DROMO: a new regularized orbital propagator

Giulio Bau,
Centro Interdipartimentale Studi e Attivita Spaziali, Padova, 35131, Italy
Alexander Hunh,
Technical University of Munich, Arcisstraße 21, 80333 Munich, Germany
Hodei Urrutxua, Claudio Bombardelli and Jesús Peláez
Technical University of Madrid (UPM)
E.T.S.I. Aeronáuticos, Pza. Cardenal Cisneros 3, E-28040 Madrid, Spain

Session: *Orbit propagation methods for artificial satellites and asteroids*

Tuesday, 27 September 2011

Outline of presentation

1. Introduction
2. Special Perturbation Methods. Cowell Method
3. Orbit Propagator. DROMO
4. Comparisons
    - Stiefel & Scheifele problem
    - Sperling-Bürdet
    - Fly-by of a Planet
    - The Tsien Problem
5. Conclusions

Introduction

This work is included in a wider research project —three years— entitled

Dynamic simulation of complex space systems

(AYA2010-18796)

supported by the DGI of the Spanish Ministry of Science and Innovation. Authors thank the Spanish Gov. for its financial support.

Simultaneously this project is closely related with some European activities on Space Situational Awareness (SSA). According with ESA, **the Space Situational Awareness (SSA) can be preliminarily defined as a comprehensive knowledge of the population of space objects, of existing threats/risks, and of the space environment**.

Introduction

During last years our group GSD-UPM —Group of Space Dynamics of the UPM— has been working on different research lines:

- dynamics of tethers (electrodynamic and/or inert tethers)
- orbit propagation (DROMO)
- attitude propagation (axisymmetric bodies)
- new propulsive concepts as the **Ion Beam Shepherd**, described in the following references.

• *[1] Ion beam shepherd for contactless space debris removal*, by

C. Bombardelli and J. Peláez,

Journal of Guidance, Control and Dynamics, 34(3):916–920, May 2011. DOI: 10.2514/1.51832.

• *[2] Ion beam shepherd for asteroid deflection*, by

C. Bombardelli and J. Peláez,

Journal of Guidance, Control and Dynamics, 34(4):1270–1272, July 2011. DOI: 10.2514/1.51640.

As a consequence of this previous work, a new research project has been started: **NEODROMO**. The main objective of **NEODROMO** is to produce a **numerical** tool to study the long term dynamics of asteroids, specially tailored for the NEO's dynamics, including models of increasing complexity that can be used for the determination of orbits and the prediction of trajectories. There are two main reasons why we opted by a numerical option:

- to benefit from our previous numerical tools and
- each 10 years, more or less, the speed of the computers is multiplied by 1000. Any propagator based on numerical techniques and sound algorithms will enjoy these future advantages practically for free.

NEODROMO Project

The **NEODROMO** projects aims to produce a numerical tool to study the long term dynamics of asteroids, specially tailored for the NEO's dynamics. To do that the following requirements should be fulfilled:

- a propagator for the prediction of the dynamics of the center of mass of the asteroid
- a propagator for the prediction of the attitude dynamics of the asteroids (if needed)
- a model for the ephemeris of the involved celestial bodies (planets, asteroids, Moon, ...)
- a model for the gravitational actions acting on the asteroid (forces and torques)
- a model for the thermal behavior of the asteroid
- a model for the surface of the asteroid
- a model for the Yarkovsky force
- a model for the YORP torque (if needed)
- models for the kinematic aspect of the problem (references frames, SOFA routines, ...)

Due to the large populations and families of asteroids, the design of this tool should facilitate the generation of particular propagation tools deliberately tailored for a particular asteroid.

NEODROMO team

In order to develop the NEODROMO project a team has been formed in the UPM:

- **Javier Herrera-Montojo**, undergraduate student (Master thesis)
- **Hodei Urrutxua**, Ph.D. student
- **Isabel Pérez-Grande**, associated professor of aerospace engineering (thermal aspects)
- **Manuel Ruiz Delgado**, associated professor of aerospace engineering
- **José Manuel Hedo Rodriguez**, associated professor of aerospace engineering
- **Claudio Bombardelli**, Ramón y Cajal Scientist
- **Jesús Peláez**, professor of aerospace engineering

Special Perturbation Methods. Cowell method

The propagation of the orbit of a celestial body or a spacecraft involves the integration of the equations of motion:

$$\ddot{\boldsymbol{r}} = -\frac{\mu}{r^3}\boldsymbol{r} + \boldsymbol{a}_p \tag{1.1}$$

where $\boldsymbol{r}$ is the position vector of the satellite and $\boldsymbol{a}_p$ is the total perturbing acceleration. The Special Perturbation Methods perform a numerical propagation of these equations. Perhaps the most classical is the Cowell method.

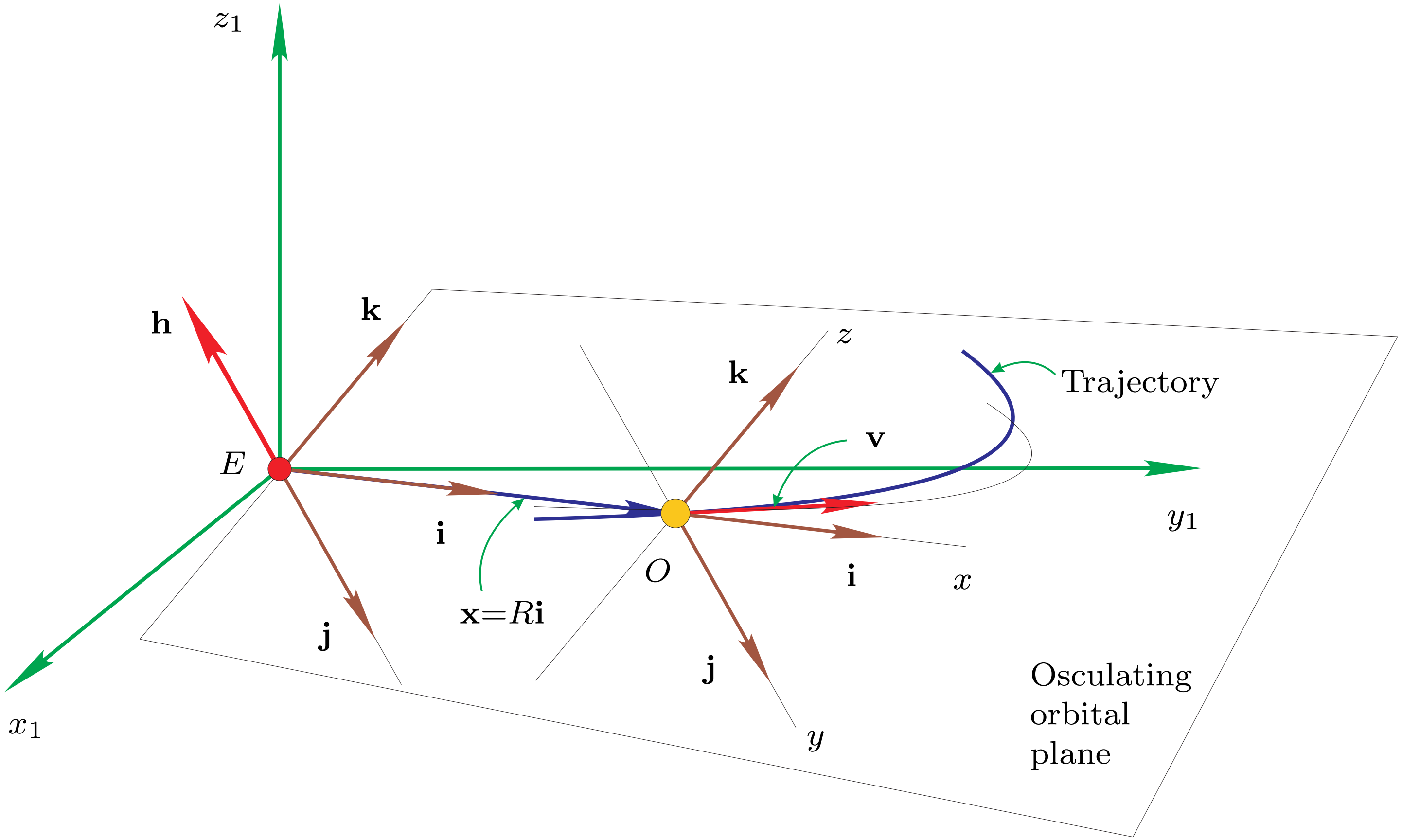
The Cowell method is a Special Perturbation method which provides a numerical solution of this problem by integrating the following set of ODE's:

$$\begin{aligned}
\ddot{x} &= -\mu\frac{x}{r^3} + a_{p_x}(t, x, y, z, \dot{x}, \dot{y}, \dot{z}), \qquad r = \sqrt{x^2 + y^2 + z^2} \\
\ddot{y} &= -\mu\frac{y}{r^3} + a_{p_y}(t, x, y, z, \dot{x}, \dot{y}, \dot{z}) \\
\ddot{z} &= -\mu\frac{z}{r^3} + a_{p_z}(t, x, y, z, \dot{x}, \dot{y}, \dot{z})
\end{aligned}$$

where (x, y, z) are the cartesian coordinates of the position vector $\boldsymbol{r}$ in some frame (usually inertial). The name is due to its discoverer P.H. Cowell in the early 20th century.

The NEODROMO project, however, is based in other Special Perturbation Method: DROMO.

DROMO

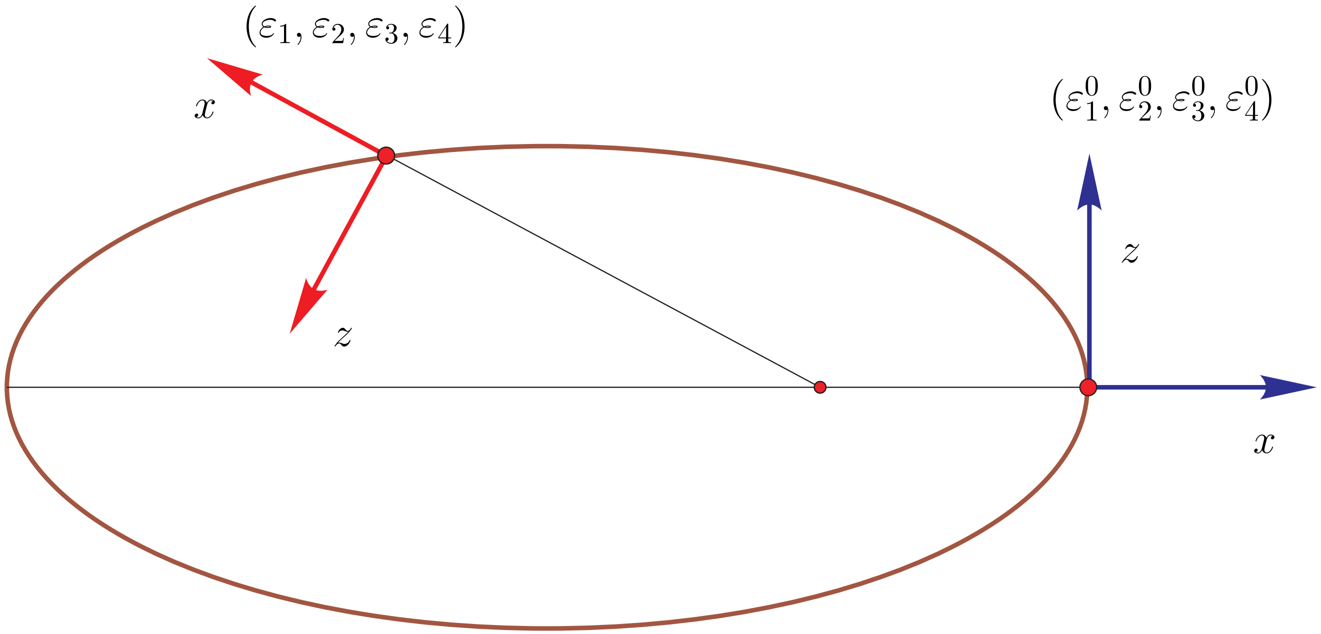


Dynamical state of the system: $\vec{\boldsymbol{x}}$, $\vec{\boldsymbol{v}}$
Order: six
Instead of $\vec{\boldsymbol{x}}$, $\vec{\boldsymbol{v}}$ we use the orbital frame $Oxyz$ and its angular velocity $\vec{\boldsymbol{\omega}}$.

$$\psi = r^2\dot{\sigma}, \qquad z = \frac{1}{r} = \frac{1}{\psi^2} + A\cos\sigma + B\sin\sigma$$

DROMO

The *departure* and the *actual* frames



• *A Special Perturbation Method in Orbital Dynamics*, by
J. Peláez, J. M. Hedo & P. Rodríguez de Andrés,
Celestial Mechanics and Dynamical Astronomy, Vol.97, pp. 131-150, 2007, doi:10.1007/s10569-006-9056-3

EQUATIONS OF DROMO

The last variable change:

$$\zeta_1 = \psi^2 A, \qquad \zeta_2 = \psi^2 B, \qquad \zeta_3 = \frac{1}{\psi}$$

leads to:

$$\begin{aligned}
\frac{d\tau}{d\sigma} &= \frac{1}{\zeta_3^3 \hat{s}^2} \\
\frac{d\zeta_1}{d\sigma} &= \frac{1}{\zeta_3^4 \hat{s}^3} \left[+\hat{s} \sin\sigma f_{px} + \{\zeta_1 + (1+\hat{s})\cos\sigma\} f_{pz}\right] \\
\frac{d\zeta_2}{d\sigma} &= \frac{1}{\zeta_3^4 \hat{s}^3} \left[-\hat{s} \cos\sigma f_{px} + \{\zeta_2 + (1+\hat{s})\sin\sigma\} f_{pz}\right] \\
\frac{d\zeta_3}{d\sigma} &= -\frac{1}{\zeta_3^3 \hat{s}^3} f_{pz} \\
\frac{d\varepsilon_1^0}{d\sigma} &= -\frac{\lambda(\sigma)}{2}\{\sin(\sigma-\sigma_0)\varepsilon_2^0 + \cos(\sigma-\sigma_0)\varepsilon_4^0\} \\
\frac{d\varepsilon_2^0}{d\sigma} &= +\frac{\lambda(\sigma)}{2}\{\sin(\sigma-\sigma_0)\varepsilon_1^0 - \cos(\sigma-\sigma_0)\varepsilon_3^0\} \\
\frac{d\varepsilon_3^0}{d\sigma} &= +\frac{\lambda(\sigma)}{2}\{\cos(\sigma-\sigma_0)\varepsilon_2^0 - \sin(\sigma-\sigma_0)\varepsilon_4^0\} \\
\frac{d\varepsilon_4^0}{d\sigma} &= +\frac{\lambda(\sigma)}{2}\{\cos(\sigma-\sigma_0)\varepsilon_1^0 + \sin(\sigma-\sigma_0)\varepsilon_3^0\}
\end{aligned}$$

These equations should be integrated, taking into account the relations:

$$\begin{aligned}
\lambda(\sigma) &= \frac{1}{\zeta_3^4 \hat{s}^3} f_{py} \\
\hat{s} &= 1 + \zeta_1 \cos\sigma + \zeta_2 \sin\sigma \\
z &= \frac{1}{r} = \zeta_3^2 \{1 + \zeta_1 \cos\sigma + \zeta_2 \sin\sigma\} \\
\frac{dr}{d\tau} &= \zeta_3(\zeta_1 \sin\sigma - \zeta_2 \cos\sigma) \\
\chi &= \frac{\sigma - \sigma_0}{2} \\
\begin{pmatrix} \varepsilon_1 \\ \varepsilon_3 \\ \varepsilon_2 \\ \varepsilon_4 \end{pmatrix} &= \begin{pmatrix} \cos\chi & \sin\chi & 0 & 0 \\ -\sin\chi & \cos\chi & 0 & 0 \\ 0 & 0 & \cos\chi & -\sin\chi \\ 0 & 0 & \sin\chi & \cos\chi \end{pmatrix} \begin{pmatrix} \varepsilon_1^0 \\ \varepsilon_3^0 \\ \varepsilon_2^0 \\ \varepsilon_4^0 \end{pmatrix}
\end{aligned}$$

and starting from the appropriated initial conditions.

Checking the method (I)

In order to check the method we chose the *Example 2b* (page 122) of the book of Stiefel & Scheifele. It is about a satellite with $e = 0.95$, $i = 30°$, perturbed by an oblate Earth and the Moon. For the Earth perturbation they take the following values:

$$J_2 = 1.08265 \cdot 10^{-3}, \qquad R_E = 6371.22\ \text{Km}, \qquad \mu = 398601.0\ \text{Km}^3\text{s}^{-2}$$

The lunar perturbation is modeled through the force:

$$\vec{\boldsymbol{F}}_{PL} = -m\mu_L\left\{\frac{\vec{\boldsymbol{R}} - \vec{\boldsymbol{\rho}}}{|\vec{\boldsymbol{R}} - \vec{\boldsymbol{\rho}}|^3} + \frac{\vec{\boldsymbol{\rho}}}{\rho^3}\right\} \qquad \begin{cases} \vec{\boldsymbol{R}} & \text{satellite position vector} \\ \vec{\boldsymbol{\rho}} & \text{Moon position vector} \end{cases}$$

with the following Moon ephemeris:

$$\vec{\boldsymbol{\rho}} = \rho\left\{\sin\Omega_L t\,\vec{\boldsymbol{i}}_1 - \frac{\sqrt{3}}{2}\cos\Omega_L t\,\vec{\boldsymbol{j}}_1 - \frac{1}{2}\cos\Omega_L t\,\vec{\boldsymbol{k}}_1\right\}$$

where ρ, Ω_L, and μ_L are constants:

$$\rho = 384400\ \text{Km}, \quad \Omega_L = 2.665315780887 \cdot 10^{-6}\ \text{s}^{-1}, \quad \mu_L = 4902.66\ \text{Km}^3\text{s}^{-2}$$

Linear and Regular Celestial Mechanics, **Stiefel & Scheifele**, Springer-Verlag, NY, 1971

Checking the method (II)

In order to calculate the trajectory the following initial condition should be given:

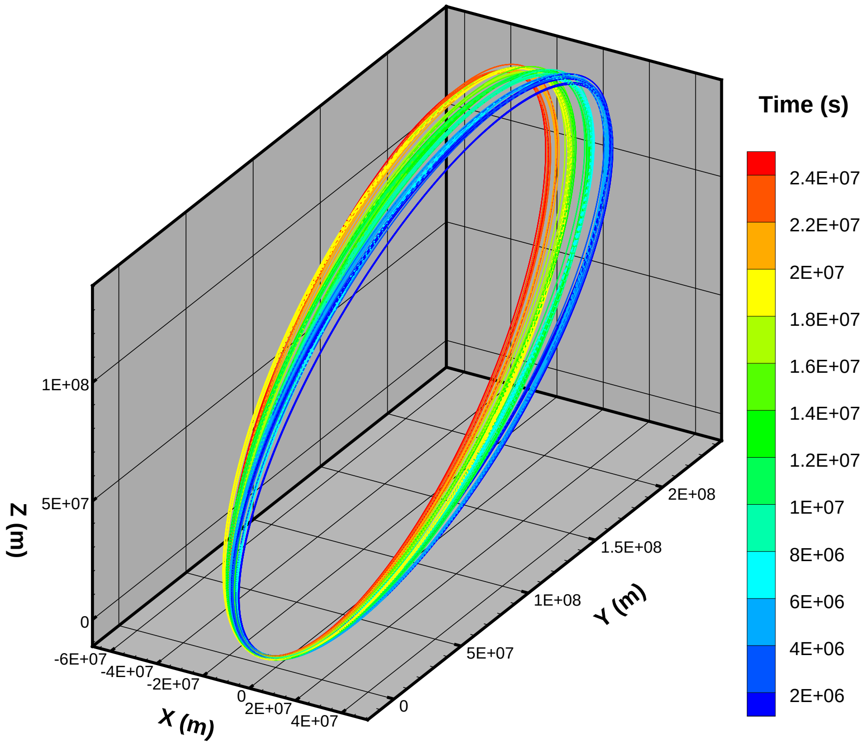
$$x_1 = 0.0, \qquad y_1 = -5888.9727 \text{ km}, \qquad z_1 = -3400.0000 \text{ km}$$
$$\dot{x}_1 = 10.691338 \text{ km/s}, \qquad \dot{y}_1 = 0.0, \qquad \dot{z}_1 = 0.0$$

Initially the satellite is at the perigee (the distance is $R = 6800$ km). The position of the satellite is propagated after 50 orbits, which correspond to 288.12768941 mean solar days. The most accurate position given in the book of Stiefel & Scheifele is:

$$x_{1f} = -24219.0503 \text{ km}$$
$$y_{1f} = 227962.1064 \text{ km}$$
$$z_{1f} = 129753.4424 \text{ km}$$

which has been obtained using 498 steps per orbit. We recalculated the orbit and we obtained:

$$x_{1f} = -24219.0501159 \text{ km}$$
$$y_{1f} = 227962.1063730 \text{ km}$$
$$z_{1f} = 129753.4424001 \text{ km}$$



Trajectory of the Example 2b
extracted from the book of Stiefel & Scheifele

First Comparison

Table taken from [1]

Method	Stiefel Scheifele [2]	Sperling Burdet [3]	Kustaanheimo Stiefel [4]	Cowell [5]	DROMO
x(Km)	-24219.050	-24218.818	-24219.002	-24182.152	-24219.279
y(Km)	227962.106	227961.915	227962.429	227943.989	227962.207
z(Km)	129753.442	129753.343	129753.822	129744.270	129753.492
Steps/rev	500	62	62	240	62
Error		0.318	0.501	42.5	0.250

[1] *Modern Astrodynamics*, **Victor R. Bond & Mark C. Allman**, Princeton U.P., NJ 1996

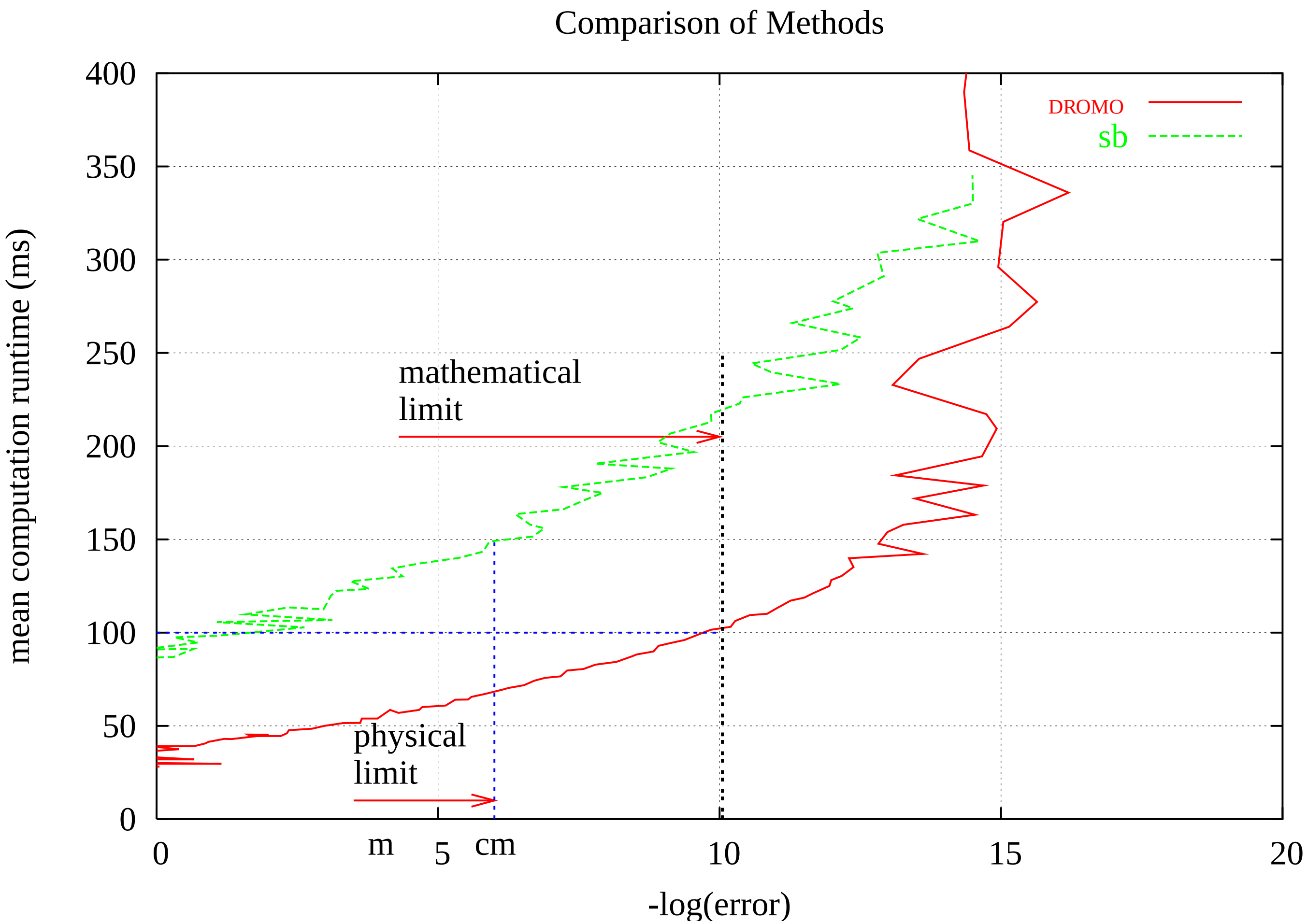
[2] *Linear and Regular Celestial Mechanics*, **Stiefel & Scheifele**, Springer-Verlag, NY, 1971

[3] *Elimination of Secular Terms from the Differential Equations for the Elements of the Perturbed Two-Body Problem*, **V. R. Bond & M. F. Fraietta**, Proceed. of the Flight Mechanics and Estimation Theory Symposium, NASA, 1991

[4] *The uniform, regular differential equations of the KS transformed perturbed two-body Problem*, **V. R. Bond**, Celestial Mechanics, 10, 1974

[5] *The Burdet formulation of the perturbed two body problem with total energy as an element*, **V. R. Bond & M. K. Horn**, JSC Internal 73-FM-86, NASA, 1973

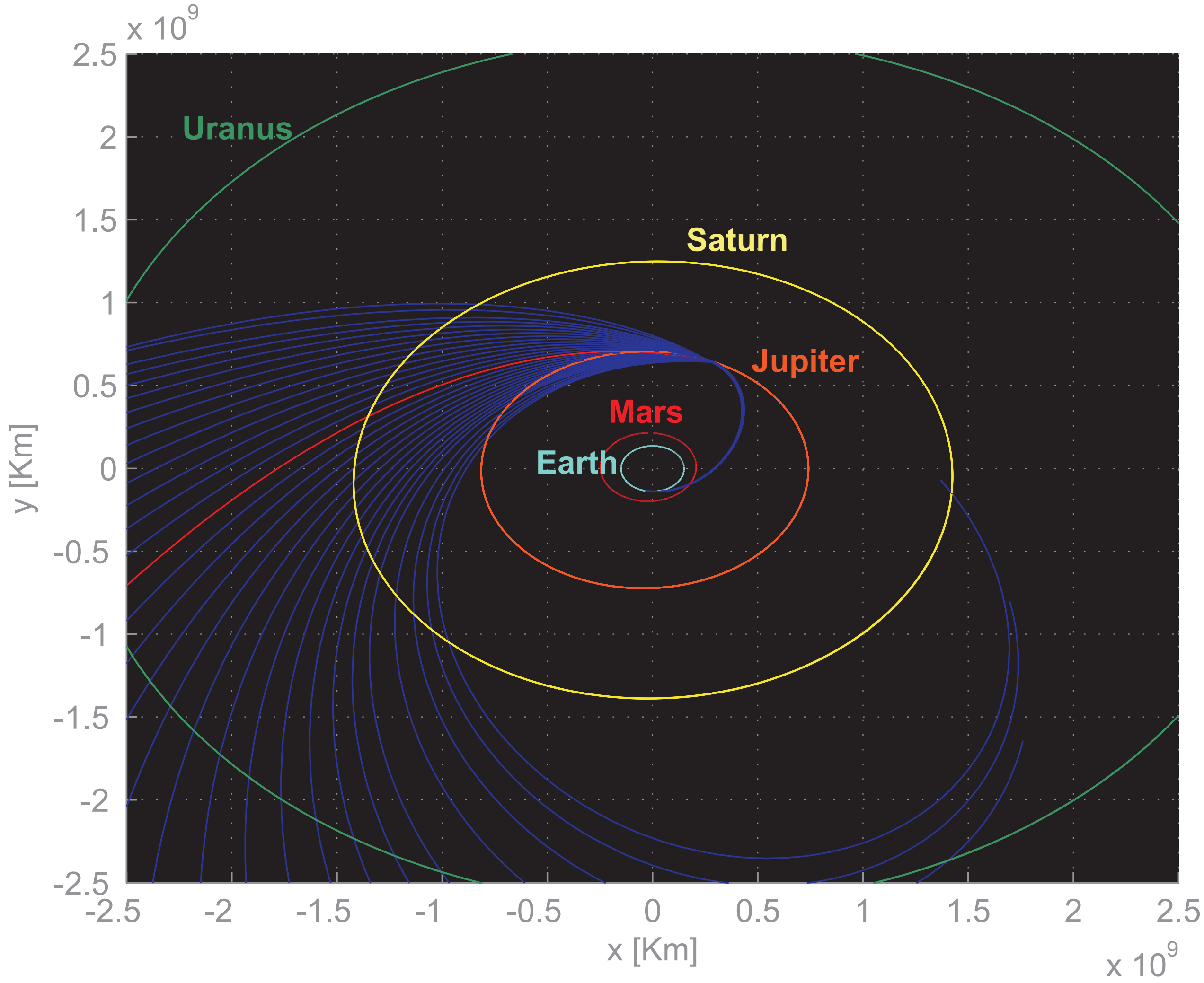
First Comparison



Integration routine: Runge-Kutta-Fehlberg 7(8) with step control.

In all zones of interest our method results quicker than Sperling-Burdet's method, or equivalently, it gives more accurate results than the SB method for the same runtime, with similar computational effort.

Fly-by of a planet



Consider a S/C which perform a fly-by of a given planet. The trajectory of the S/C, after the fly-by, has a very large sensitivity with the initial conditions at the entrance to the sphere of influence of the planet. High-fidelity propagation is needed.

Fly-by of a planet

In order to describe the fly-by of a planet with some accuracy we prepared **TWO** propagation tools —based on DROMO and Cowell method— with the following characteristics:

- Numerical integrator: 8th order embedded Runge-Kutta method (Dormand & Prince)
- Earth Orientation Parameters: SOFA routines
- JPL ephemerides based on the mean orbital parameters
- Earth's gravitational potential EGM96 (up to degree and order 360)
- Third body gravitational perturbation (Sun, Moon and all the planets)
- Solar radiation pressure

With these propagations tools some fly-by's were described.

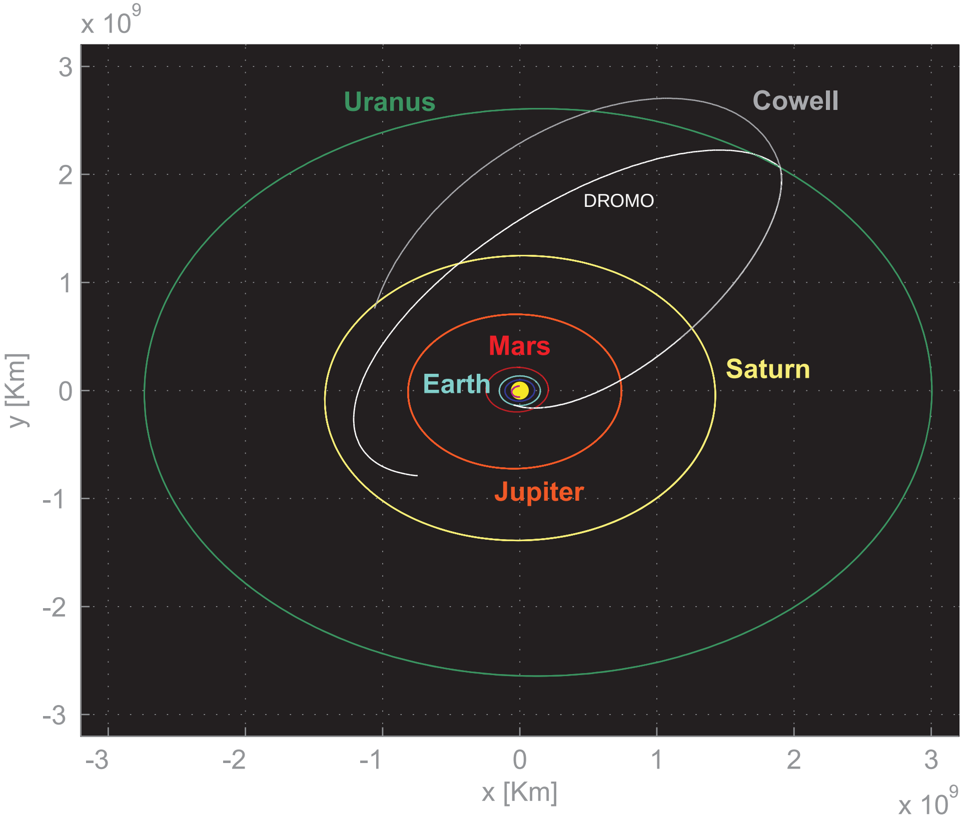
- *Advanced Propagation of Interplanetary Orbits in the Exploration of Jovian Moons*, by

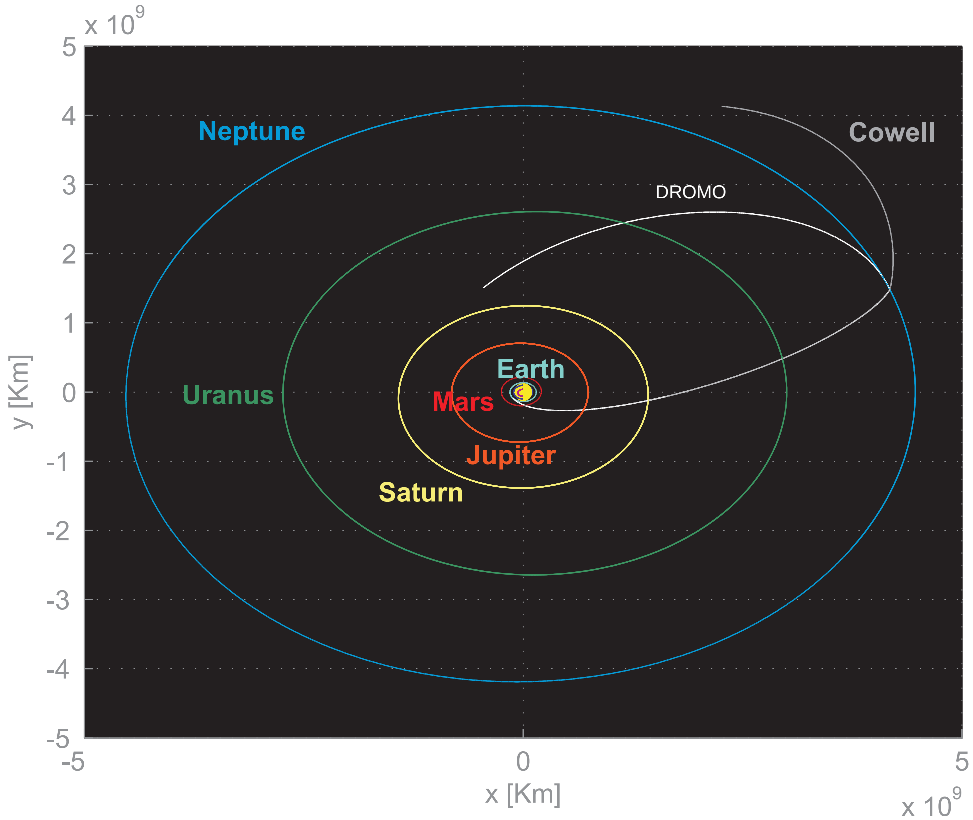
J. Esteban-Dones and J. Peláez,

4th Int. Conference on Astrodynamics Tools and Techniques, 3-6 May 2010, ESAC Villafranca, Madrid, Spain

Fly-by of a planet

- Comparison GDT method vs Cowell's method





Störmer-Cowell formulation

INTEGRATION ALGORITHMS				
Method	Single-step/ Multi-step	Fixed-step/ Variable-step	1st order/ 2nd order	Summed/ Non-summed
Runge-Kutta	Single	Fixed	1st order	NA
Runge-Kutta-Fehlberg	Single	Variable	1st order	NA
Adams (non-summed)	Multi	Fixed	1st order	Non-Summed
Adams (summed)	Multi	Fixed	1st order	Summed
Shampine-Gordon	Multi	Variable	1st order	Non-Summed
Störmer-Cowell	Multi	Fixed	2nd order	Non-Summed
Gauss-Jackson	Multi	Fixed	2nd order	Summed
Störmer-Cowell	Multi	Variable	2nd order	Non-Summed

The performances of the Cowell method increase when the integration algorithm used in the propagation is a new Störmer-Cowell of variable step. In particular we use in our calculation the algorithm carried out by Matthew M. Berry. A Variable-Step Double-Integration Multi-Step Integrator. PhD thesis, Virginia Tech, Blacksburg, 2004.

A finer comparison of propagators

Method	Cowell	DROMO	DROMO	Cowell	Cowell
Integrator	RKF4(5)	RKF4(5)	RKF7(8)	Störmer-Cowell 5	Störmer-Cowell 9
x (km)	-24210.188	-24219.049	-24219.050	-24232.184	-24219.183
y (km)	227957.706	227962.097	227962.105	227966.173	227962.169
z (km)	129751.208	129753.437	129753.441	129755.268	129753.473
steps/rev	240	62	29	372	372
Fcalls/rev	1440	372	372	372	372
Run-Time(s)	0.232	0.094	0.050	0.065	0.12
Error (km)	10.143	0.010	0.002	13.896	0.150

Table 1.1: Results for Stiefel & Scheifele's Example 2b, using DROMO's newest formulation and different integrators for the Cowell equations.

The new comparison has been performed on the following basis:

1. improved integration routines has been used (RKF and SC)
2. for each special perturbation method the best integration routine is selected: the RKF —for the moment— for DROMO and the Störmer-Cowell for the Cowell method
3. the tolerance of the integration process is tuned in order to reach the same number of functions call per orbit: 372
4. the final error and run-time is selected as the parameters measuring the quality of the integration

A finer comparison of propagators

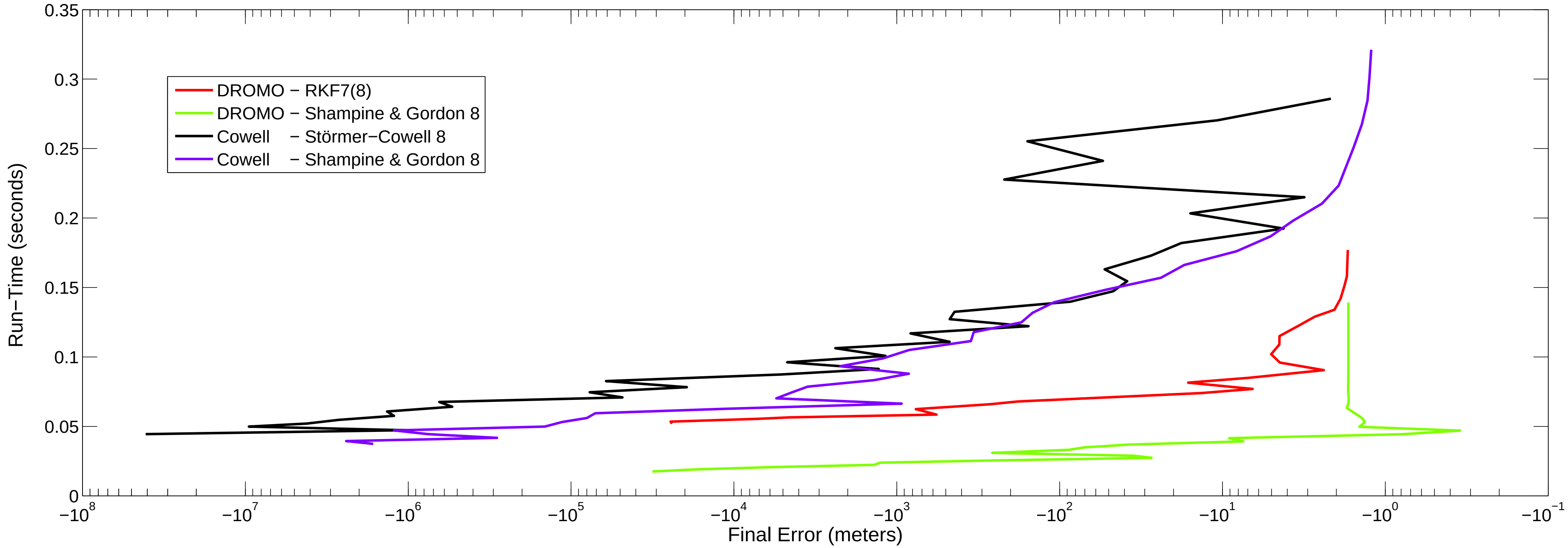


Figure 1.1: Comparative results showing the "Run-Time" vs "Final Error" relation for different propagators when used in Stiefel & Scheifele's Example 2b.

Since in a **real propagation** the time consumed in the evaluation of the right hand sides of the ODE's that govern the motion is the largest it would very interesting to use DROMO with some multistep integrator because of the number of function calls will be reduced. In our test we selected the classical Shampine & Gordon —written in 1975— usually called DE in several works on orbit propagation. The above figure summarizes the excellent behavior of DROMO with this multistep algorithm.

Problem of Tsien

DEFINITION AND CLASSICAL ANALYSIS

- circular orbit. Radius R_0. Velocity $R_0\omega_0$

$$\omega_0^2 = \frac{\mu}{R_0^3}$$

- at $t = 0$ start a constant radial thrust $\vec{\boldsymbol{a}}_p = a_R\,\vec{\boldsymbol{u}}_r$

The angular momentum is constant and the trajectory is a plane curve:

$$\vec{\boldsymbol{h}} = \vec{\boldsymbol{r}} \times \vec{\boldsymbol{v}} = \vec{\boldsymbol{r}}_0 \times \vec{\boldsymbol{v}}_0 = R_0^2\omega_0(-\vec{\boldsymbol{j}})$$

Let (r, θ) polar coordinates inside the orbital plane. The law of areas takes the form

$$r^2\,\dot{\theta} = h, \qquad \text{where} \quad h = R_0^2\omega_0 \tag{1.2}$$

The forces are conservatives and derive from the potential

$$V(r) = -\frac{\mu}{r} - a_R\,r$$

and the total energy is conserved

$$\frac{1}{2}v^2 + V(r) = E, \qquad \text{where} \quad E = \frac{1}{2}v_0^2 - \frac{\mu}{R_0} - a_R\,R_0$$

We introduce the following non-dimensional variables:

$$r = u\,R_0, \qquad \tau = \omega_0\,t, \qquad \epsilon = \frac{8\,a_R}{R_0\omega_0^2}$$

Using the law of areas (1.2), the energy equation takes the form

$$\frac{du}{d\tau} = \pm\sqrt{\mathcal{E} - V_{eff}(u)}$$

where the effective potential V_{eff} and the total energy $\mathcal{E}$ (non-dimensional values) are given by:

$$V_{eff}(u) = \frac{1}{u^2} - \frac{2}{u} - \frac{\epsilon}{4}u, \qquad \mathcal{E} = -(1 + \frac{\epsilon}{4})$$

The solution is given by the following quadratures:

$$\tau = \pm\int_1^u \frac{d\xi}{\sqrt{\mathcal{E} - V_{eff}(\xi)}} \tag{1.3}$$

$$\theta = \theta_0 \pm \int_1^u \frac{d\xi}{\xi^2\sqrt{\mathcal{E} - V_{eff}(\xi)}} \tag{1.4}$$

which lead to an analytical solution; see, for example:

• *An Introduction to the Mathematics and Methods of Astrodynamics*, by **Richard H. Battin.,** Educational Series. AIAA, revised edition, 1999.

Test solution

Depending on ϵ, two different behaviors appear:

1. $\epsilon < 1$ the thrust is *small* and the motion is bounded by two concentric circles
2. $\epsilon > 1$ the thrust is *large* and the motion is unbounded. In particular, the escape velocity is reached after a while (see Battin's book).

There is an asymptotic motion which separates these two different behaviors; it appears for $\epsilon = 1$.

$$\tau = 4 \ln \left[\frac{1 + \sqrt{u-1}}{1 - \sqrt{u-1}} \right] - 4\sqrt{u-1} \qquad (1.5)$$

Notice that the motion is tending to a circular motion along a circumference of radius $2R_0$
The *numerical* obtention of this analytical solution is not easy. In effect, the errors accumulated in the calculation prevent the numerical solution to reach the asymptotic behavior for moderately large values of the time τ. These errors *move the energy line* which is no longer tangent to the graphic of the effective potential.

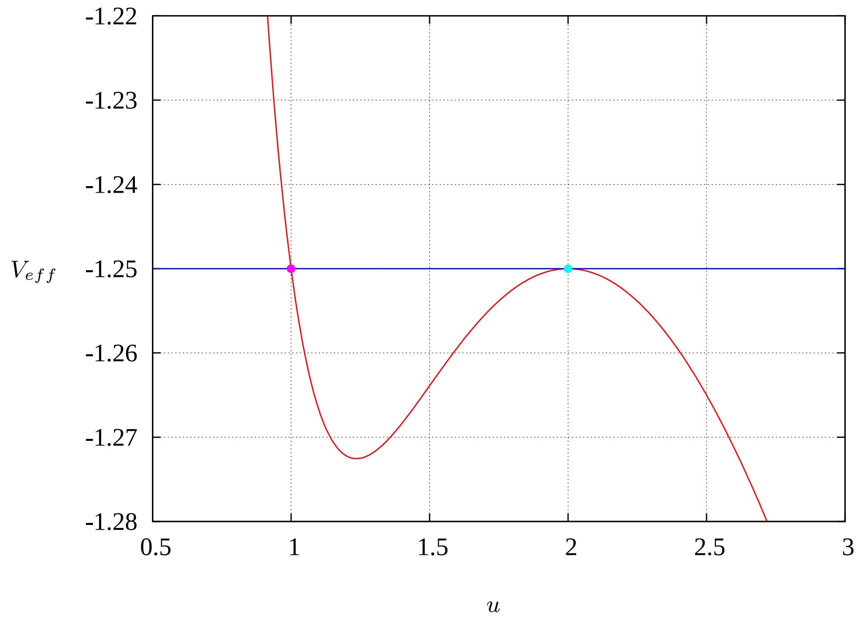


Figure 1. Effective potential V_{eff} and total energy $\mathcal{E}$ for $\epsilon = 1.0$

As a consequence, and due to the numerical errors:

1. the satellite descends towards the starting circle or
2. it escapes from the attractive body.

Thus, this well defined analytical solution of the Tsien problem is an excellent tool to compare performances of different propagators and integrators.

Comparison DROMO/Störmer-Cowell

TEST SOLUTION

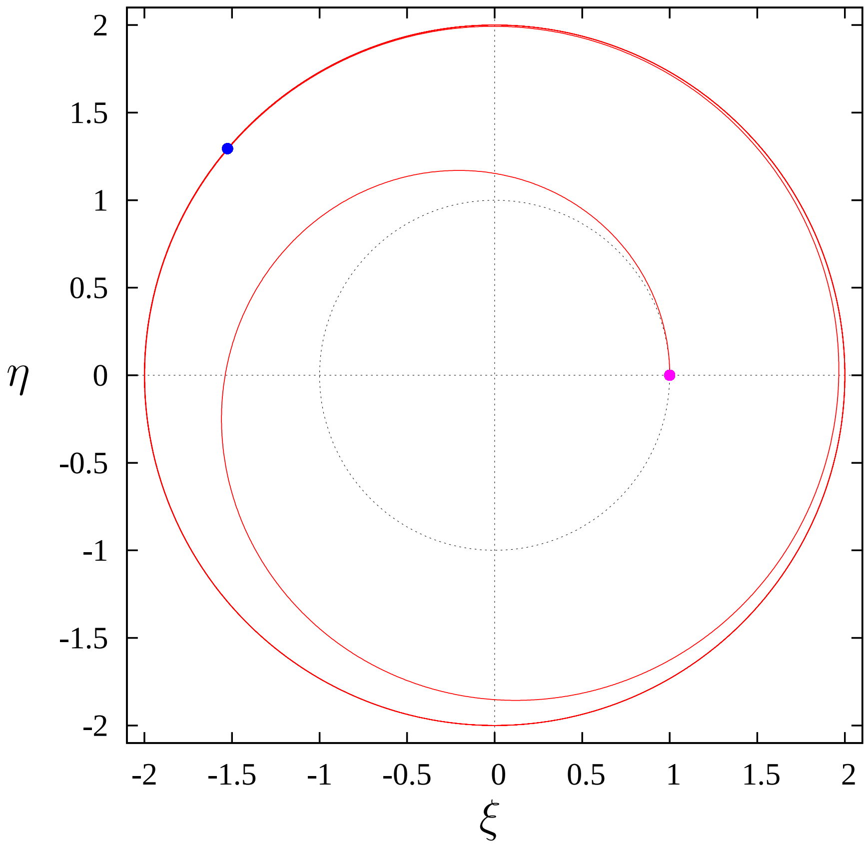


FIGURE 1.1: Satellite trajectory in the asymptotic case $\epsilon = 1$

The *numerical* obtention of the asymptotic analytical solution of the Tsien problem is not easy. In effect, the errors made in the numerical integration prevent to describe the asymptotic trajectory during long periods of time. Note that a propagator is the combination of a Special Perturbation method and an integration algorithm.

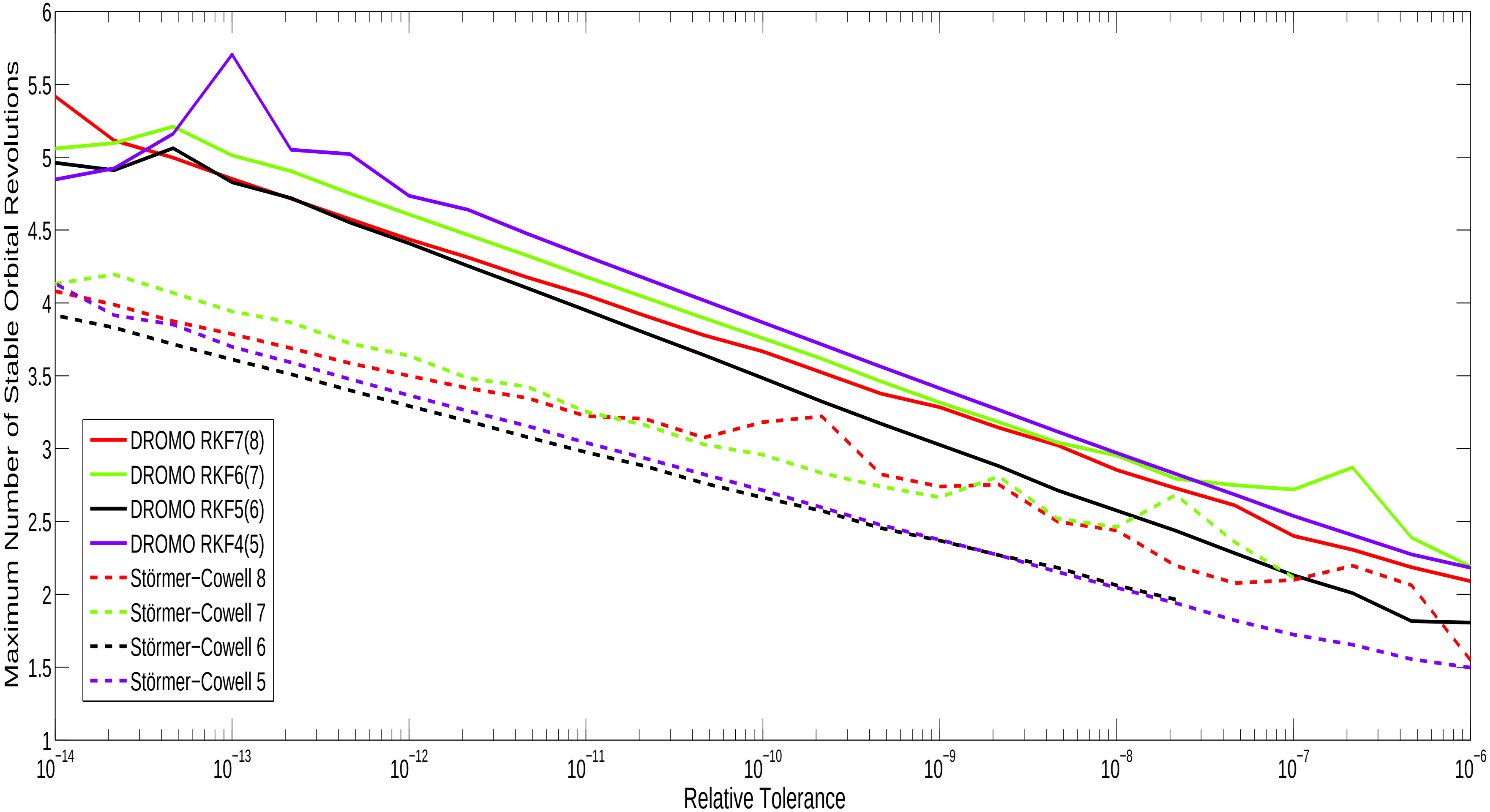
A suitable measure to evaluate performance of the presented propagators is to calculate the number of orbits until the numerical solution starts to deviate from the asymptotic orbit. A deviation is considered, when the relative error of the numerically computed position is larger than a threshold. Here R is the current orbital radius which must be compared with the radius of the asymptotic orbit $2R_0$.

$$\frac{|2R_0 - R|}{2R_0} < 10^{-3}$$

In our group we compared the results obtained with different combinations as shows the next slides.

Comparison DROMO/Störmer-Cowell

In this slide the results obtained by DROMO using Runge-Kutta-Fehlberg algorithms of given order are compared with the Cowell method using the Störmer-Cowell algorithm of the same order



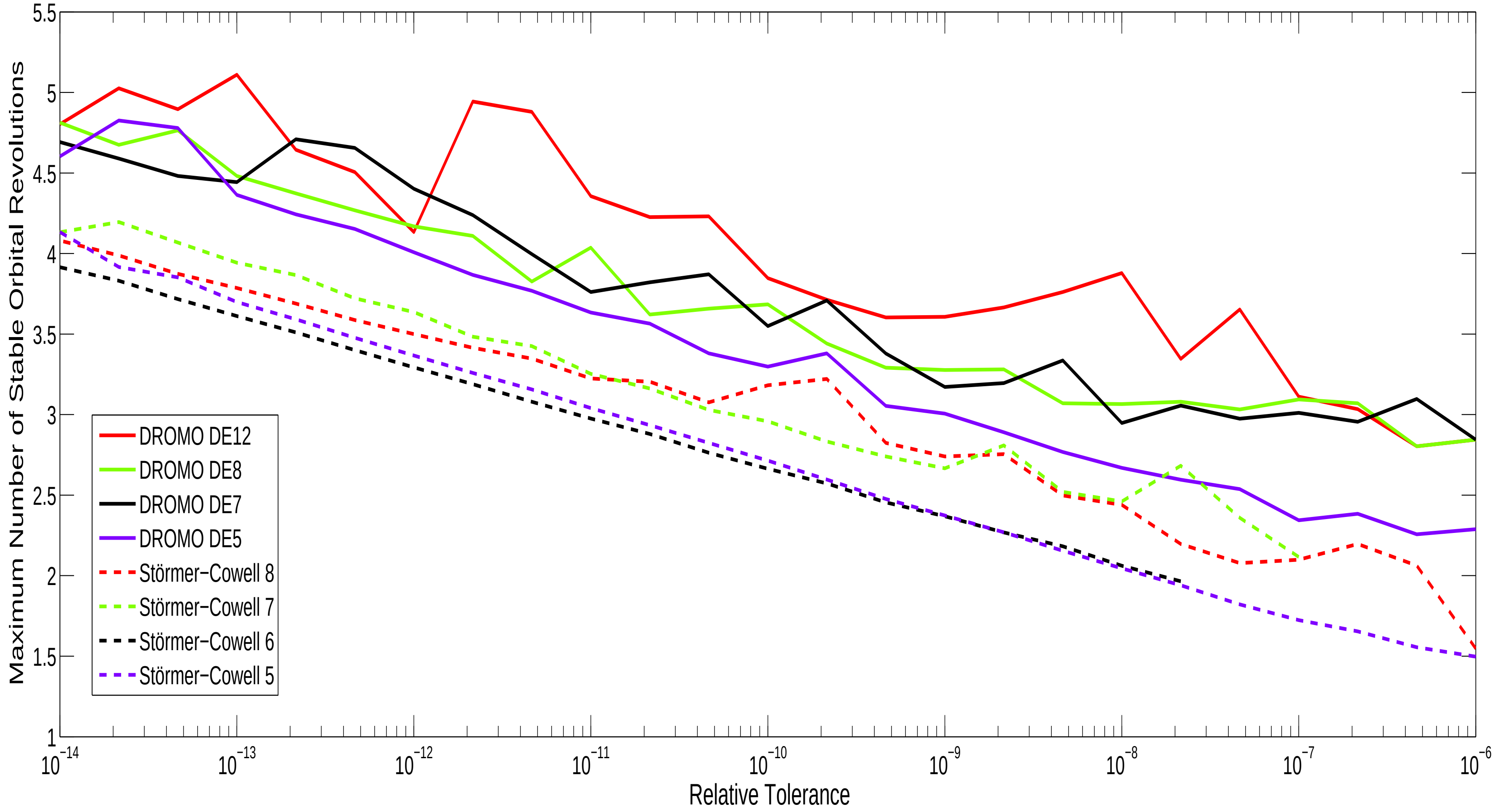
Comparison DROMO/Störmer-Cowell

Method	DROMO RKF7(8)	DROMO RKF6(7)	DROMO DE8	DROMO DE7	SC 8th order	SC 7th order
Rel tolerance	1e-11	1e-11	1e-11	1e-12	1e-14	1e-14
Runtime in s	0.21	0.47	-	-	0.24	0.24
Function calls	2004	3378	1113	1623	439	536
Number of steps	154	338	-	-	431	529

Runtime comparison for 4 complete orbits

Comparison DROMO/Störmer-Cowell

In this slide the results obtained by DROMO using Shampine & Gordon algorithms of given order are compared with the Cowell method using the Störmer-Cowell algorithm of the same order



Conclusions

- In terms of *accuracy* DROMO with the Runge-Kutta-Fehlberg routine RKF7(8) turn out to be the best combination since they provide a longer and more stable description of the asymptotic orbit (in the Tsien problem) and a much more accurate answer (in the example 2b of the book of Stiefel and Seifele).
- In terms of *function calls* the Störmer-Cowell formulation —in some cases, but not always— turns out to be the best formulation since it provides the lower number of call to the derivative functions.
- from a *global point of view*, the combination of DROMO with the multistep method of Shampine & Gordon (DE) shows excellent characteristics.

In the Tsien problem, DROMO + RKF7(8) is able to describe almost 6 times the asymptotic orbit and SC9 only 4 with a very tight tolerance. That is, DROMO + RKF7(8) reaches levels of accuracy unachievable for other propagators. This plus of accuracy makes DROMO propagator the most appropriated when a high-fidelity description of the trajectory is mandatory. This plus of accuracy, however, has a cost: the higher number of function calls due to the Runge-Kutta-Fehlberg routine used to perform the integration.

The END

THANK YOU VERY MUCH FOR YOUR ATTENTION!!