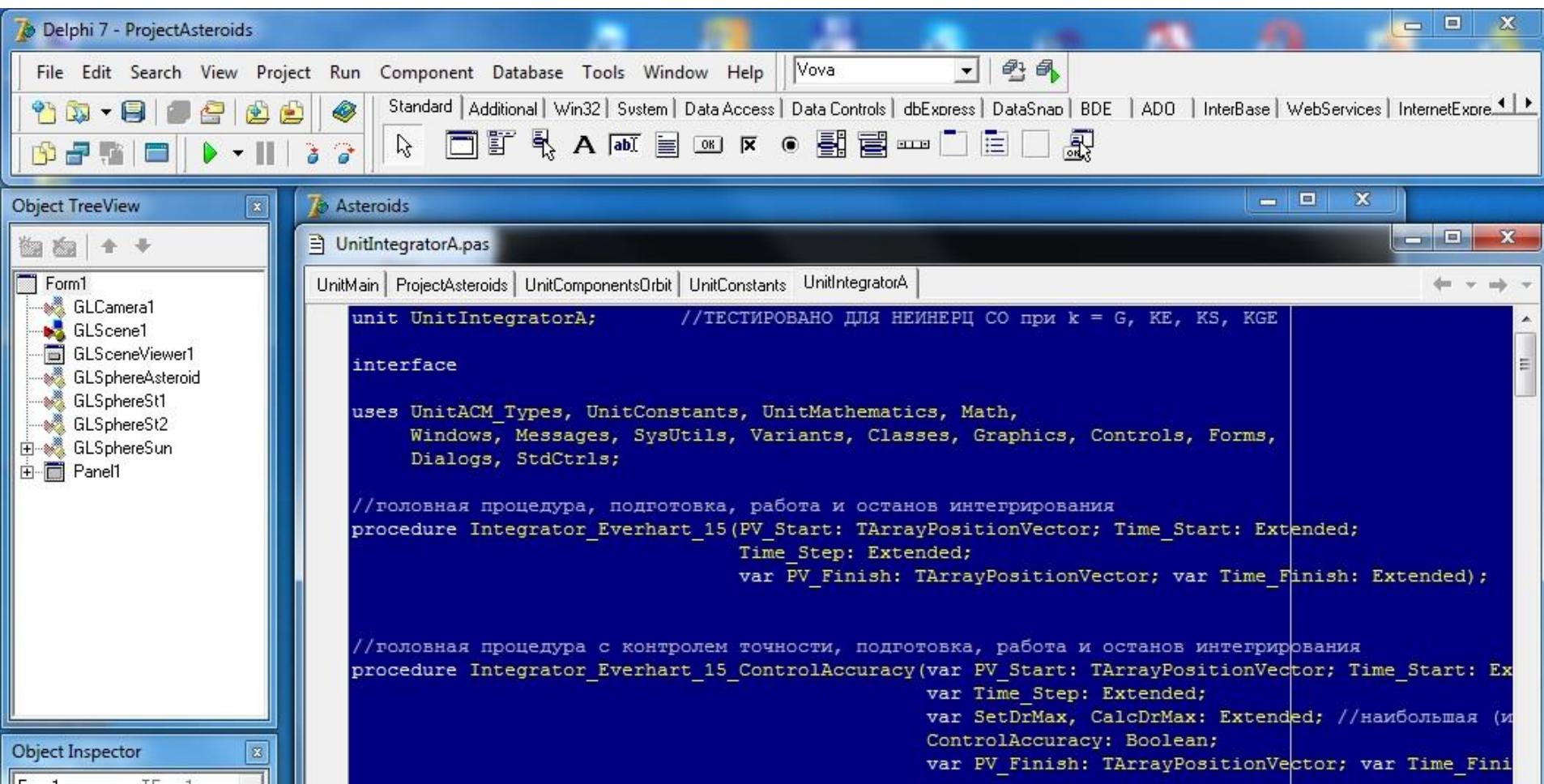
**Table 12. Mass parameters of asteroids (1 of 7).**

Number	Name	GM, au <sup>3</sup> /day <sup>2</sup>	$GM_{ast}/GM_{\odot}$	GM, km <sup>3</sup> /s <sup>2</sup>
1	Ceres	0.140047655617234400E-12	4.73E-10	62.809393
2	Pallas	0.310444819893871300E-13	1.05E-10	13.923011
3	Juno	0.361753831714793700E-14	1.22E-11	1.622415
4	Vesta	0.385475018780881000E-13	1.30E-10	17.288009
5	Astraea	0.374873628455203200E-15	1.27E-12	0.168126
6	Hebe	0.831241921267337200E-15	2.81E-12	0.3728

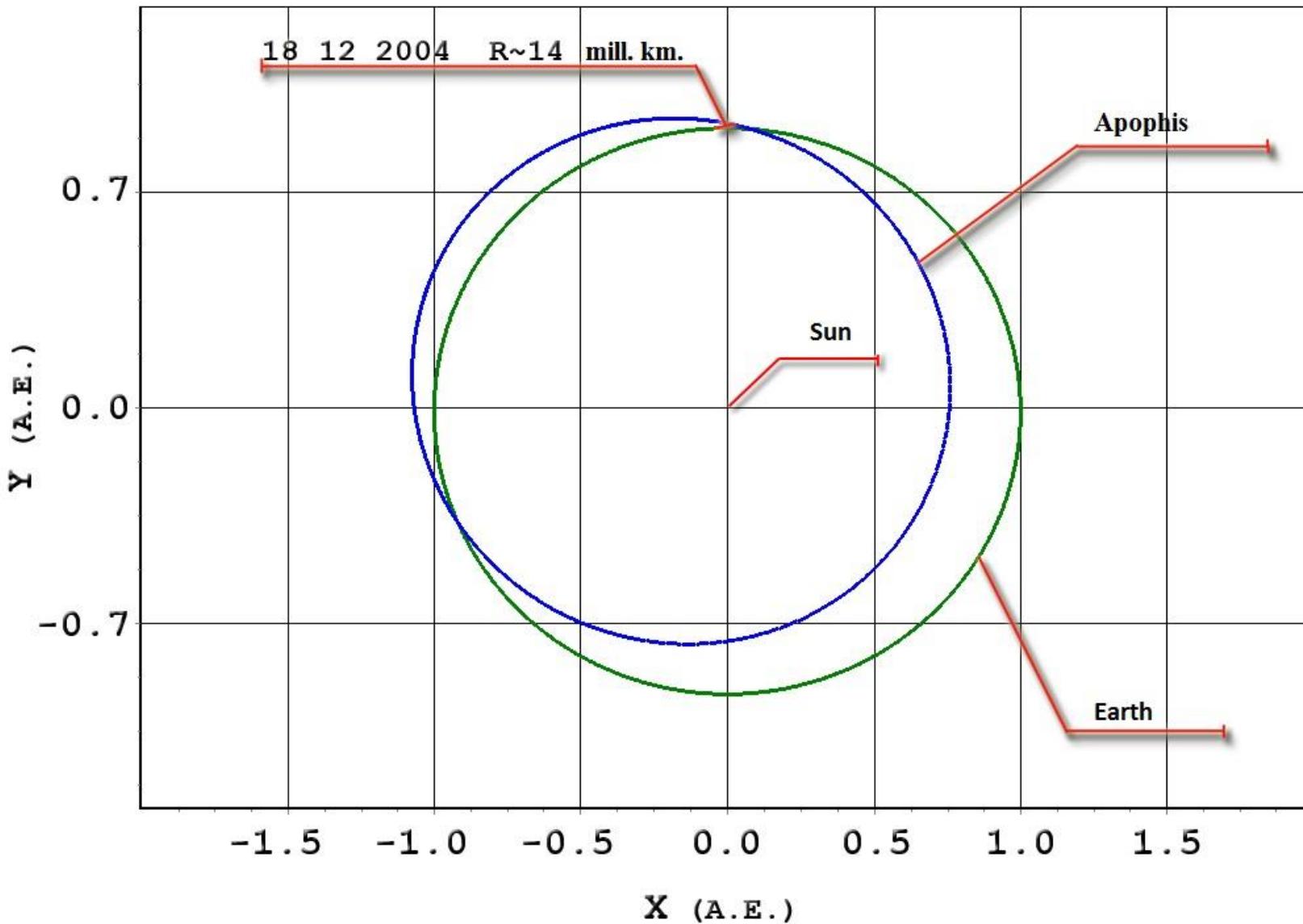
**Table 13. Initial positions (au) and velocities (au/day) of the asteroids with respect to the Sun at Julian day (TDB) 2440400.5 (June 28, 1969) in the ICRF2 frame (1 of 15).**

1	Ceres	$x, y, z$	1.438681809676469747	-2.204373633189407045	-1.326397853361325874
		$v_x, v_y, v_z$	0.008465406136316316	0.004684247977335608	0.000466157738595739
2	Pallas	$x, y, z$	0.203832272462290465	-3.209619436062307152	0.623843179079393351
		$v_x, v_y, v_z$	0.008534313855651248	-0.000860659210123161	-0.000392901992572746
3	Juno	$x, y, z$	0.461207259670432135	-3.006098959780790114	-0.580164049296942208
		$v_x, v_y, v_z$	0.008395458298285176	0.003111908045571209	0.000273059675893248
4	Vesta	$x, y, z$	0.182371836377417107	2.386628211277654010	0.924596062836265498
		$v_x, v_y, v_z$	-0.010174496747119257	0.000041478190529952	0.001344157634155624
5	Astraea	$x, y, z$	2.489297359488491956	1.036395265106434982	0.210563198822894787
		$v_x, v_y, v_z$	-0.005569115604615741	0.007959732929200320	0.003113959705731406
6	Hebe	$x, y, z$	1.339049495814490065	1.442775542206668815	0.079273672077323748
		$v_x, v_y, v_z$	-0.008775983793258694	0.009426820472364830	0.003535716141864189

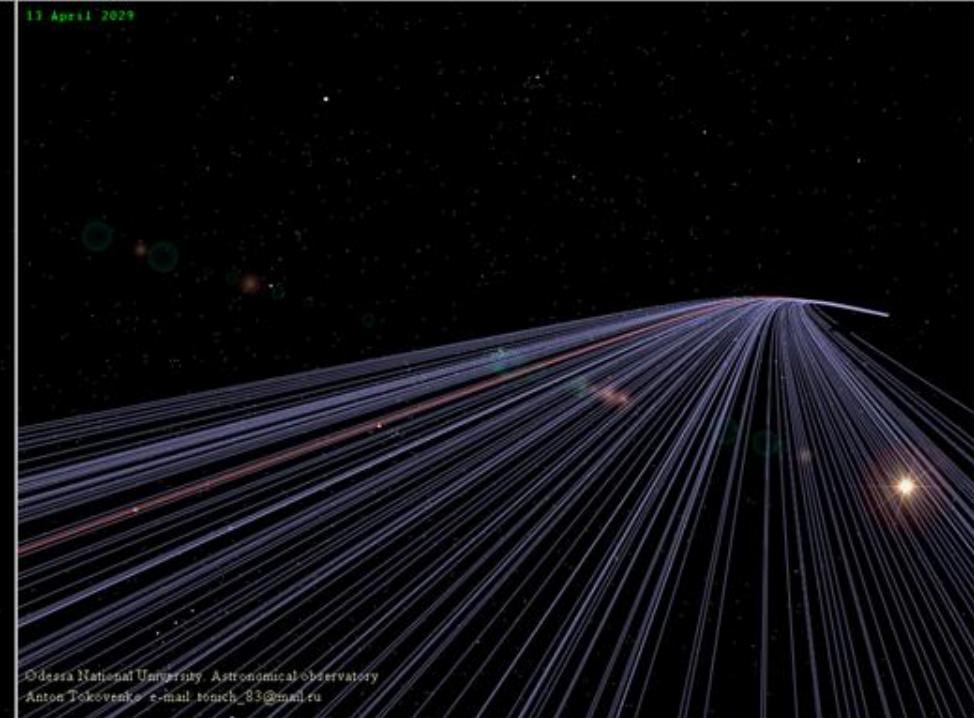
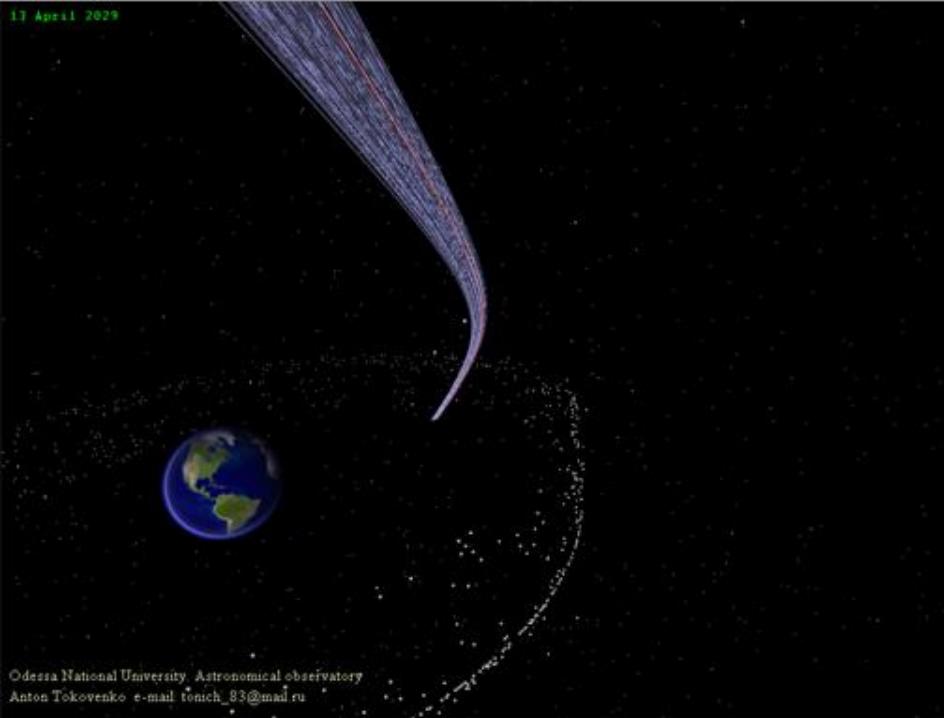
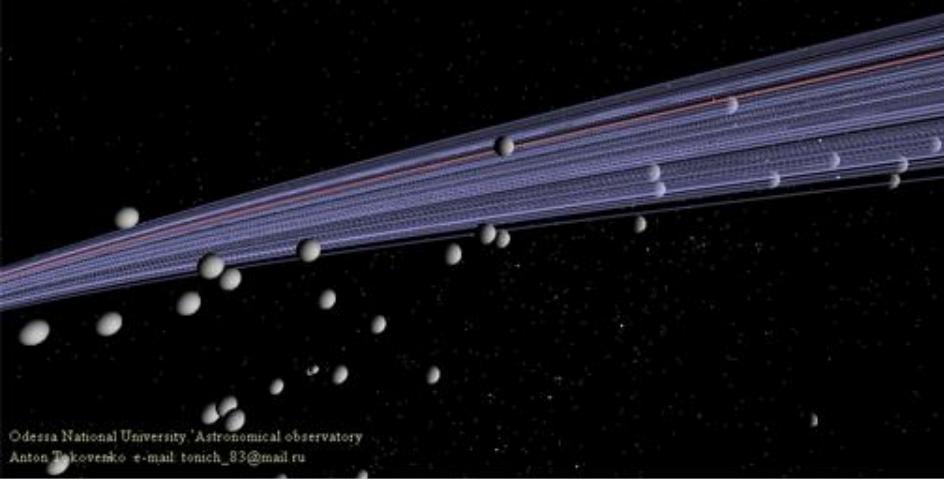
# Method of mathematical modeling in the “Delphi7” we got a change in Kepler orbit of the planets, asteroids and their chosen companions on an arbitrary time interval.



# Orbits asteroid Apophis 99942



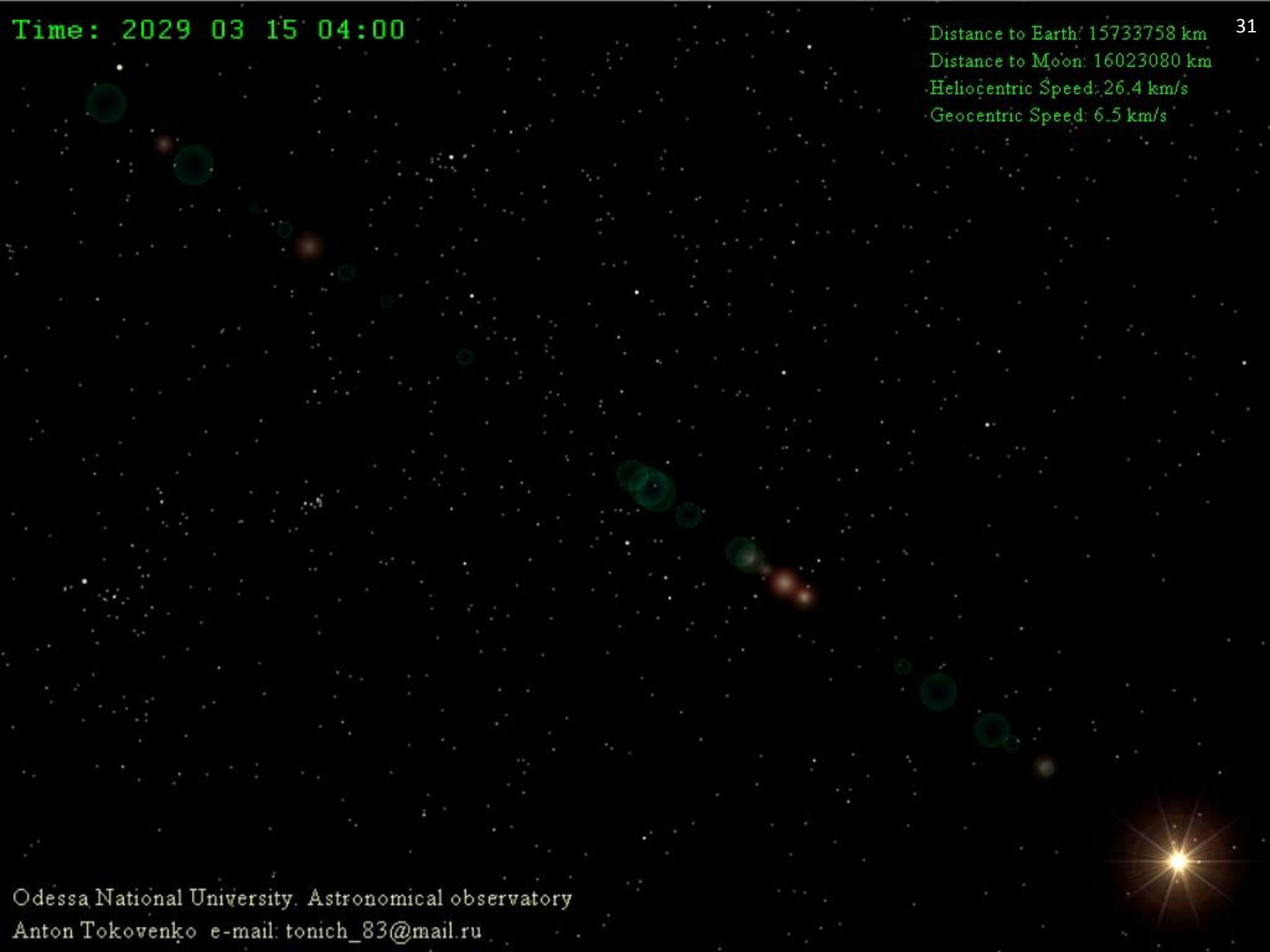
# Simulation orbits



Time: 2029 03 15 04:00

Distance to Earth: 15733758 km  
Distance to Moon: 16023080 km  
Heliocentric Speed: 26.4 km/s  
Geocentric Speed: 6.5 km/s

31



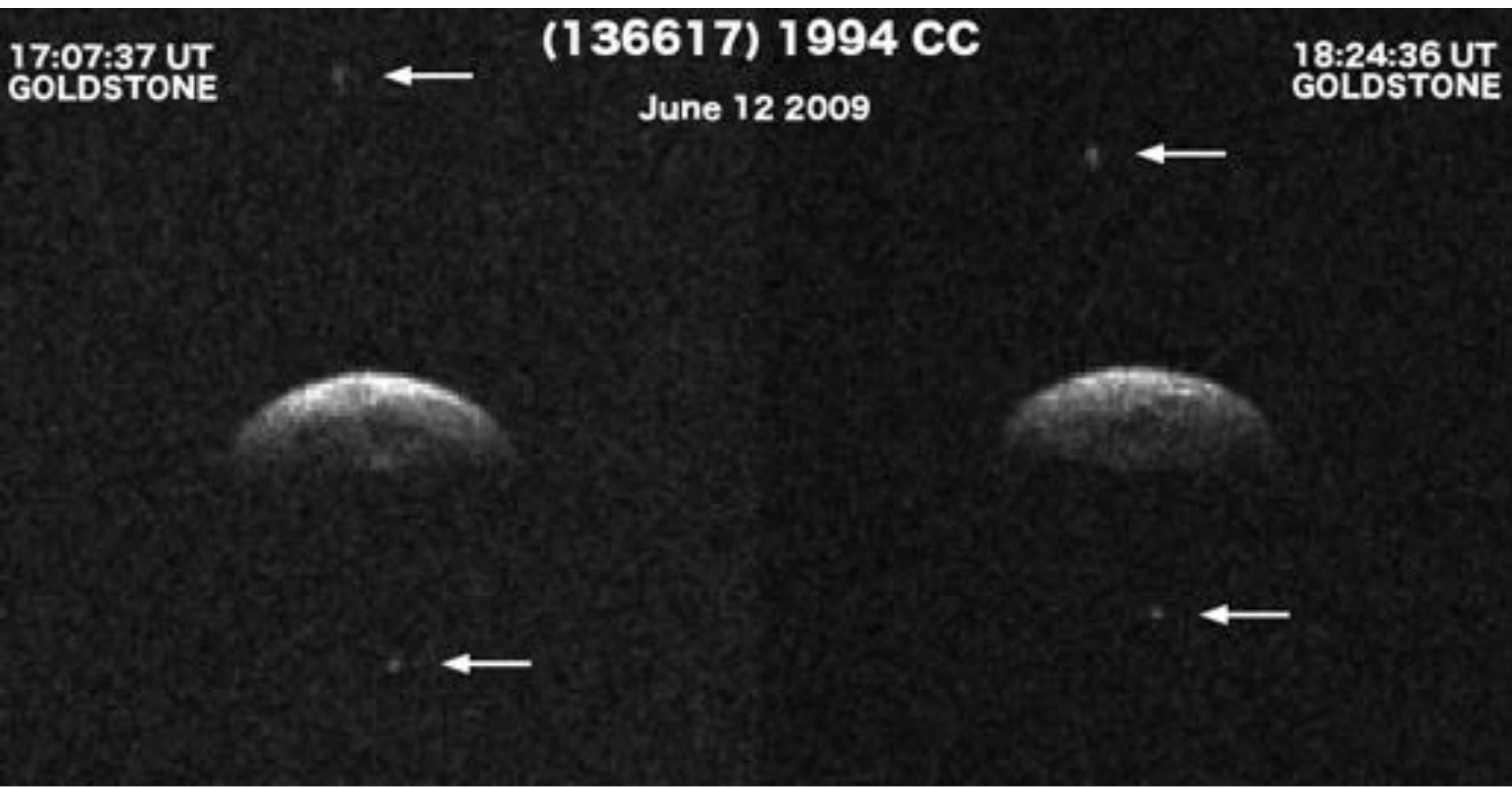
13 Apr. 2029

# For an observer from Odessa

32

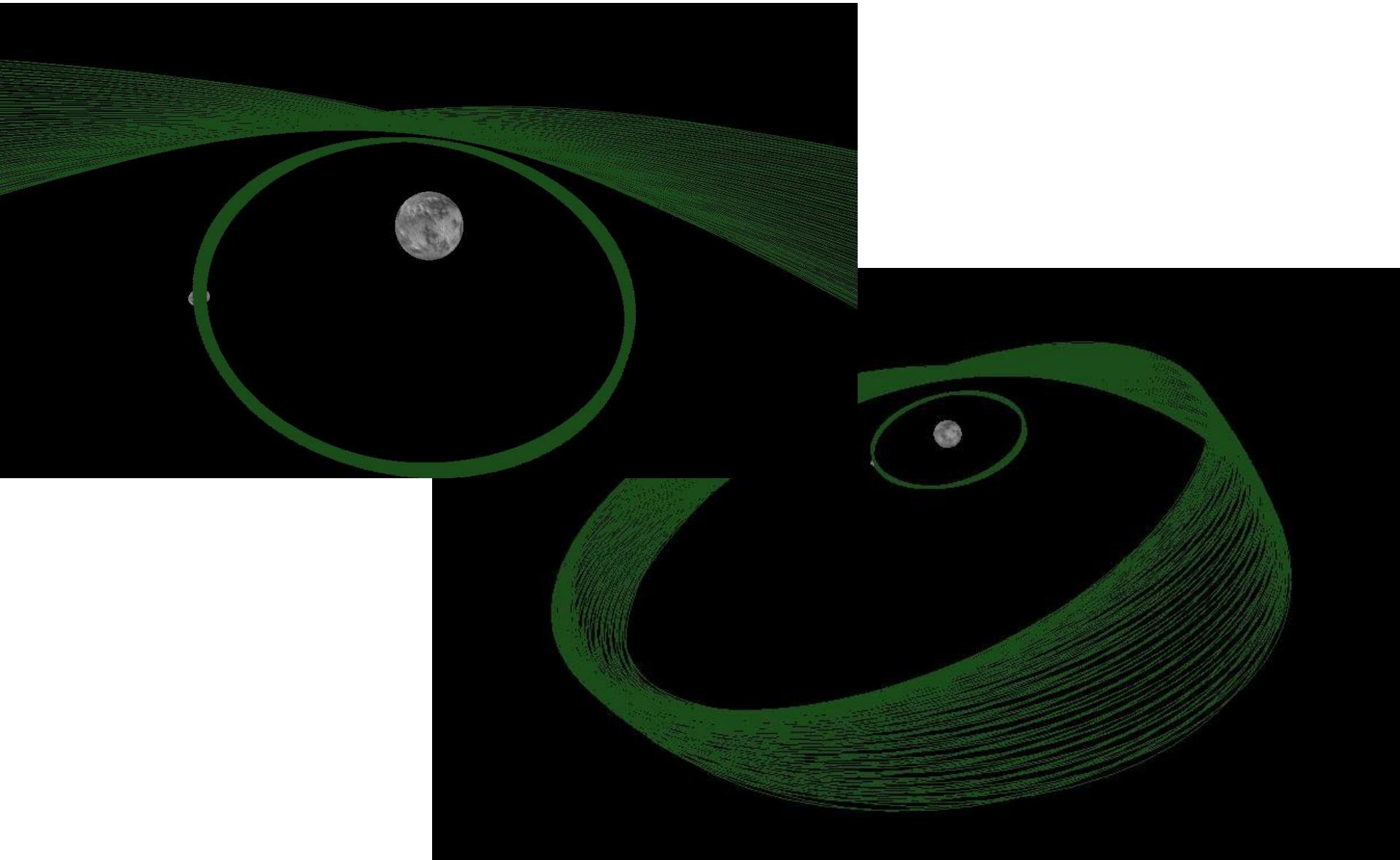


# (136617) 1994 CC (*Apollo*)



Radar imaging at NASA's Goldstone Solar System Radar on June 12 and 14, 2009, revealed that near-Earth asteroid 1994 CC is a triple system. Image Credit: NASA.

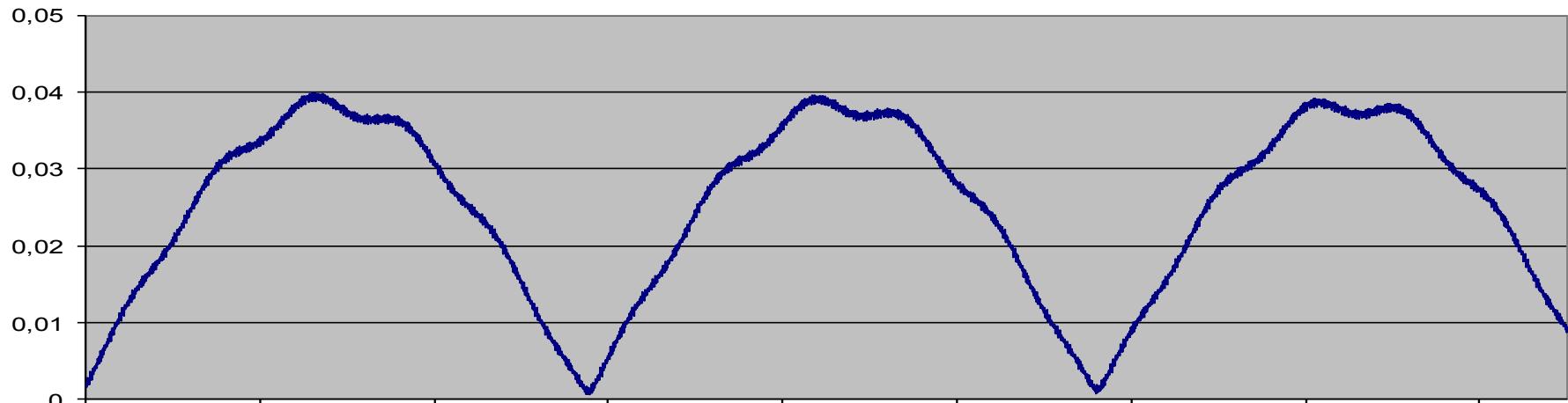
# Evolution orbits satellites (136617) 1994 CC



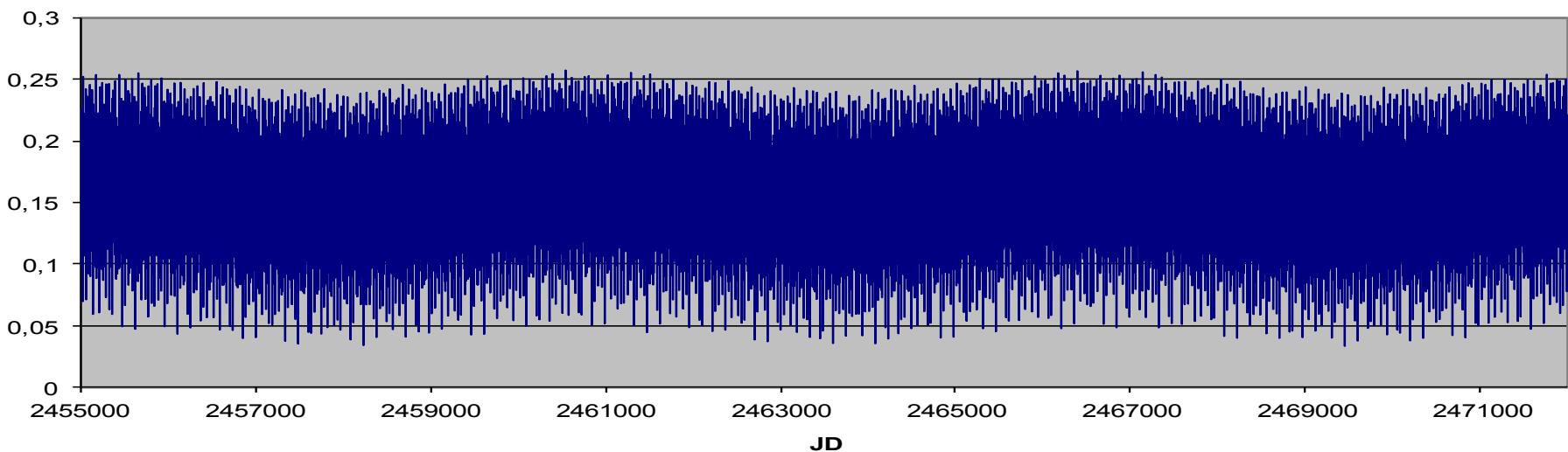
# Evolution of the eccentricity of the asteroid satellites

35

Satellite Alpha



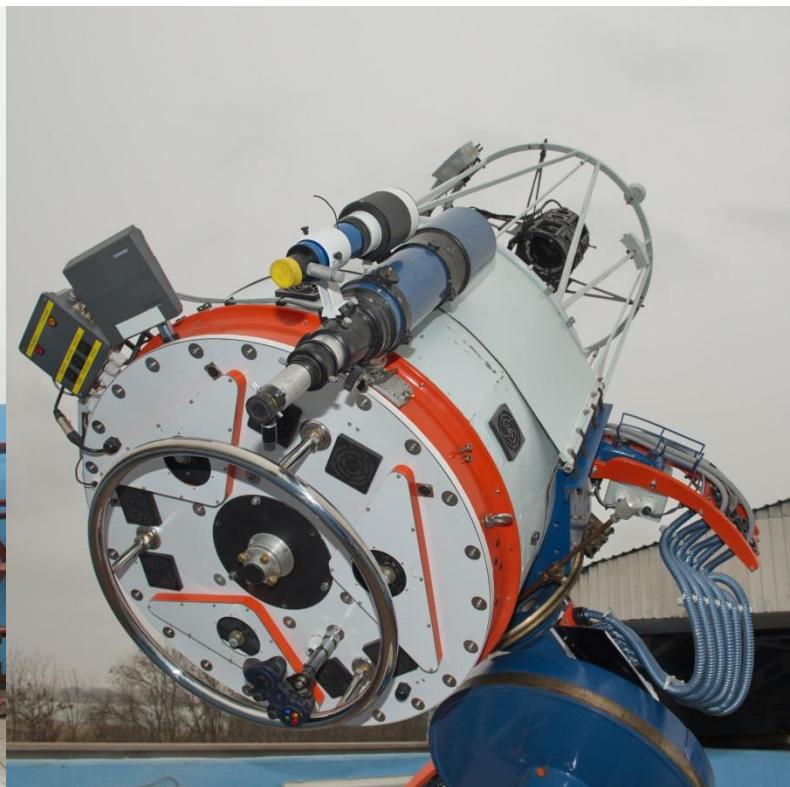
Satellite Beta



# Thanks for attention !!!



# Questions



Volodymyr Troianskyi  
[v.troianskyi@onu.edu.ua](mailto:v.troianskyi@onu.edu.ua)