GRT AND CONTEMPORARY FUNDAMENTAL ASTRONOMY LANGUAGE

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1. Introduction

In developing the results obtained earlier (Brumberg et al., 1996 ; Brumberg, 1995, 1997) this paper deals with the contemporary fundamental astronomy language considered from the point of view of general relativity theory (GRT). This language is meant therewith as a glossary of astronomical terminology used, for example, in IERS Standards (1992), IERS Conventions (1996), JPL DE/LE documents, Astronomical Almanac Supplement (1992), etc. To be more specific it is sufficient to consider IERS Conventions (1996) and the new IAU (1991) resolutions on reference systems (RSs) and time scales (TSs). At first sight it may seem that these resolutions are completely carried out and contemporary fundamental astronomy is becoming essentially relativistic. But such a conclusion would be too premature. The language of modern fundamental astronomy still remains Newtonian even in discussing GRT effects. The present status of fundamental astronomy, several years later after the adoption of the IAU (1991) resolutions, forces one to follow three rules, as follows :

1) one may often forget about the quantitative characteristics of the GRT effects due to their smallness but the qualitative differences between the GRT spacetime and Newtonian absolute space and absolute time should be never forgotten ;

2) one should not trust the currently employed terminology due to its GRT inconsistency ;

3) to understand the meaning of any fundamental astronomy quantity one should analyse an algorithm adopted for this quantity.

As an illustration of the present status it will be shown that the replacement of IERS Standards (1992) by IERS Conventions (1996) being aimed to achieve the better compatibility of the international RSs, ICRS and ITRS, with the IAU (1991) resolutions resulted in a step backward in the practical realization of these resolutions.

2. International Celestial (ICRS) and Terrestrial (ITRS) Reference Systems

First of all, it is to be noted that the IAU (1991) resolutions might be formulated more rigorously in the GRT framework. There is no exact separation of the roles of TCG and TT, no exact formulation of the difference between dynamically and kinematically non-rotating RSs, whereas only initial terms of order c^{-2} are reproduced in the metric forms describing the solar system barycentric (BRS) and geocentric (GRS) reference systems. But even within the adopted accuracy the practical use of IAU (1991) resolutions may involve some additional relativistic ambiguities.

Two basic RSs maintained by IERS are ICRS and ITRS. In accordance with IERS Standards (1992) and IERS Conventions (1996) language, ICRS represents mathematically a BRS based on VLBI observations (BRSV in terms of Brumberg et al., 1996, with V reminding VLBI). Its approximate metric as given by IAU (1991) resolutions is

$$ds^2 = \left[1 - 2c^{-2}U(t,\mathbf{x})\right]c^2dt^2 - \left[1 + 2c^{-2}U(t,\mathbf{x})\right]\left(dx^{1^2} + dx^{2^2} + dx^{3^2}\right), \tag{1}$$

$U(t,\mathbf{x})$ being Newtonian potential of all solar system bodies. Instead of t=TCB and spatial coordinates $\mathbf{x}$ JPL DE/LE, IERS Standards (1992) and IERS Conventions (1996) use (rather implicitly)

$$\mathrm{TDB} = (1 - L_B)\,\mathrm{TCB}, \qquad (\mathbf{x})_{\mathrm{TDB}} = (1 - L_B)\,\mathbf{x}, \qquad (GM)_{\mathrm{TDB}} = (1 - L_B)\,GM\,, \tag{2}$$

with $L_B = 1.5505197 \times 10^{-8}$ given in IAU (1991) resolutions for the relationship between TCB and TDB of JPL DE/LE ephemerides. Actually each version of JPL DE/LE provides its own L_B. Scaling of spatial coordinates and mass factors is produced to keep invariance of the light velocity and the equations of motion of the solar system bodies under transformation from TCB to TDB. This transformation involves also the relationship

$$(ds^2)_{\mathrm{TDB}} = (1 - L_B)^2 ds^2\,. \tag{3}$$

where $(ds^2)_{\mathrm{TDB}}$ keeps the same form in terms of TDB, $(\mathbf{x})_{\mathrm{TDB}}$ and $(GM)_{\mathrm{TDB}}$ as expression (1) in terms of t, $\mathbf{x}$ and GM. Hence, BRS of IAU (1991) resolutions and ICRS differ by introducing L_B factor. From the theoretical point of view, ITRS maintained by IERS is supposed to be a realization of GRS$^+$, a geocentric system rotating with the Earth and resulting from application of Newtonian rotation of the spatial axes of GRS envisaged by IAU/IUGG (1991) resolutions. The latter system (GRSV in terms of Brumberg et al., 1996) is described by the metric of the form

$$\begin{aligned} ds^2 &= \left\{1 - 2c^{-2}\left[\hat{U}_E + Q_j w^j + T(u,\mathbf{w})\right]\right\}c^2du^2 \\ &+ 2c^{-3}\left[(1-q)\varepsilon_{ijk}\dot{F}^j w^k + \ldots\right]cdudw^i \\ &- \left\{1 + 2c^{-2}\left[\hat{U}_E + Q_j w^j + T(u,\mathbf{w})\right]\right\}\left(dw^{1^2} + dw^{2^2} + dw^{3^2}\right). \end{aligned} \tag{4}$$

Here $\hat{U}_E$ is the GRS geopotential, $T(u,\mathbf{w})$ is the tidal potential vanishing at the GRS origin (only these two quantities are indicated in IAU (1991) resolutions), Q_j is the non-geodesic acceleration in the BRS motion of the Earth (caused by the distinction of the actual Earth from the point-mass model), F^j is the geodesic rotation vector of the Earth, $\varepsilon_{ijk} = (i-j)(j-k)(k-i)/2$ is the Levi-Civita symbol and Einstein summation rule from 1 to 3 is applied to each repeated latin index. Dots stand for the additive c^{-3}-order terms in the mixed components of the metric. The terms indicated explicitly enable one to make an option between dynamically (D version, $q=1$) or kinematically (K version, $q=0$) GRSs.

Instead of coordinate time u=TCG, spatial coordinates $\mathbf{w}$ and mass factors $G\hat{M}$ related to GRS, IERS Standards (1992) use actually

$$\mathrm{TT} = (1 - L_G)\,\mathrm{TCG}, \quad (\mathbf{w})_{\mathrm{TT}} = (1 - L_G)\,\mathbf{w}, \quad (G\hat{M})_{\mathrm{TT}} = (1 - L_G)\,(G\hat{M}) \tag{5}$$

with $L_G = 6.9629 \times 10^{-10}$ as given by IAU (1991) resolutions. Of course, L_G is different for various geoid models. Again, the scaling of the spatial coordinates and mass factors is aimed to keep invariance of the light velocity and the GRS equations of motion (of the Moon or Earth's artificial satellites) under the transformation from TCG to TT. This transformation accompanied by the scaling of spatial coordinates and mass factors involves the invariance of the metric (up to the constant factor)

$$(ds^2)_{TT} = (1 - L_G)^2 ds^2 \tag{6}$$

where $(ds^2)_{TT}$ has the same form in terms of TT, $(\mathbf{w})_{\mathrm{TT}}$ and $(G\hat{M})_{\mathrm{TT}}$ as expression (4) in terms of u, $\mathbf{w}$ and $G\hat{M}$. Both transformations (2) and (5) are often considered as the change of unit of time from SI unit to TDB or TT units, respectively.

Hence, GRS^+ system corresponding to IAU/IUGG (1991) resolutions differs from ITRS as given by IERS Standards (1992) by introducing L_G factor. IERS Conventions (1996) did not stop on that. ITRS realized by a new VLBI formula of Conventions (1996) is, in fact, in contradiction to IAU/IUGG (1991) resolutions although by its language it claims to be in better compatibility with these resolutions. First of all, one should decide what kind of distances are demanded for ITRS and then to make choice between two basic options :

(1) calculated coordinate distances associated to TCG, or

(2) locally measurable distances in the infinitesimal vicinity of an observer on the surface of the Earth.

The first option corresponds to the GRS metric adopted by IAU/IUGG (1991) resolutions. The second option corresponds to the new VLBI formula of IERS Conventions (1996) although this is not stated explicitly and may be demonstrated only by analysing the new VLBI formula. In principle, one may realize the second option in two different ways. The first one is based on a local transformation of any GRT metric to the flat (Minkowski) metric with proper time and proper distances. Indeed, any metric

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \,, \qquad g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \tag{7}$$

with Einstein summation from 0 to 3 for any repeated greek index and Minkowski tensor $\eta_{\mu\nu}$ ($\eta_{00} = 1$, $\eta_{0i} = 0$, $\eta_{ij} = -\delta_{ij}$, δ_{ij} being the Kronecker symbol) may be locally transformed to the Minkowski metric (see, e.g., Brumberg, 1991)

$$ds^2 = c^2 d\tau^2 - dl^2, \qquad dl^2 = \delta_{ik} dx^{(i)} dx^{(k)} \tag{8}$$

with elementary proper time

$$c d\tau = \left(1 + \frac{1}{2} h_{00} - \frac{1}{8} h_{00}^2\right) c dt + \left(h_{0i} - \frac{1}{2} h_{00} h_{0i}\right) dx^i \tag{9}$$

and elementary proper distances

$$dx^{(i)} = dx^i + \frac{1}{2}(-h_{ik} + h_{0i} h_{0k}) dx^k . \tag{10}$$

These formulas are accurate up to the third order with respect to c^{-1} provided that h_{00} and h_{ik} are of the second order and h_{0i} are of the first order.

Now let us apply these formulas to $GRS^+(u, \mathbf{y})$. This metric resulted from (4) by three-dimensional rotation of the spatial axes is as follows :

$$\begin{aligned} ds^2 = &\left\{1 - 2c^{-2}\left[\hat{U}_E^+ + \frac{1}{2}(\boldsymbol{\omega}_E \times \mathbf{y})^2 + Q_k^+ y^k + \ldots\right]\right\} c^2 du^2 - 2c^{-1}(\boldsymbol{\omega}_E \times \mathbf{y})_i c du dy^i \\ &- \left\{1 + 2c^{-2}\left[\hat{U}_E^+ + Q_k^+ y^k + \ldots\right]\right\} ({dy^1}^2 + {dy^2}^2 + {dy^3}^2). \end{aligned} \tag{11}$$

Here $\boldsymbol{\omega}_E$ is the Earth' rotation angular velocity and $\hat{U}_E^+$ and Q_k^+ have the same meaning as $\hat{U}_E$ and Q_k, respectively, but expressed in the rotating coordinates. Hence, one has for the infinitesimal intervals of proper time and proper distances

$$c d\tau = (1 - c^{-2} W_E) c du - c^{-1} (\boldsymbol{\omega}_E \times \mathbf{y})_i dy^i \tag{12}$$

and

$$dy^{(i)} = (1 + c^{-2} \hat{U}_E^+) dy^i + \frac{1}{2} c^{-2} (\boldsymbol{\omega}_E \times \mathbf{y})_i (\boldsymbol{\omega}_E \times \mathbf{y})_k dy^k \tag{13}$$

with the potential of the force of gravity

$$W_E = \hat{U}_E^+ + \frac{1}{2}(\boldsymbol{\omega}_E \times \mathbf{y})^2 . \tag{14}$$

Only main terms are indicated explicitly in (11)–(14). Remembering that $L_G = c^{-2} W_0$, W_0 being the value of W_E on the geoid, and neglecting all the terms with the Earth's rotation velocity one may see in

(12) and (13) the factors $1 - L_G$ and $1 + L_G$ used implicitly in the new VLBI model of IERS Conventions (1996). Even if it is good for a local network it cannot be applied successfully for the whole Earth taking into account the variability of $\hat{U}_E$ on the surface of the Earth.

The same VLBI model of IERS Conventions (1996) may be constructed in describing the VLBI procedures in a topocentric reference system (TRS). Indeed, at the spatial origin of any TRS its metric takes the Minkowski form enabling one again to operate with the infinitesimal intervals of proper time and proper distances. In such a way this model was obtained as formula (6.4.37) in (Brumberg, 1991) to illustrate the possibility to construct a local TRS network.

Hence, GRS$^+$ envisaged by IAU/IUGG (1991) resolutions and ITRS of VLBI model of IERS Conventions (1996) differ not only by TCG/TT dualism but also by different types of the employed distances (coordinate/proper distances dualism). Again the existing contradiction between the adopted terminology and the algorithm itself may lead to GRT ambiguities.

3. ICRS⟶ITRS Transformation and Fundamental Astronomy Concepts

In comparing ICRS and ITRS of IERS with theoretical BRSV and GRS$^+$ of the relativistic hierarchy of RSs one should always remember that ICRS and ITRS are practically employed as three-dimensional spatial RSs whereas BRSV and GRS$^+$ represent four-dimensional relativistic RSs. As stated above, the four dimensional relativistic transformation between BRSV and GRS$^+$ involves an intermediate RS, i.e. GRSV in D or K versions. The main planes of GRS$^+$ and GRSV are close to each other and the mutual orientation of these systems, i.e. ERP problem, demands one more intermediate RS. Indeed, the ICRS⟶ITRS transformation is meant in fundamental astronomy as three-dimensional Newtonian rotation transformation

$$[\mathrm{ICRS}] = PNRW[\mathrm{ITRS}] \tag{15}$$

with matrices P, N, R, W responsible, respectively, for polar motion, diurnal rotation, nutation and precession. In particular, the nutation matrix is represented in the form

$$[N] = D_1(-\varepsilon)D_3(\Delta\psi)D_1(\varepsilon + \Delta\varepsilon) \tag{16}$$

with standard designations D_i for the elementary rotations and the nutation angles $\Delta\psi$ and $\Delta\varepsilon$ in longitude and obliquity. The obliquity angle ε appears just as an intermediate quantity to facilitate ERP analysis. In the GRT framework transformation (15) should be formulated more precisely defining, in particular, such concepts as ecliptic in GRS, equator in BRS, obliquity ε for different BRSs and GRSs, Earth's rotation angular velocity in different RSs, etc. These procedures may be rigorously formulated by introducing BRSC, an ecliptical BRS, and GRSQ, an equatorial GRS. The first system is constructed by fitting planetary theories to BRSV. The second system may be determined by performing ERP analysis in GRSC, resulted from BRSC by the four-dimensional relativistic transformation. These procedures are described in detail in (Brumberg et al., 1996 ; Brumberg, 1995, 1997). More refined questions concerning GRT Earth's rotation problem are studied in (Klioner, 1995, 1996, 1997). Practical realization of this relativistic approach has been started in (Bretagnon et al., 1997). In such a way the traditional concepts of fundamental astronomy may be unambiguously re-formulated in the GRT framework.

From practical point of view many relativistic effects related to the fundamental astronomy concepts may be presently ignored but the rigorous GRT language enables one to avoid relativistic ambiguities in not so distant future.

4. References

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