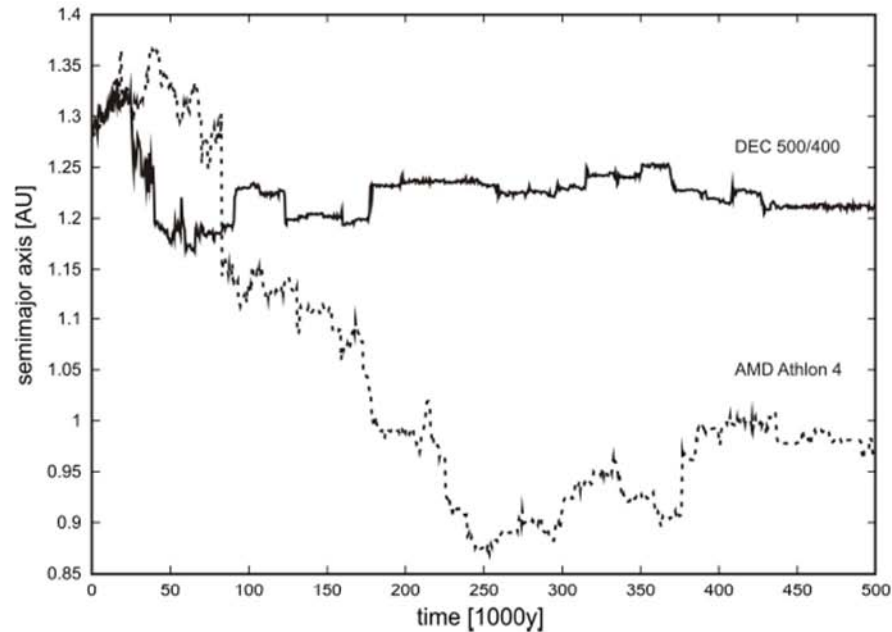


## UN PROBLEME SEVERE: 2 Ordinateurs → 2 orbites differentes



**Fig. 1** Evolution of the semimajor axis of a NEA. The two curves were obtained with the same method and initial conditions but on different computers.

Meme dans un modele simple – loi de Newton  
–masses ponctuelles - asteroide sans masse

# AsteRisk

## Atelier CIA MEUDON Juin 2011



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AstroDynamicsGroup  
Institute for Astronomy, University of Vienna

# ORBITES ET PROBABILITES D'IMPACTS

- Prologue (Un probleme principale)
  - Comparison des integrateurs
  - Les Hungarias et les impacts
    - Le role de la Lune
      - Resume

Eggl, Dvorak: 2010: *An Introduction to Common Numerical Integration Codes Used in Dynamical Astronomy*, in Dynamics of Small Solar System Bodies and Exoplanets by J. Souchay and R. Dvorak (Eds.), Lecture Notes in Physics Vol. 790 pp.431-480

Comparison of 6 different Numerical Integration methods:

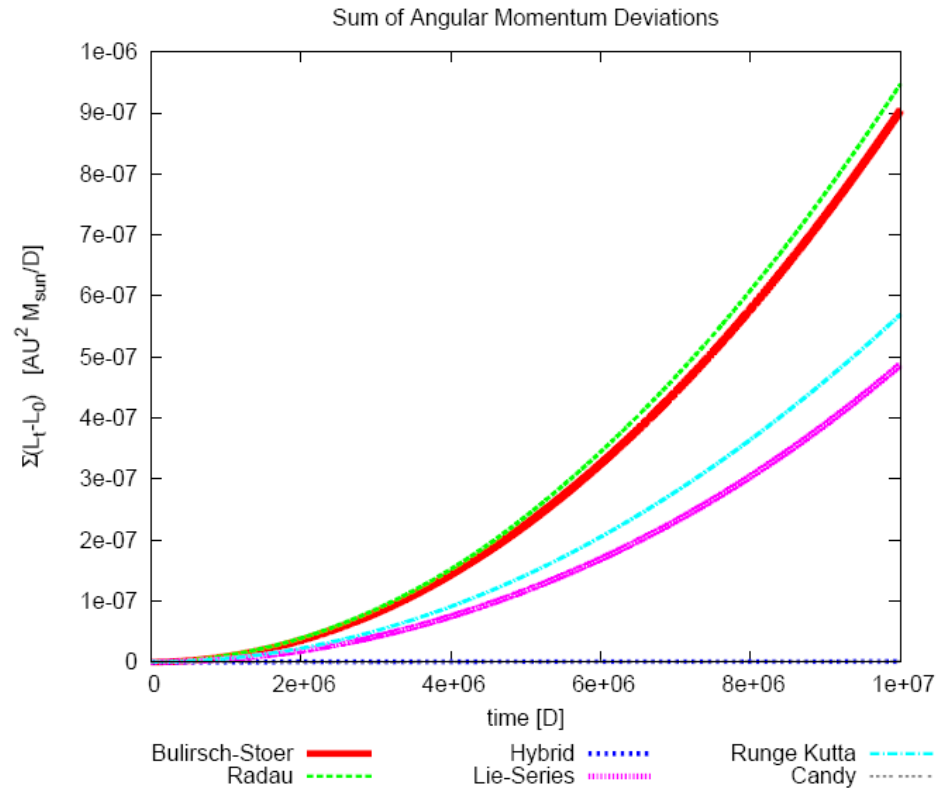
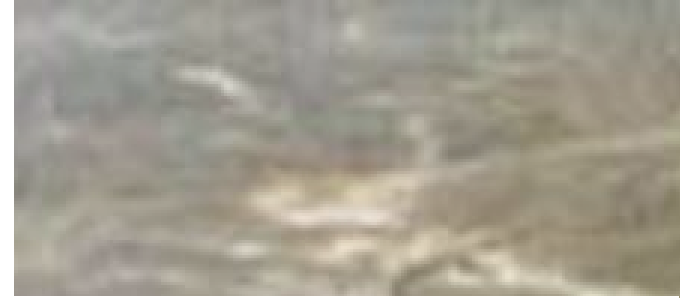
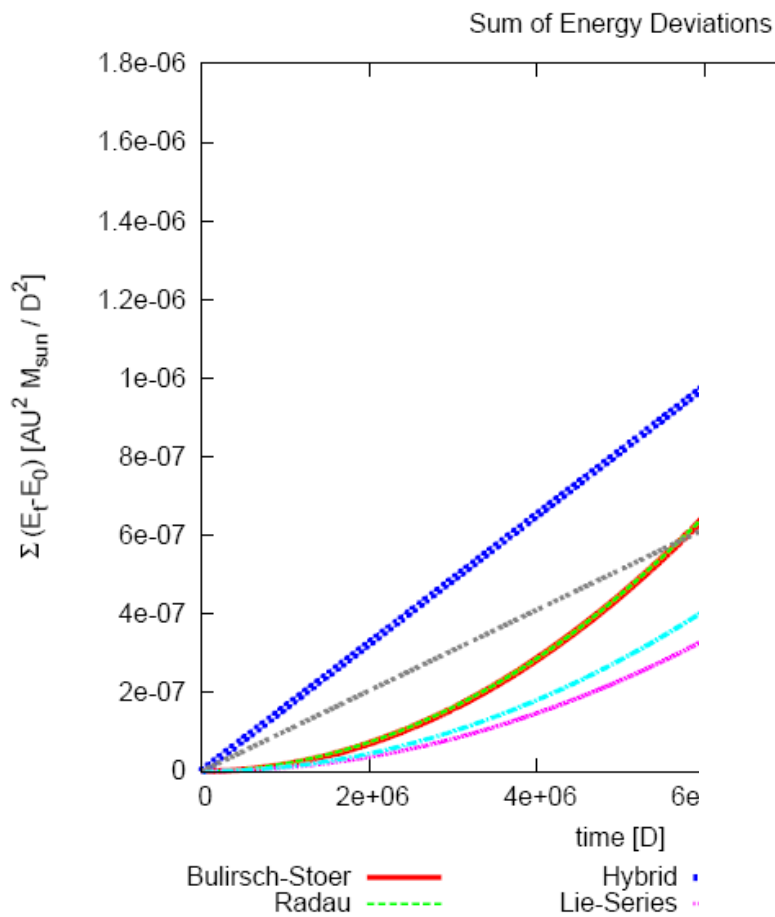
**Bulirsch-Stoer**  
**Radau**  
**Hybrid**  
**Lie-Series**  
**Runge-Kutta**  
**Candy**



Mercury 6



Nie-Integrator



**Test :**  
**Keplerproblem ->**

**Energie**  
**Moment cinetique**

**Fig. 3:** Sum of deviations of total energy from its initial value for all integration algorithms (above); sum of deviations of total angular momentum from its initial value for all integrators (below); the accuracy parameters of all algorithms were chosen, such that the total sum of deviations of energy at time  $10^7$  [D] are roughly in the same order of magnitude. The symplectic algorithms show a linear trend in the deviation of total energy, whereas the non-symplectic show a quadratic trend.

- 
- Test :
  - Keplerproblem ->
    - Energie
    - Moment cinétique
  - Demie grande axe
    - Eccentricité
    - Inclinaison
    - Noeud
    - Perihelie

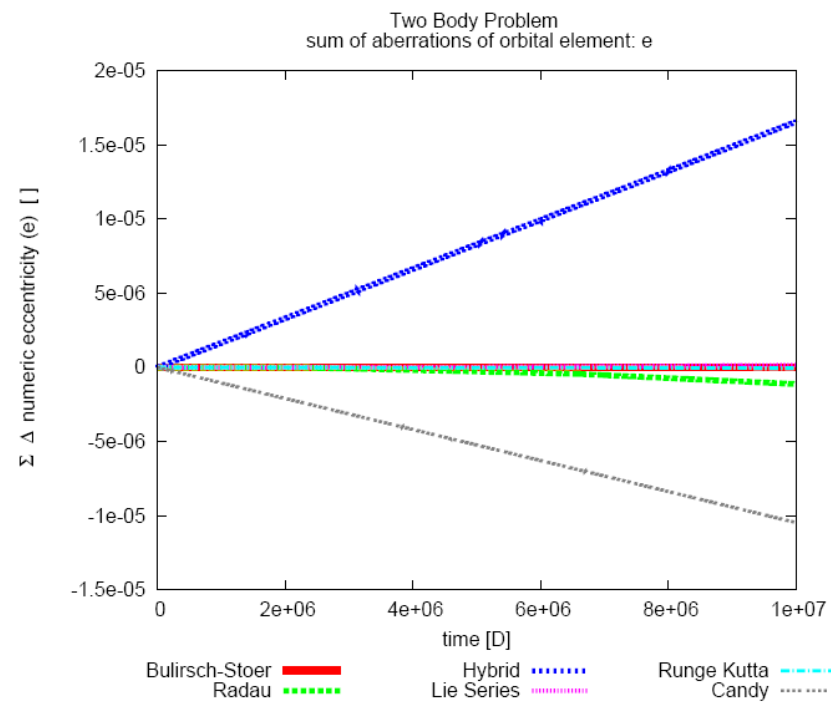
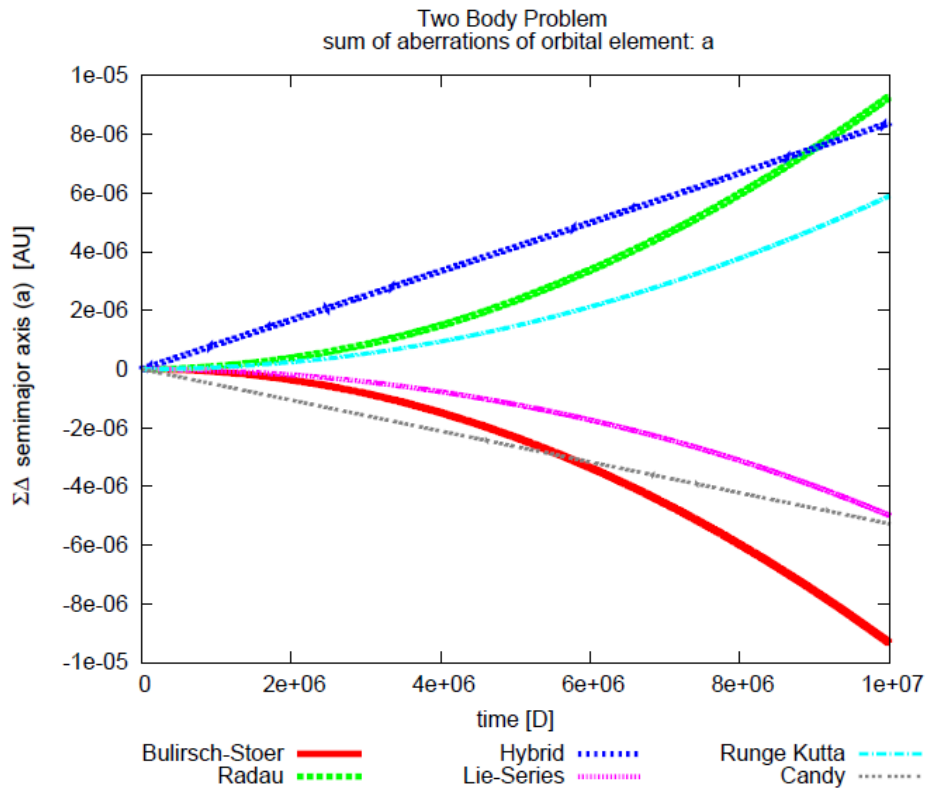
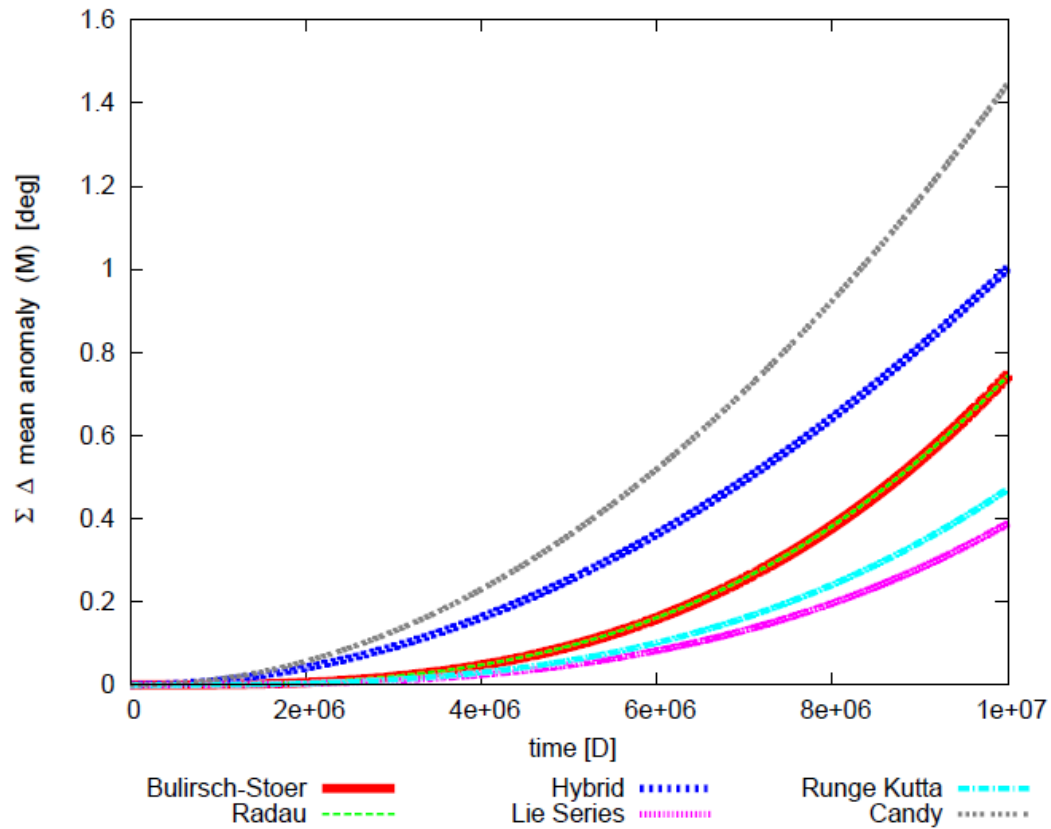
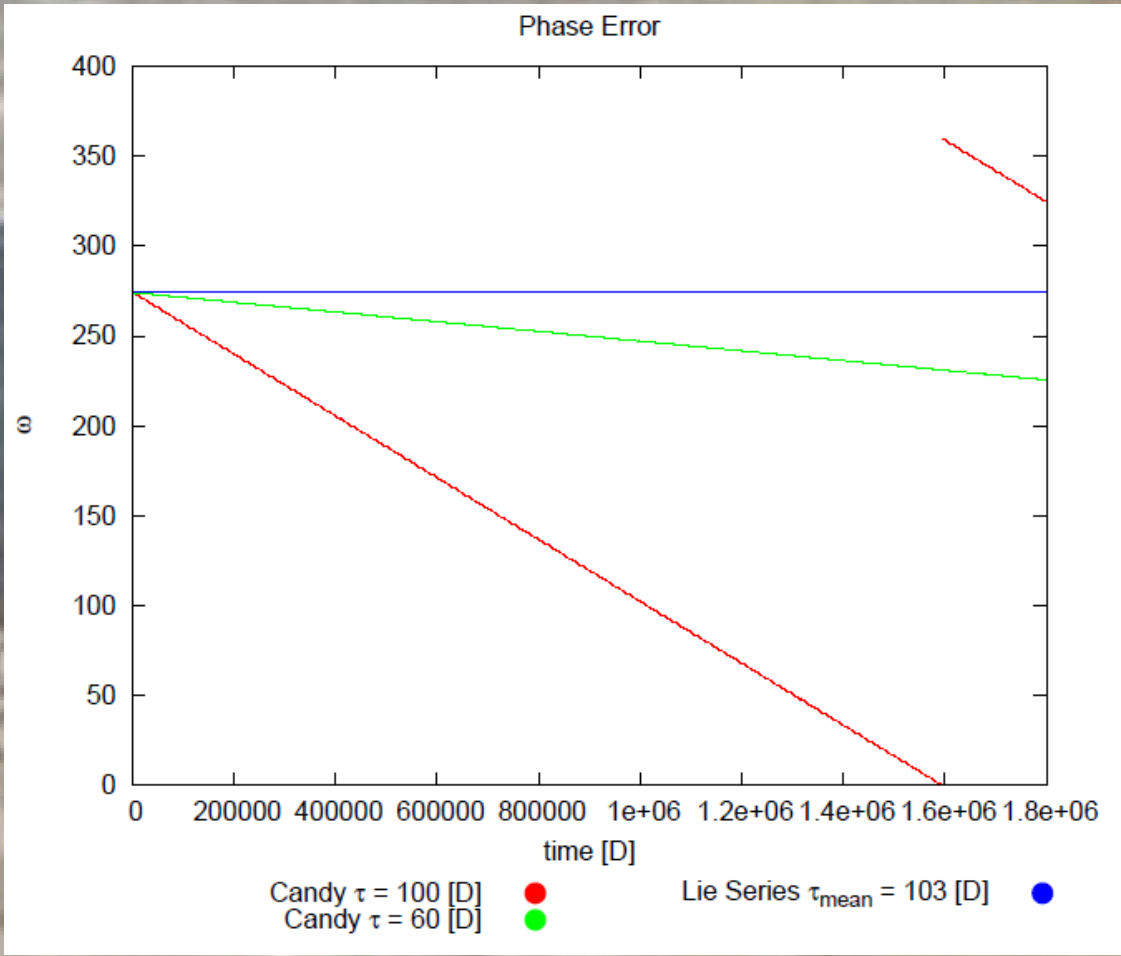


Fig. 6: Sum of deviations of orbital elements  $a$  and  $e$  from their initial values for all integration algorithms.

Two Body Problem  
sum of aberrations of orbital element: M

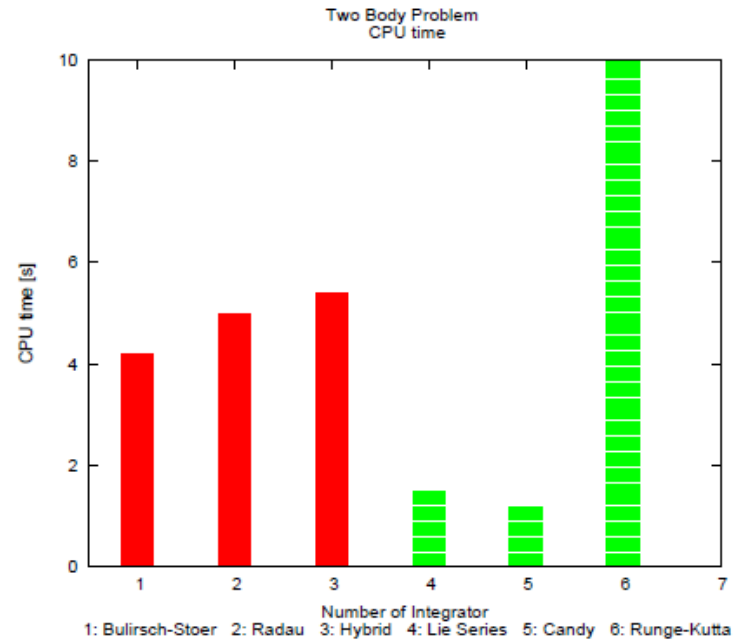




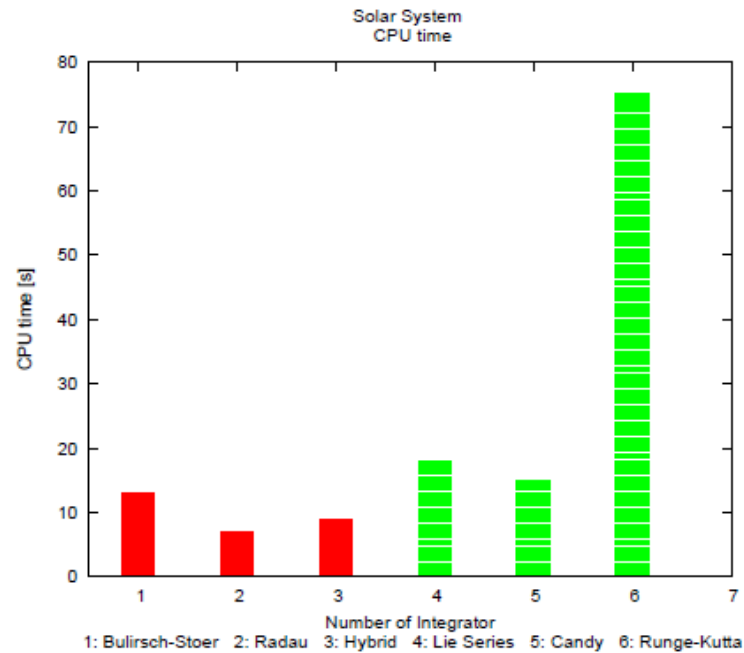


Integrator	symplectic	variable step-size	pros	cons
Cash-Karp Runge-Kutta	no	yes	easy implementation adaptability	performance
Gauss Radau	no	yes	accuracy above order, stability	adaptability
Bulirsch-Stoer	no	yes	fast	weak performance when many substeps are required
Lie-Series	no	yes	high performance	specifically designed for a given problem
Candy	yes	no	energy and angular momentum conservation	fixed step-size, symplectic phase-error
Hybrid	yes	yes	energy and angular momentum conservation	specifically designed for a given problem, symplectic phase-error

**Table 3:** A rough summary of the main properties of the 6 common N-body integration algorithms described in this paper.



**Fig. 11:** CPU-time needed to calculate the Sun-Jupiter system up to  $10^7$  [D].  
 red histogram: CPU-time consumption of algorithms contained in the *mercury6* package  
 green histogram: CPU-time consumption of algorithms contained in the *nie* package



**Fig. 12:** CPU-time needed to calculate the Solar System up to  $10^6$  [D].  
 red histogram: CPU-time consumption of algorithms contained in the *mercury6* package  
 green histogram: CPU-time consumption of algorithms contained in the *nie* package



La famille des Hungarias et  
les impacts

Le role de la Lune

Doctoral School

*“Planetology: From Asteroids to Impact Craters (NEO asteroids and Impact Crater Studies)”*

at the University of Vienna

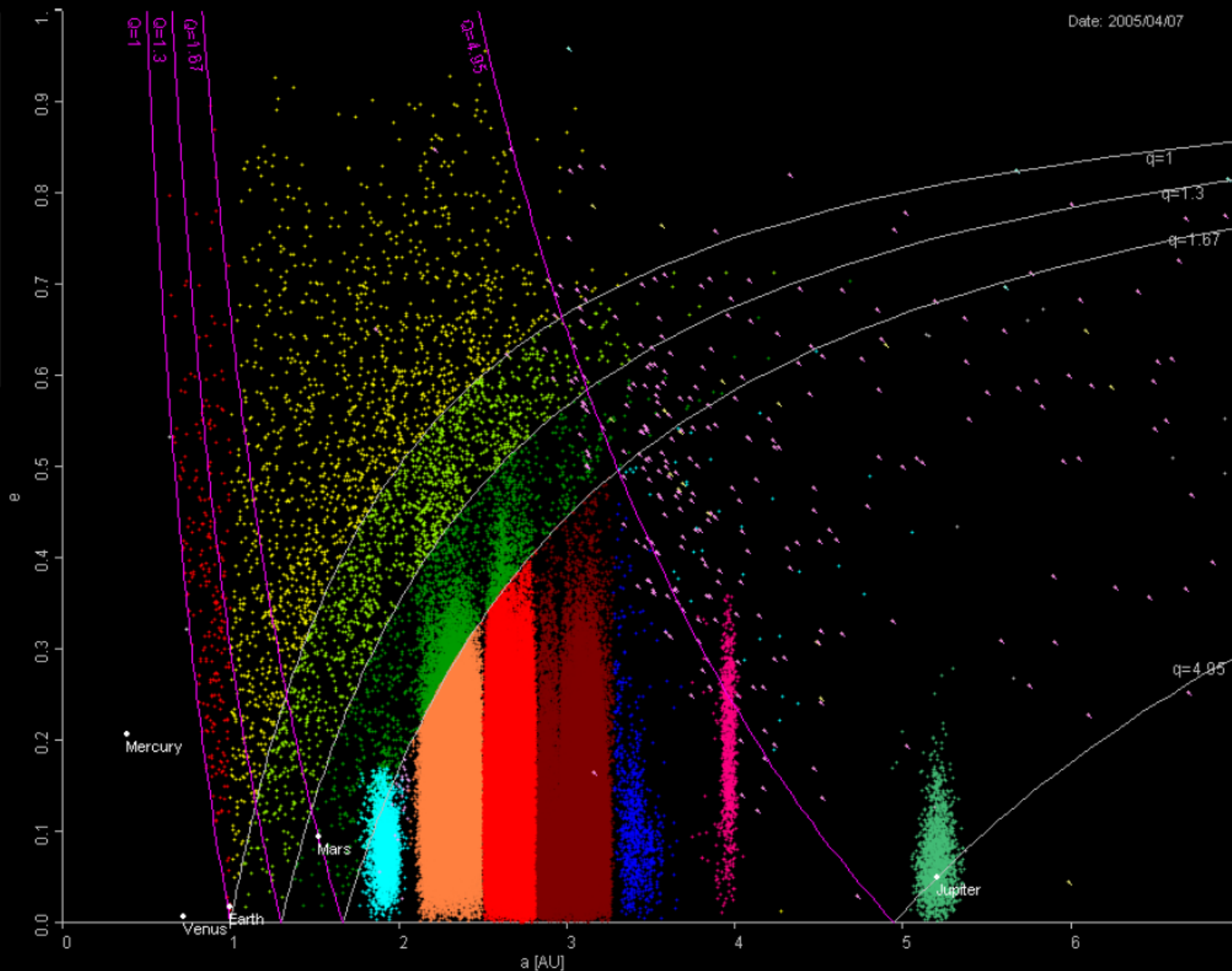
■ **Goals**

- collaboration with Department of Earth Sciences
- NEAs and their threat to Earth
- connection of NEAs and parameters for impact crater studies
- possible effects of the Moon at present & in the past

**M. Galliazzo et A. Bazso**

# MBA's → Dangerous Families → Hungarias

- IEA
- Aten
- Apollo
- Amor
- Mars-Crosser
- Hungaria
- Pre-Main-belt
- MB Zone I
- MB Zone II
- MB Zone III
- Cybele
- Hilda
- Thule
- Int. Jup.-crosser
- Trojan
- Ext. Jup.-crosser
- Comets
- Jup. fam. comets
- Hal. fam. comets

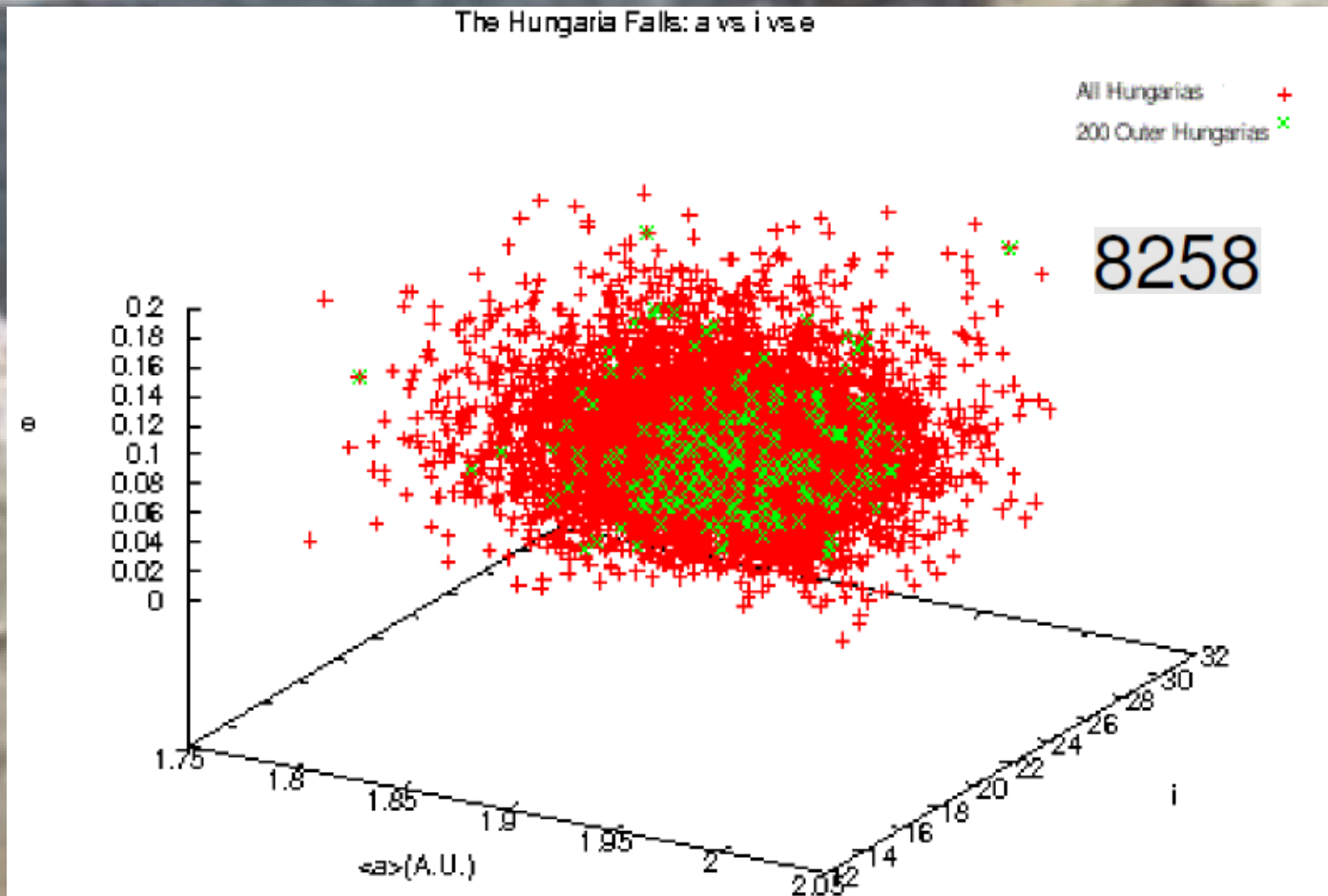


➤  $a=1.78-2.03\text{A.U.}$

➤  $i=16^\circ-31^\circ$

➤  $e=0-0.19$

# Hungaria





Integration – Lie prog.

$10^8$ y -Outp.: $5 \cdot 10^3$ y

4 Groups of 50 massless bodies with the highest metric+Sun,Mars,Jupiter and Saturn

11 large deviations

➤ Clones-deviations:

- $a_{H,i} \pm 0.005$
- $e_{H,i} \pm 0.003$
- $i_{H,i} \pm 0.005$

➤ Clones-Integration - SuperLie (Zooming steps for close-encounters:  $<0.0025$ A.U.)

➤  $10^8$ y -Outp.: $10^3$ y

➤ 5 Groups of 10 massless bodies +Sun,Venus,Earth,Mars,Jupiter Saturn.

➤ First group: "Real asteroid" + 9clones.  
Other groups: 10 clones each one.

➤ 49 clones for all 11 asteroids (largest deviation in  $\langle a \rangle$ ) in the First principal integration.

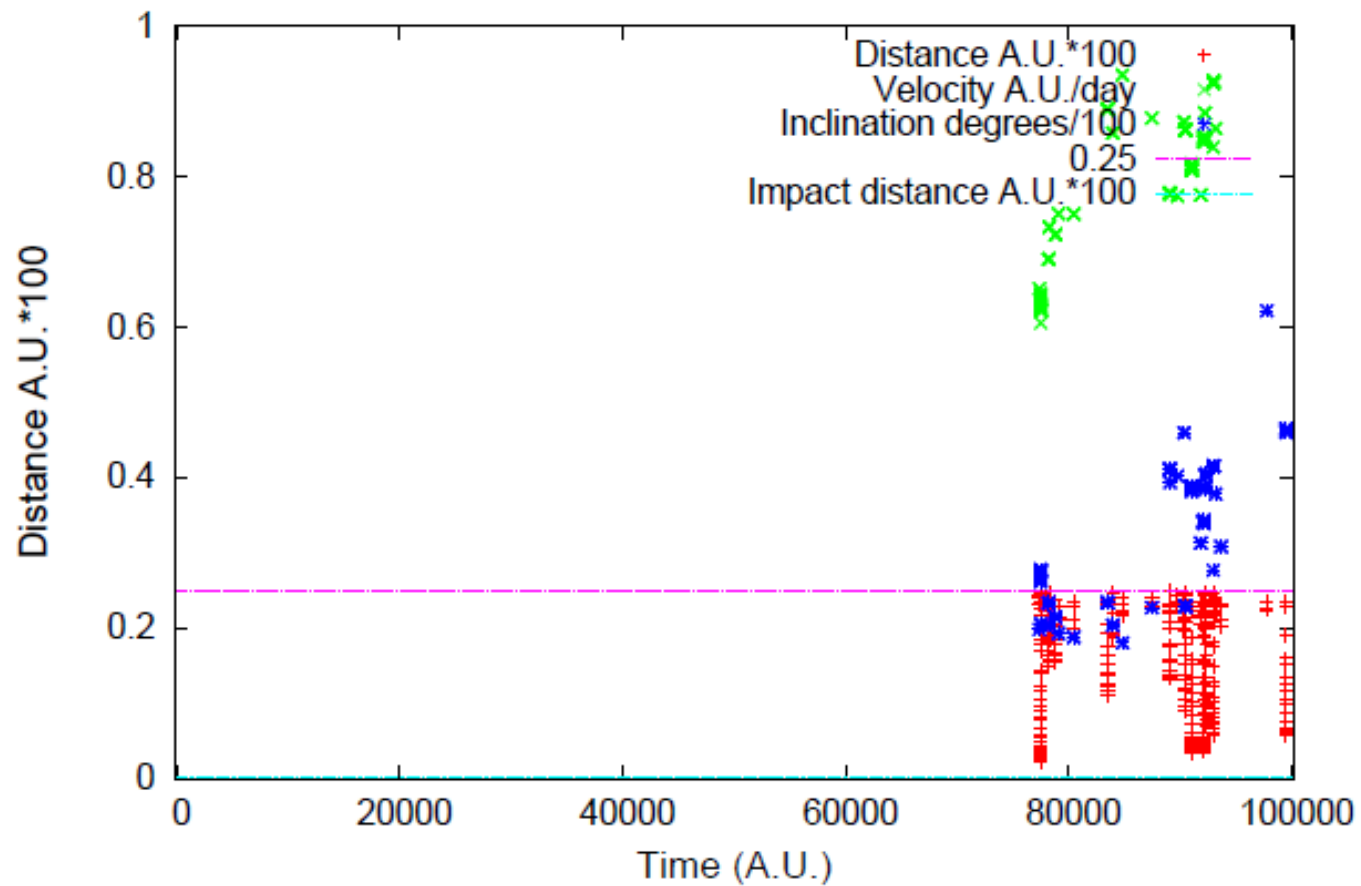
Planet	Asteroid	<Enc>	Max Enc	Min Enc	<Dur>	Max Dur	Min Dur
Earth	2002 RN137	0	-	-	-	-	-
	<i>Clones<sub>av</sub></i>	70	207	2	0.3812266	0.8193286	0.152660
	2000 WN 124 0	-	-	-	-	-	-
	<i>Clones<sub>av</sub></i>	0	-	-	-	-	-
	1992 QA	8	-	-	0.73986012	-	-
	<i>Clones<sub>av</sub></i>	60	205	3	0.3550350	0.7398601	0.110918
Mars	2002 RN137	147	-	-	0.59237680	-	-
	<i>Clones<sub>av</sub></i>	184	427	36	0.5257913	0.6990184	0.354989
	2000 WN 124 0	-	-	-	-	-	-
	<i>Clones<sub>av</sub></i>	69	181	2	0.5331294	0.9510737	0.251630
	1992 QA	407	-	-	0.63218410	-	-
	<i>Clones<sub>av</sub></i>	104	407	2	0.4562290	0.7512960	0.088847

$\langle Enc \rangle$  = Mean number of close encounters per the clones who have a close encounter per relative planet,  $MaxEnc$  = Max number of close encounter per the clones who have a close encounter per relative planet,  $MinEnc$  = Min number of close encounter per the clones who have a close encounter per relative planet,  $\langle Dur \rangle$  = Interval of time of the asteroids inside the distance of 0.0025 A.U. per relative planet,  $MaxDur$  = Maximum of  $Dur$  and  $MinDur$  = minimum of  $Dur$ .

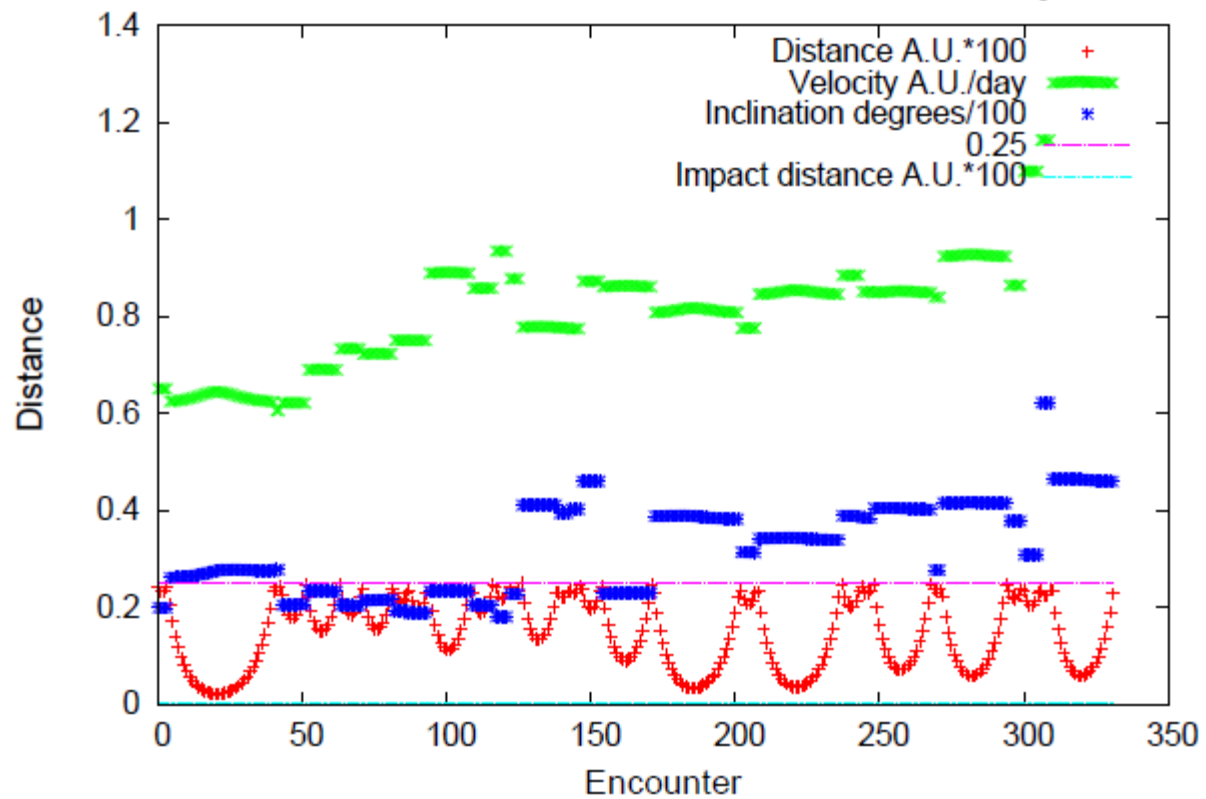
Planet	Asteroid	$\langle \Theta \rangle$	$\langle \Theta_{max} \rangle$	$\Theta_{max}$	$p_{\Theta_{max}}(10^{-4})$
Earth	2002 RN137	-	-	-	-
	<i>Clones<sub>av</sub></i>	0.23	3.33	16.63	1.1245
	2000 WN 124	-	-	-	-
	<i>Clones<sub>av</sub></i>	-	-	-	-
	1992 QA	3.37	11.48	-	5.897
	<i>Clones<sub>av</sub></i>	0.94	9.07	44.06	1.1580
Mars	2002 RN137	0.27	-	2.03	13.6000
	<i>Clones<sub>av</sub></i>	6.12	48.13	176.86	18.785
	2000 WN 124	-	-	-	-
	<i>Clones<sub>av</sub></i>	0.22	1.15	1.96	2.402
	1992 QA	0.39	-	10.47	1.490
	<i>Clones<sub>av</sub></i>	2.21	17.58	171.34	6.574

$\langle \Theta \rangle$  = mean angle of deflection,  $\langle \Theta_{max} \rangle$  = Mean of the maximum of the angle of deflection,  $\Theta_{max}$  = maximum angle of deflection,  $p_{\Theta_{max}}(10^{-4})$  = perigeum of the close encounter at the maximum angle of deflection in  $10^{-4}$  A.U. .

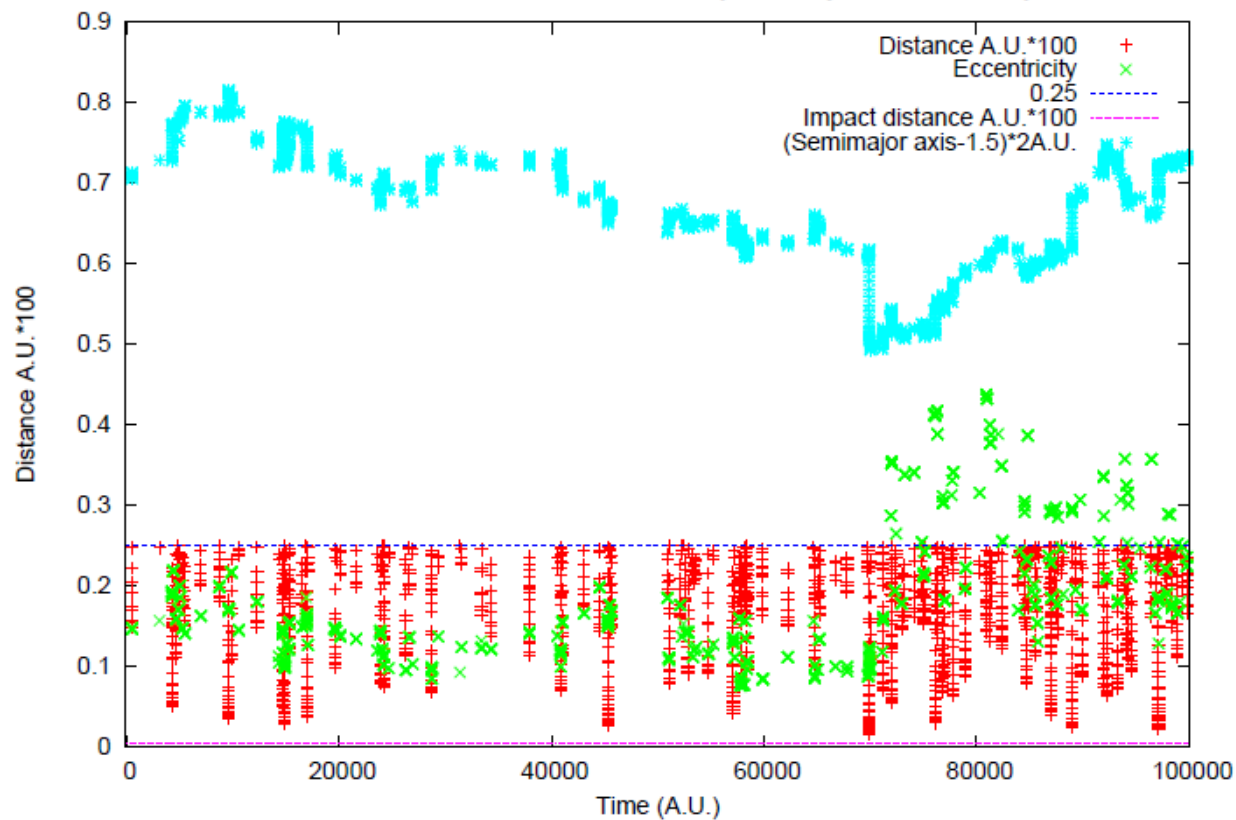
CG1a2-7-DistanceEncounters-Inclination-Velocity vs time



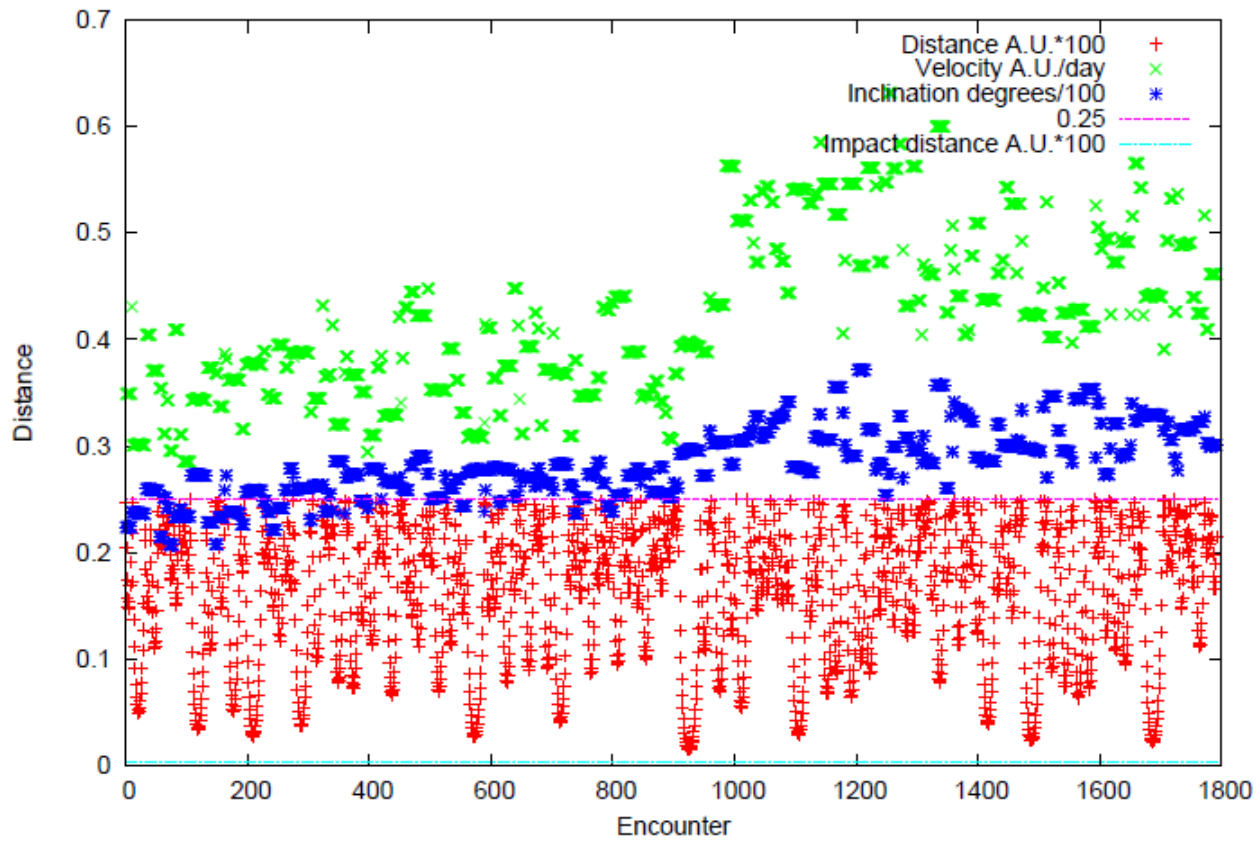
CG1a2-7-Distance-Encounters-Inclination-Velocity

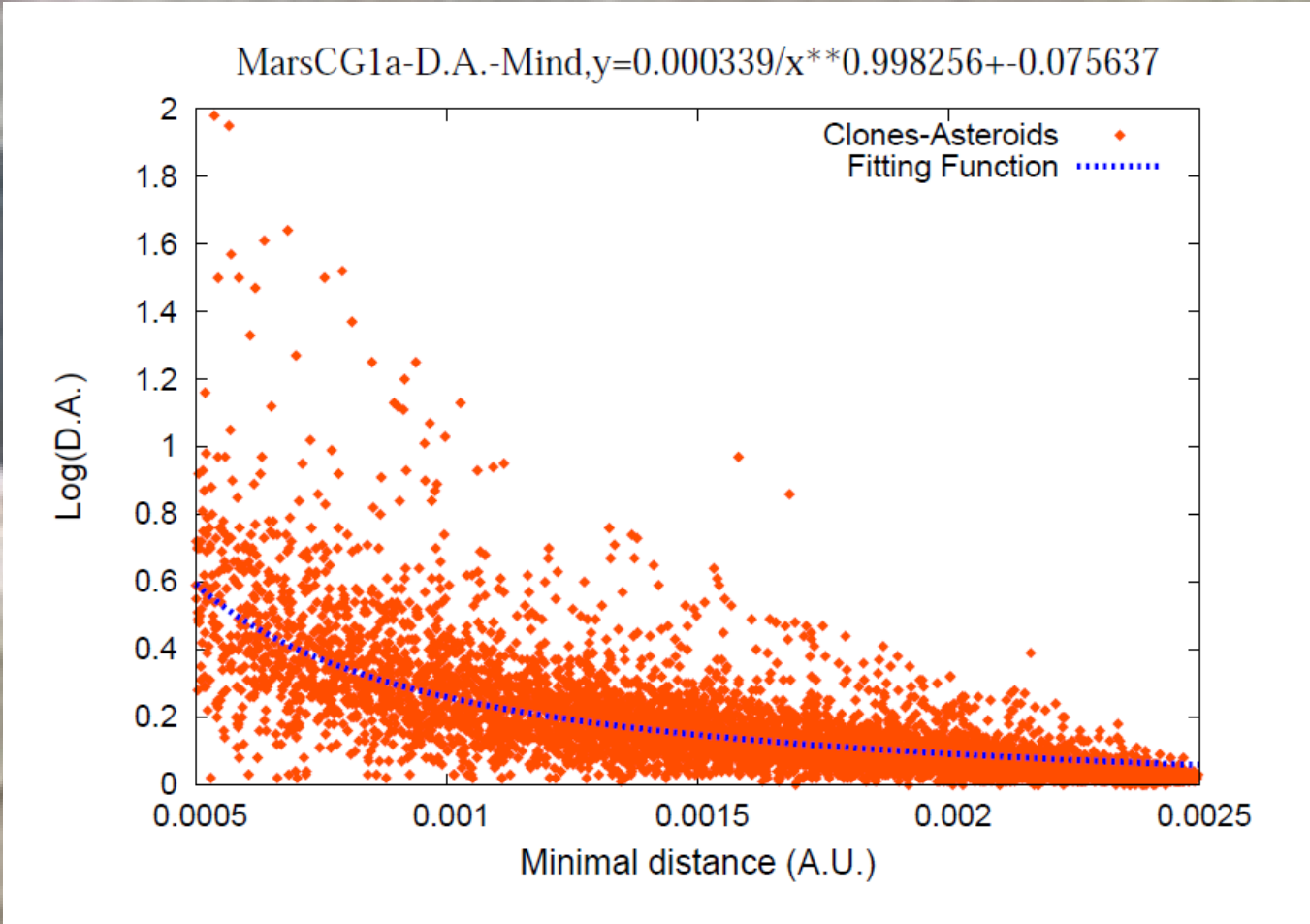


CG1a3-9-DistanceEncounters-Inclination-Velocity-Semimajor axis-eccentricity vs time



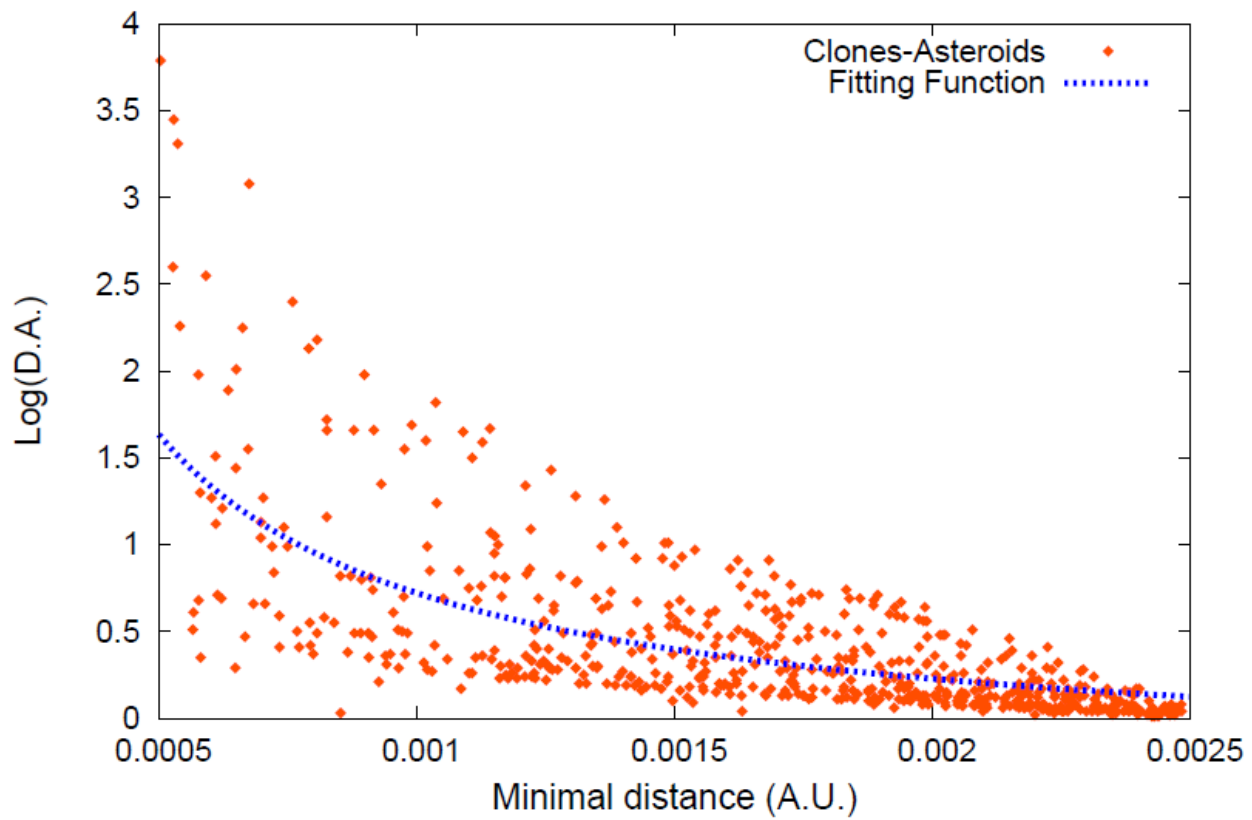
CG1a3-9-Distance-Encounters-Inclination-Velocity







EarthCG1a-D.A.-Mind, $y=0.002307/x^{0.889101}-0.351250$



## About NEAs & the Moon

### ■ Known facts

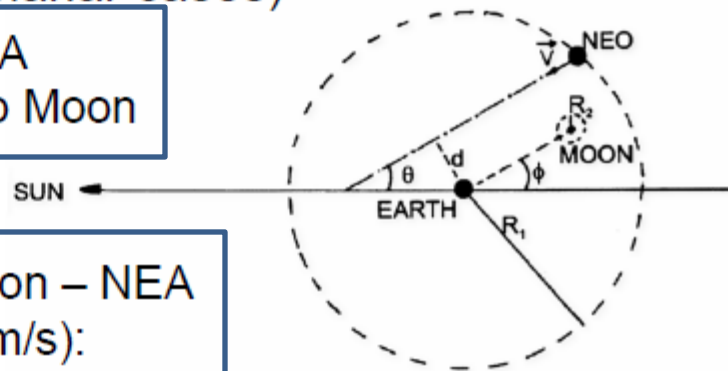
- ❑ NEAs as the major source for impact craters in past 3 Ga
- ❑ Continuous replenishment from MBA
- ❑ Time scale to become Earth crosser  $\leq 10^6$  yr
- ❑ Dynamical lifetime in NEO region  $\leq 10^7$  yr

(from: Morbidelli, Bottke, Froeschlé, Michel, 2002)

- Domingos, Winter & Neto (2004)
  - assessment of Moon's contribution on impacts on Earth
  - comparison of two models (planar cases)

- model 1 = Sun – Earth – NEA  
sample of collision orbits w/o Moon

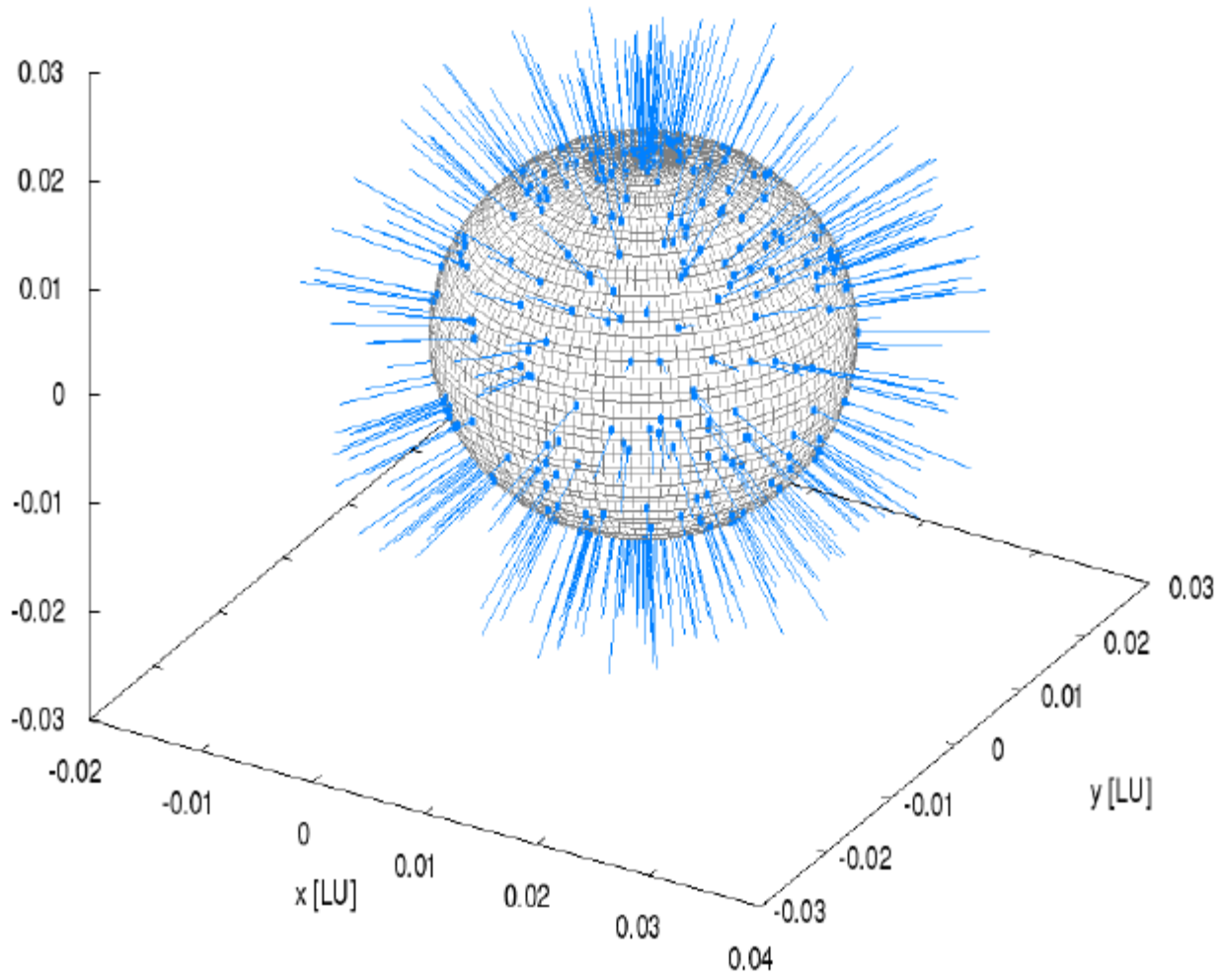
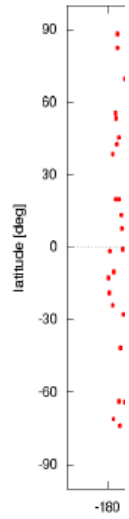
- model 2 = Sun – Earth – Moon – NEA  
low relative velocities ( $\leq 5$  km/s):  
Moon on average deflects 2.6%  
of NEAs on collision trajectories

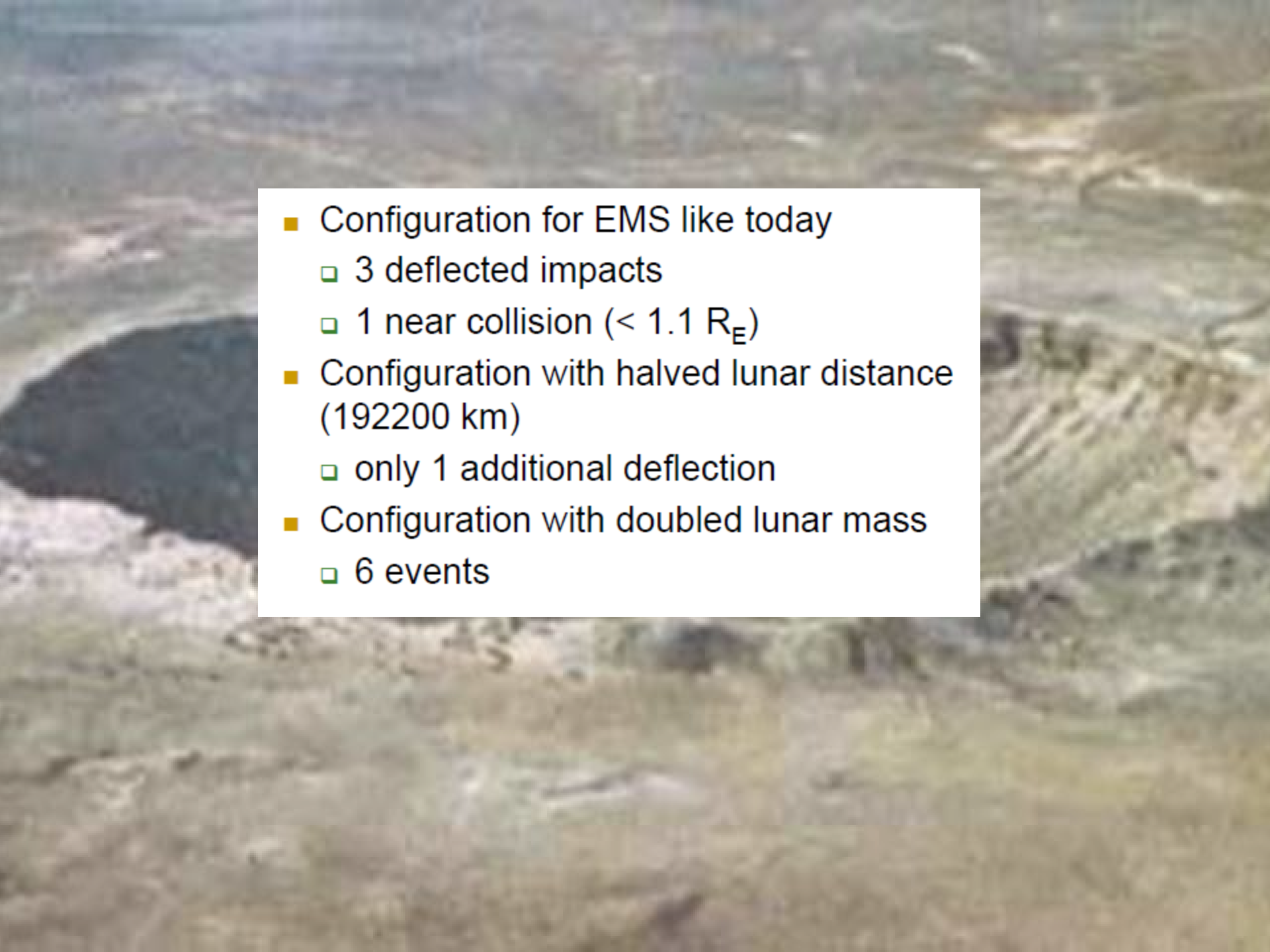


- 3D CR3BP Earth – Moon – NEA
- rotating barycentric coordinate system (Szebehely, 1967)
- normalized units
  - $G = 1, M = 1$
  - length unit                      LU = 384400 km
  - time unit                              TU = 4.37 d
  - mass parameter                       $\mu = 1/82.30057$

- Starting on Earth surface
- Uniform random distribution of “impact sites”
  - velocity in range  $1 < v/v_{\text{esc}} < 2$
  - normal to surface
- Phase 1: no Moon – follow orbit until outside Sphere of Influence (SOI) of  $150 R_E$
- Phase 2: add Moon & reverse time direction – impact still taking place?
- 30000 initial conditions

Distribution of random initial positions on Earth surface



- 
- Configuration for EMS like today
    - 3 deflected impacts
    - 1 near collision ( $< 1.1 R_E$ )
  - Configuration with halved lunar distance (192200 km)
    - only 1 additional deflection
  - Configuration with doubled lunar mass
    - 6 events

An aerial photograph showing a large, dark, irregularly shaped area on a light-colored, textured ground surface. The dark area has a mottled appearance, suggesting it might be a spill of a dark liquid or a large stain. The surrounding ground is light brown and tan, with some darker patches and textures. The word "EPILOG" is overlaid in white, bold, sans-serif capital letters across the center of the dark area.

**EPILOG**



## Symmetric Lie Series Integration?

...take a step forward...

$$\vec{z}_{n+1} = e^{\tau D} \vec{z}_n$$

...take a step back...

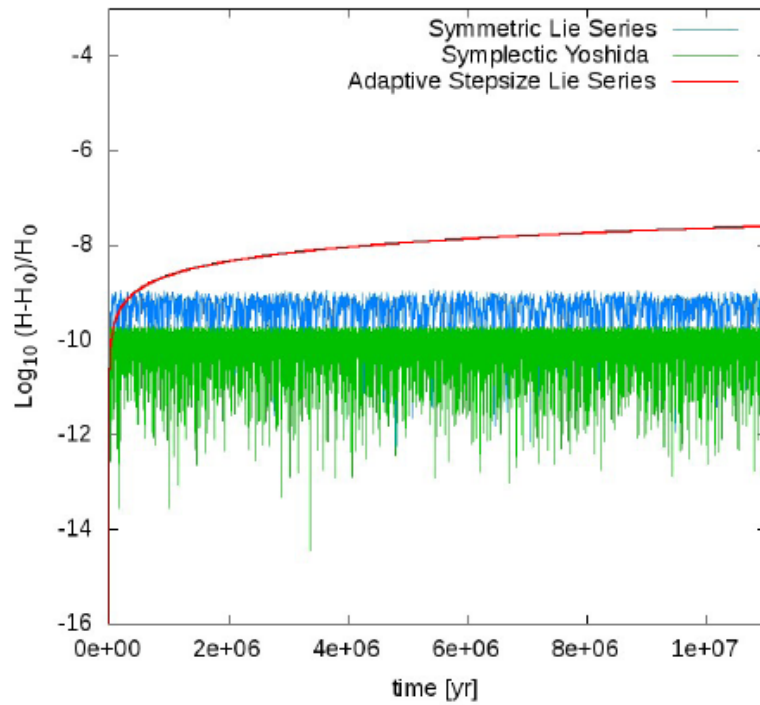
$$\vec{z}_{n-1} = e^{-\tau D} \vec{z}_n$$

...a symmetric two step method

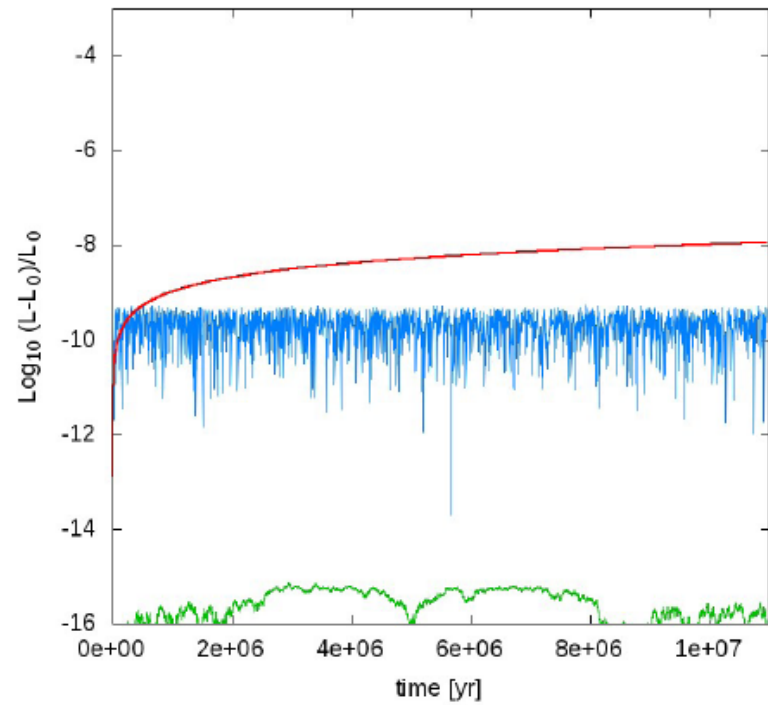
$$\vec{z}_{n+1} = 2 \sum_{m=0}^O \frac{(\tau D)^{2m}}{(2m)!} \left[ \vec{z}_n + \vec{z}_{n-1} \right]$$

where  $O \in \mathbb{N}_{+, \text{even}}$  is the order of truncation of the symmetric Lie Series algorithm.

Sun-Jupiter-Saturn System  
Step-size  $h = 40$  [D], Order 8



Sun-Jupiter-Saturn System  
Step-size  $h = 40$  [D], Order 8



Order	Yoshida	Symmetric Lie Series
6	8	6
8	16	8
10	32	10

relative CPU-Time for Sun-Jup-Sat,  $t = 10^7$  [yrs], Order 8:

Symmetric Lie Series	Yoshida	Adaptive Stepsize Lie Series
1	2	2.8

Dans notre groupe ADG a Vienne on travaille sur les trois aspects

-> Construire un integrateur qui est precis et rapid

-> La contribution des familles des Asteroides (Hungarias, Flora et Themis) pour le NEAs et determiner les collisions avec la terre

-> Examiner le role de la Lune

MERCI!  
THANK YOU!



FURUK.