Method of Determining Small Bodies' Orbits Based on an Exhaustive Search of Orbital Planes

D. E. Vavilov, Yu. D. Medvedev

Institute of Applied Astronomy of Russian Academy of Science

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Introduction

- November 6 2014 there are 415688 numbered asteroids and 249089 unnumbered asteroids.
- More than 10000 near-Earth objects.
- From April 15 2014 till November 6 2014 12088 asteroids were discovered! (≈ 1700 asteroids per month)

Task

Let's assume that we have $n \geq 3$ positional observations of a body right ascensions α_j and declinations δ_j at times t_j, $(j = \overline{1, n})$.
Generally this problem can be divided into 2 subproblems:

- determine the unperturbed Keplerian orbit
- improve the orbital elements

Description of the orbit determination method

Our idea is not to determine the unperturbed Keplerian orbit.
The method consists of two stages:

- Determination of the first approximation of the orbit using exhaustive search of orbital planes
- Improving the orbit by the differential method.

Description of the method

$$\vec{H_j} = \rho_j \vec{L_j} + \vec{E_j},\ \ j = \overline{1, n}, where$$

$\vec{H_j}$ are heliocentric vectors of the body positions at time t_j,
ρ_j are the topocentric distances,
$\vec{L_j}$ are unit vectors pointing to the body (in the topocentric equatorial coordinate system
$\vec{L_j} = (\cos\alpha_j \cos\delta_j, \sin\alpha_j \cos\delta_j, \sin\delta_j))$
$\vec{E_j}$ are the heliocentric vectors of the observer's position.

- We don't know ρ_j,
- Generally ρ_j are determined using iterations, which can diverge or give unsatisfactory results (e.g. $\rho_j < 0$).

The first stage

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- determine the ρ_j values as lengths of vector sections L_j pointing from the observer to the object till the intersection with the plane.

$$\rho_j = \left| \frac{(\vec{N}, \vec{E}_j)}{(\vec{N}, \vec{L}_j)} \right|,$$

where $\vec{N} = (\sin i \cdot \sin\Omega, -\sin i \cdot \cos\Omega, \cos i)$ is the normal vector to the orbital plane.

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- rms of the observation fit is determined:

  $$\sigma = \sqrt{\frac{1}{n}\sum_{j=0}^{n}(\alpha_j - \alpha_j^c)^2\cos^2\delta_j + (\delta_j - \delta_j^c)^2},$$

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The orbit associated with the least σ is considered as the first approximation.

The second stage

The orbital elements obtained in the first stage are improved by differential method taking the following perturbations into account:

- Gravitational perturbations from major planets, Moon and Pluto.
- Relativistic corrections from the Sun.

The coordinates of major planets are calculated by DE423.

Advantages and disadvantages

Advantages:

- Iterations are not carried out to calculate ρ_j.
- The first approximation is always exists.
- If the orbit is not improved we can choose the orbit with the least σ and also we can consider set of appropriate orbits.

Disadvantages:

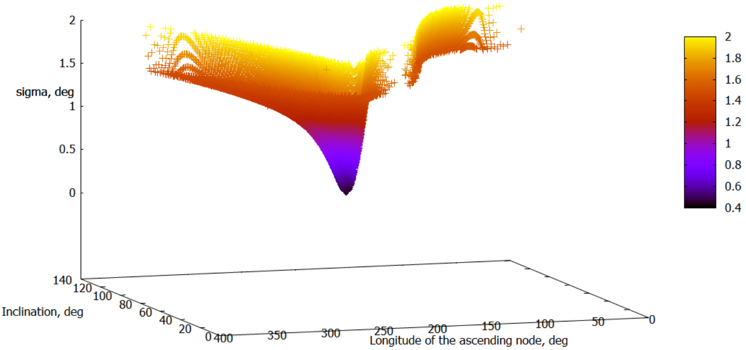
- Calculation time is about several seconds.
- We don't know what step we should use at first stage.

Comparison with the Gauss method

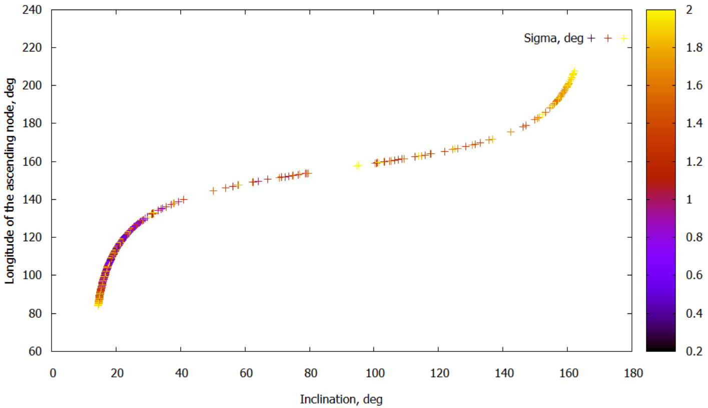
We calculated orbits for 34 asteroids by proposed method and compared with the Gauss method. For 11 asteroids we couldn't obtain the orbit by the Gauss method which can be improved by differential method.

- For 9 asteroids the observation times are two groups separated by a relatively large time interval.
- The mean observation of 2011 KK15 asteroid was inaccurate.
- In calculating the orbit of 2010 SG13 the iterations used to determine ρ diverged. $\rho < 0$.

2010 SG13


sigma, deg
2
1.5
1
0.5
0
140
120
100
80
60
40
20
0
Inclination, deg
400
350
300
250
200
150
100
50
0
Longitude of the ascending node, deg
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4

2010 PT66


Sigma, deg
Longitude of the ascending node, deg
Inclination, deg
240
220
200
180
160
140
120
100
80
60
0
20
40
60
80
100
120
140
160
180
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2

Conclusion

- We developed a new method to determine orbits of celestial bodies.
- The appropriate orbits of all considered asteroids have been calculated by this method.
- For some asteroids the Gauss method didn't determine appropriate orbit which can be improved by differential method.

Reference

- Yu. S. Bondarenko, D. E. Vavilov, and Yu. D. Medvedev , *Method of Determining the Orbits of the Small Bodies in the Solar System Based on an Exhaustive Search of Orbital Planes // Solar System Research, 2014, vol. 48, № 3, pp. 212-216.*
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Thank you for attention!