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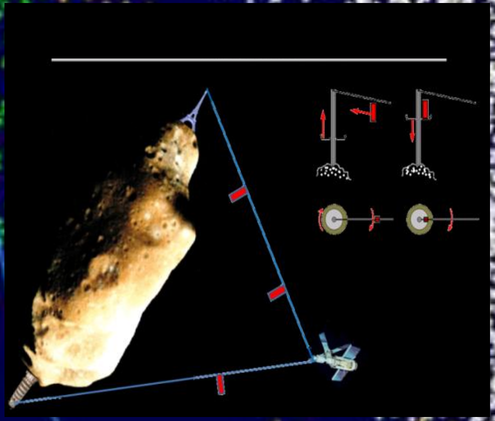
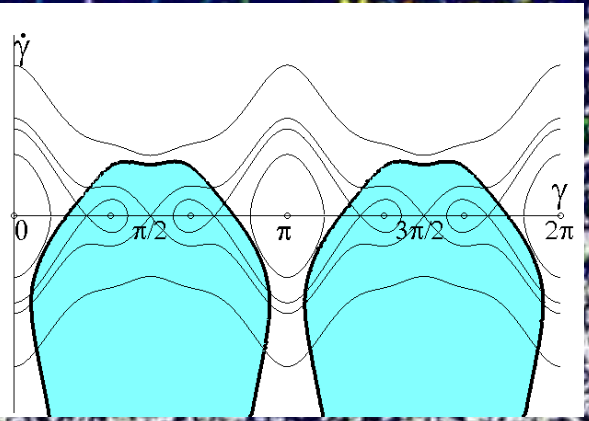
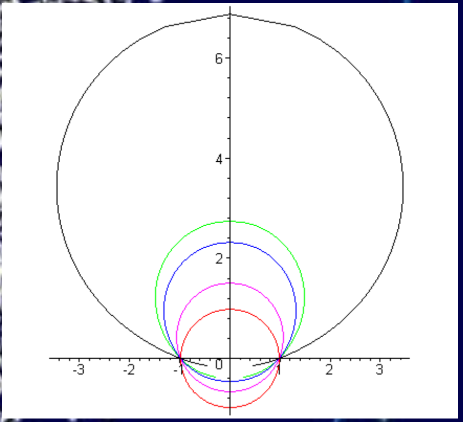
On motions of a space station tethered to an asteroid (On space elevators for asteroids)



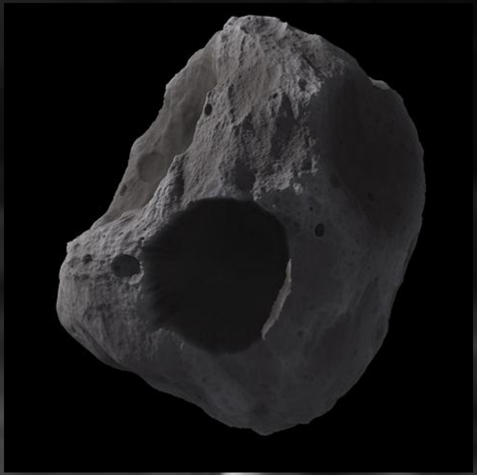




Why should we do something excessive if we could do something better?

Difficulties

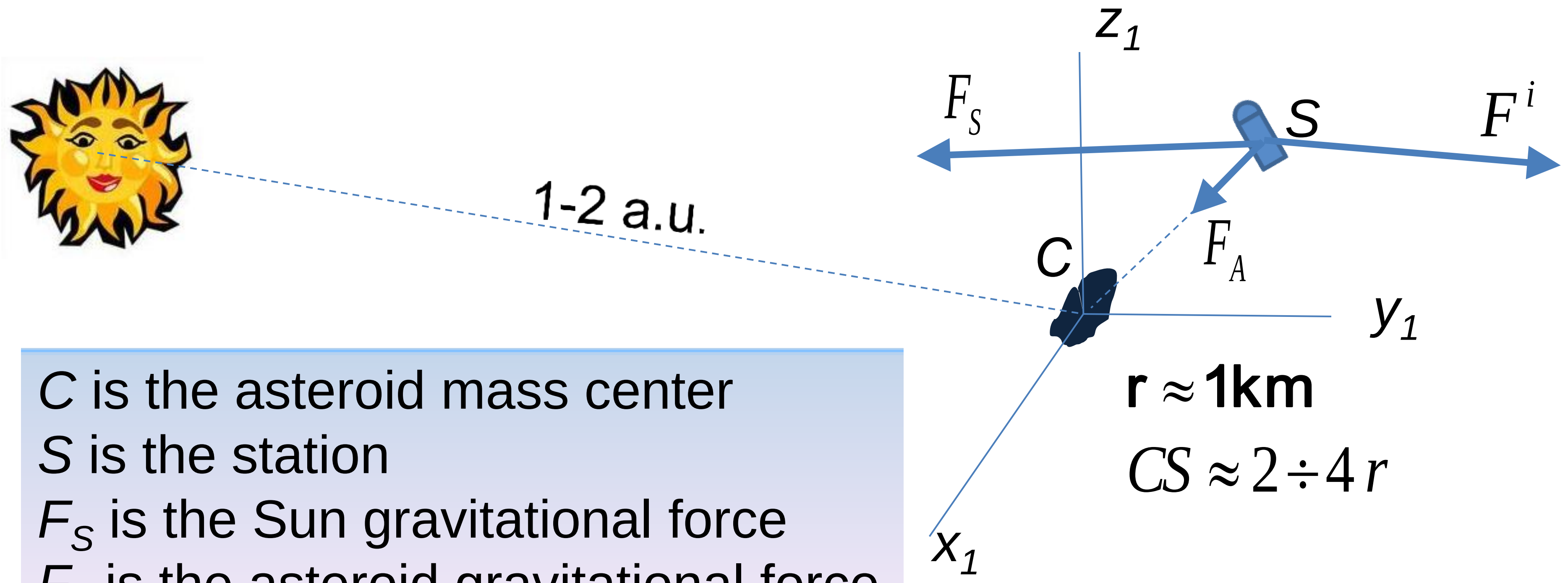
1. Asteroids have very complicated shapes ⇒
2. The asteroid rotation about mass center is not a pure rotation about one of the principal axes. ⇒
3. Excepting particular cases, asteroids have no stationary orbits in traditional sense
4. Influences of the Sun, the Jovi and etc. could be estimated.



Tumbling stones

Forces.

$Cx_1y_1z_1$ are König's axes



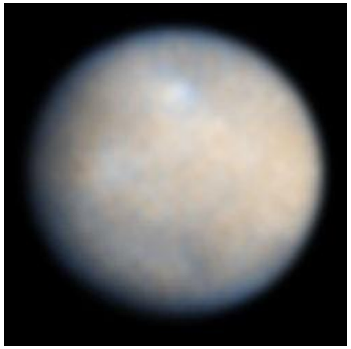
$\mathbf{r} \approx \mathbf{1km}$

$CS \approx 2 \div 4\, r$

C is the asteroid mass center
S is the station
F_S is the Sun gravitational force
F_A is the asteroid gravitational force
F^i is the force of moving space

$$F_S >> F_A, \quad \text{but} \quad \frac{\left|F_S - F^i\right|}{\left|F_A\right|} \sim 10^{-4} \div 10^{-5}$$

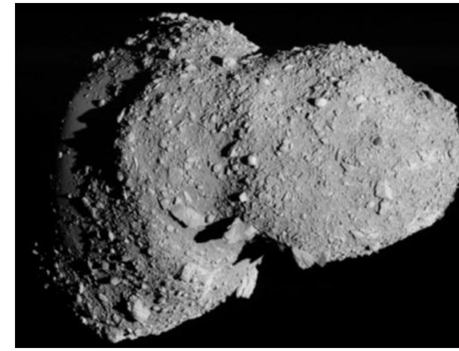
Shapes.



(1)Ceres

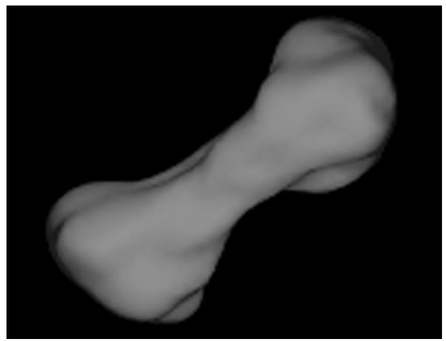
(433)Eros (?)

(951)Gaspra



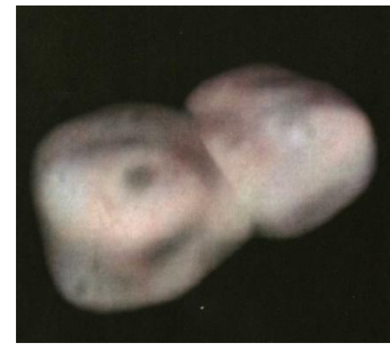
(25143)Itokawa (?)

(243)Ida

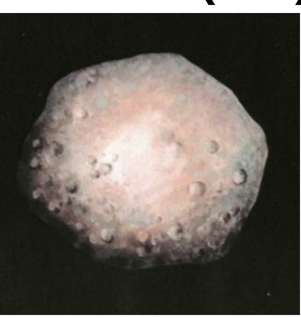


(216)Kleopatra

(107)Camilla



(4769)Castalia



(762)Pulcova

(90)Antiope

(4179)Toutatis (?)

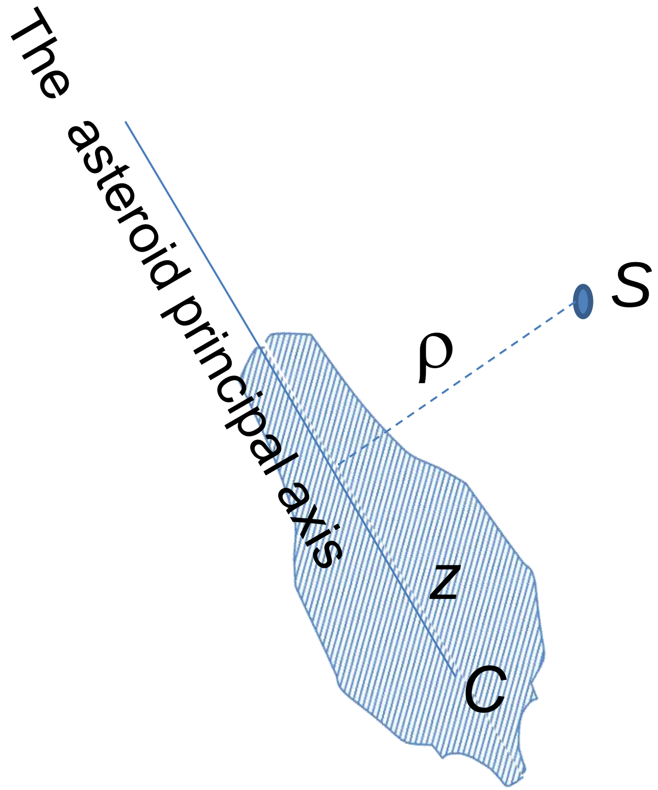
(624)Hector

The dumb-bell

There is a set of dynamically symmetric or dumbbell-shaped asteroids

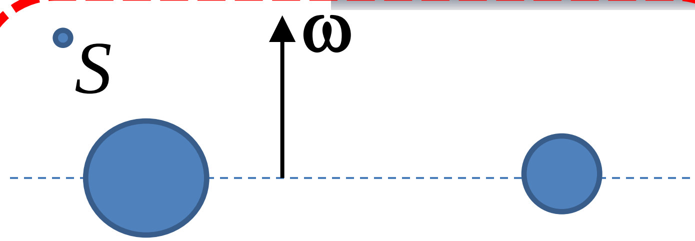
Assumptions

1. The Sun influence on the station relative motion can be neglected
2. The asteroid is a dynamically-symmetric rigid body and its motion about the mass center is a regular precession
3. The asteroid gravitational potential depends only on *z* and ρ.

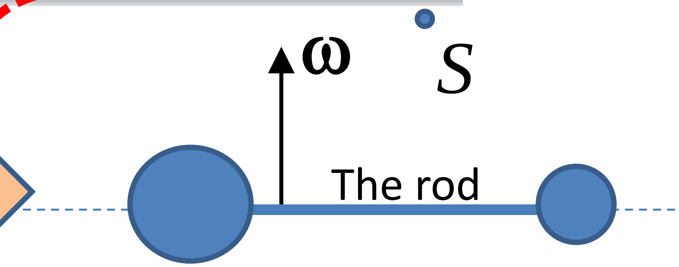


If the asteroid gravitational field is the gravitational field of two particles then the station motion describes by equations of V.V.Beletsky's problem named the ***Generalized Restricted Circular Problem of Three Bodies* (GRCP3B)**. In this case *Librations Points of GRCP3B* are analogies of stationary orbits.

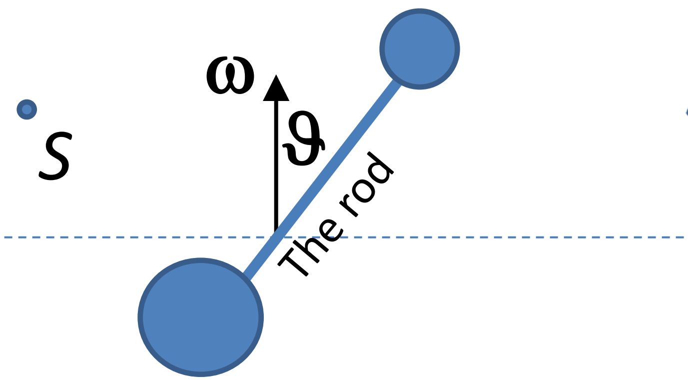
Comments for GRCP3B



Classical RCP3B. **ω=1**
5 libration points



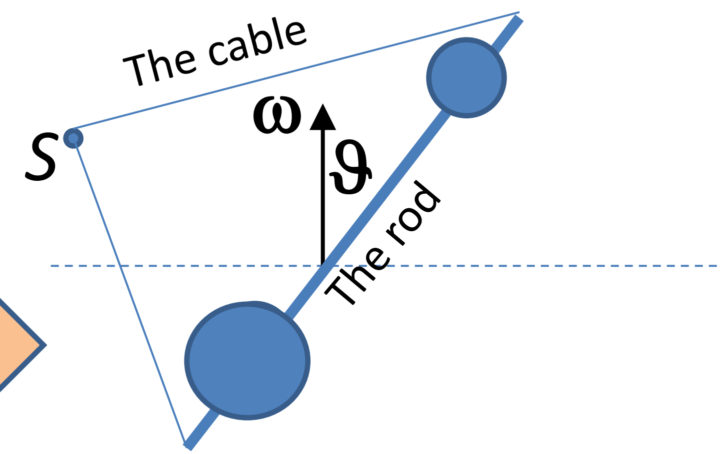
One-parametric generalization of RCP3B. **ω>0,** there are 5 or 3 libration points (If **ω** is sufficiently big then Lagrangian points do not exist)



V.V.Beletsky's *two-parametric generalization* of RCP3B.

$\omega>0 \quad 0\le\vartheta\le\pi/2$.

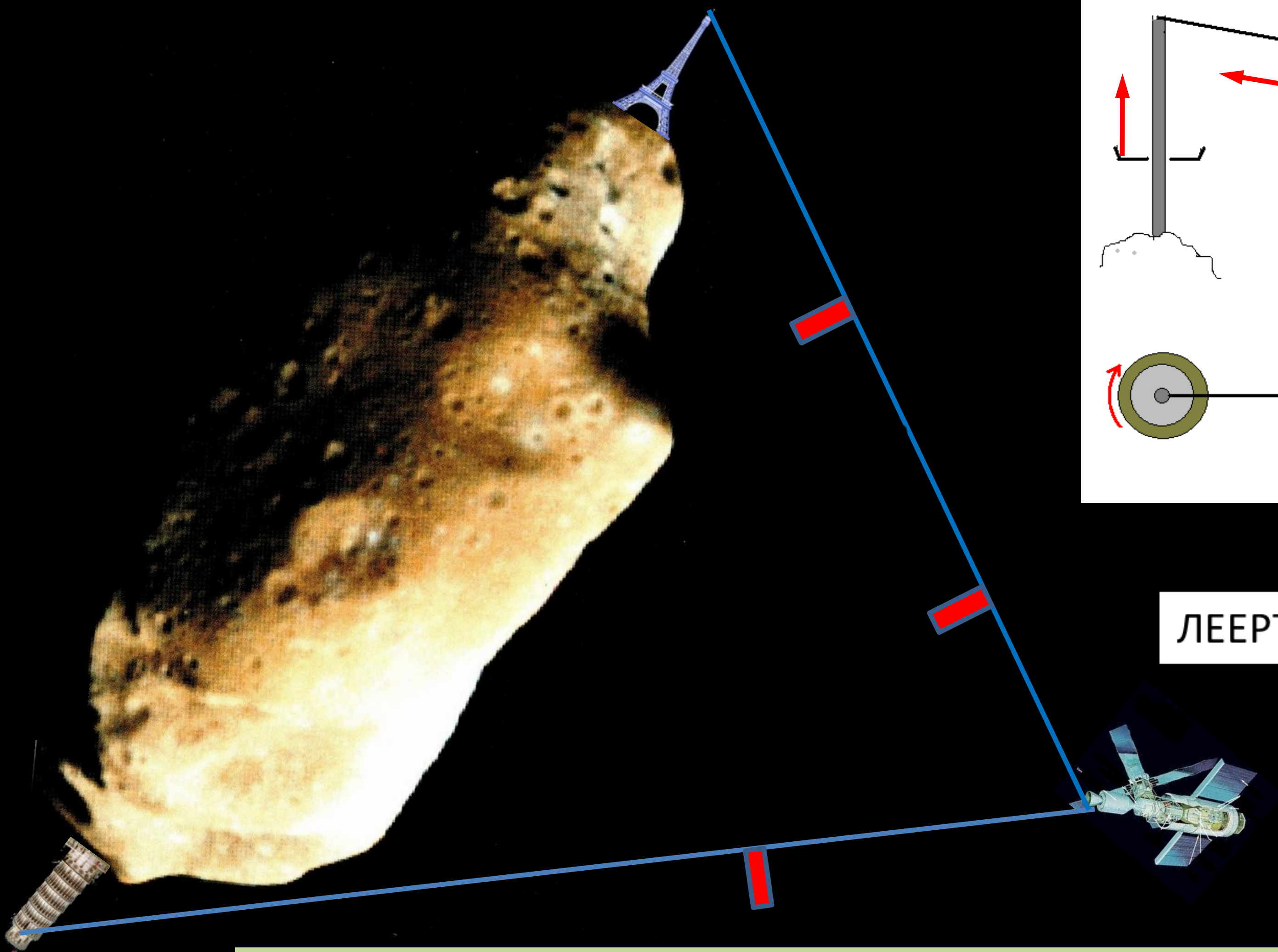
From 3 up to 9 libration points

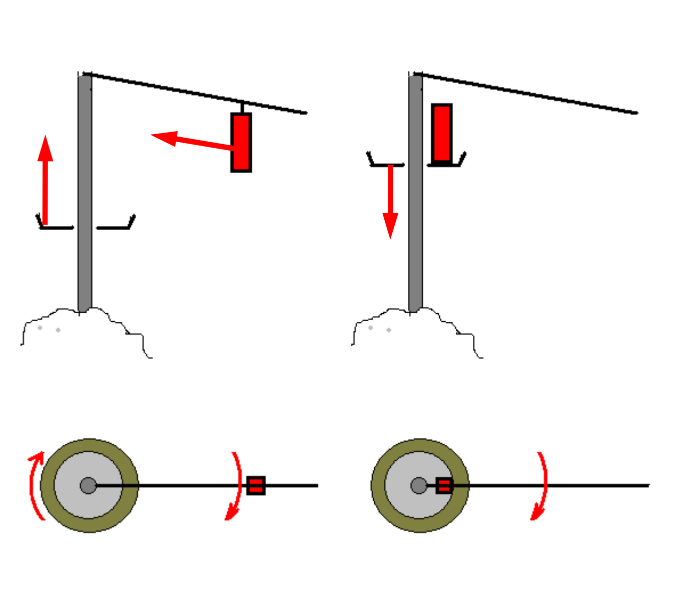


The station equilibria are not only in libration points

V.V. Beletsky //Cosmic researches, 2007, 45,3 , V.V.Beletsky, A.V.Rodnikov //Cosmic researches, 2008, 46, 1, etc.

Space elevator based on ‘the leier constraint’ for an asteroid



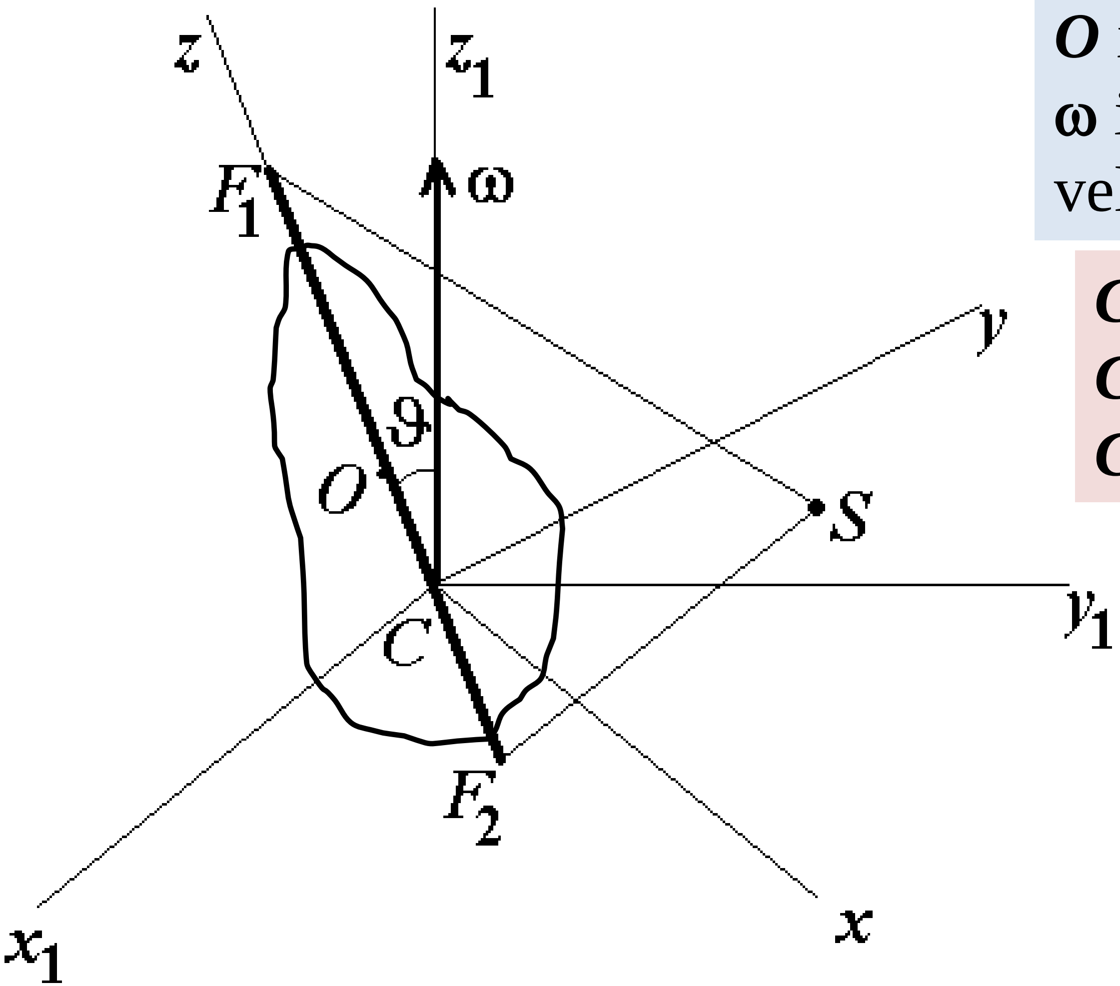


ЛЕЕРЪ

Such writing of this word is exotic even for modern Russians!

Dutch maritime term ‘**leier**’ means the rope with both fixed ends (*Glossary of foreign terms in Russian, Moscow, 1959*)

Designations, axes, parameters



$\boldsymbol{S}(x,y,z)$ is the station,
$\boldsymbol{F}_1$ and $\boldsymbol{F}_2$ are the towers,
$\boldsymbol{C}$ is the asteroid mass center,
$\boldsymbol{O}$ is $\boldsymbol{F_1F_2}$ midpoint,
ω is the precession angular velocity

$\boldsymbol{Cx_1y_1z_1}$ are König's axes,
$\boldsymbol{Cz_1}$ is the precession axis,
$\boldsymbol{Cx}$ belong to $\boldsymbol{Cx_1y_1}$

$$e = \frac{F_1F_2}{F_1S + F_2S}$$

$$d = \frac{OC}{F_1F_2}$$

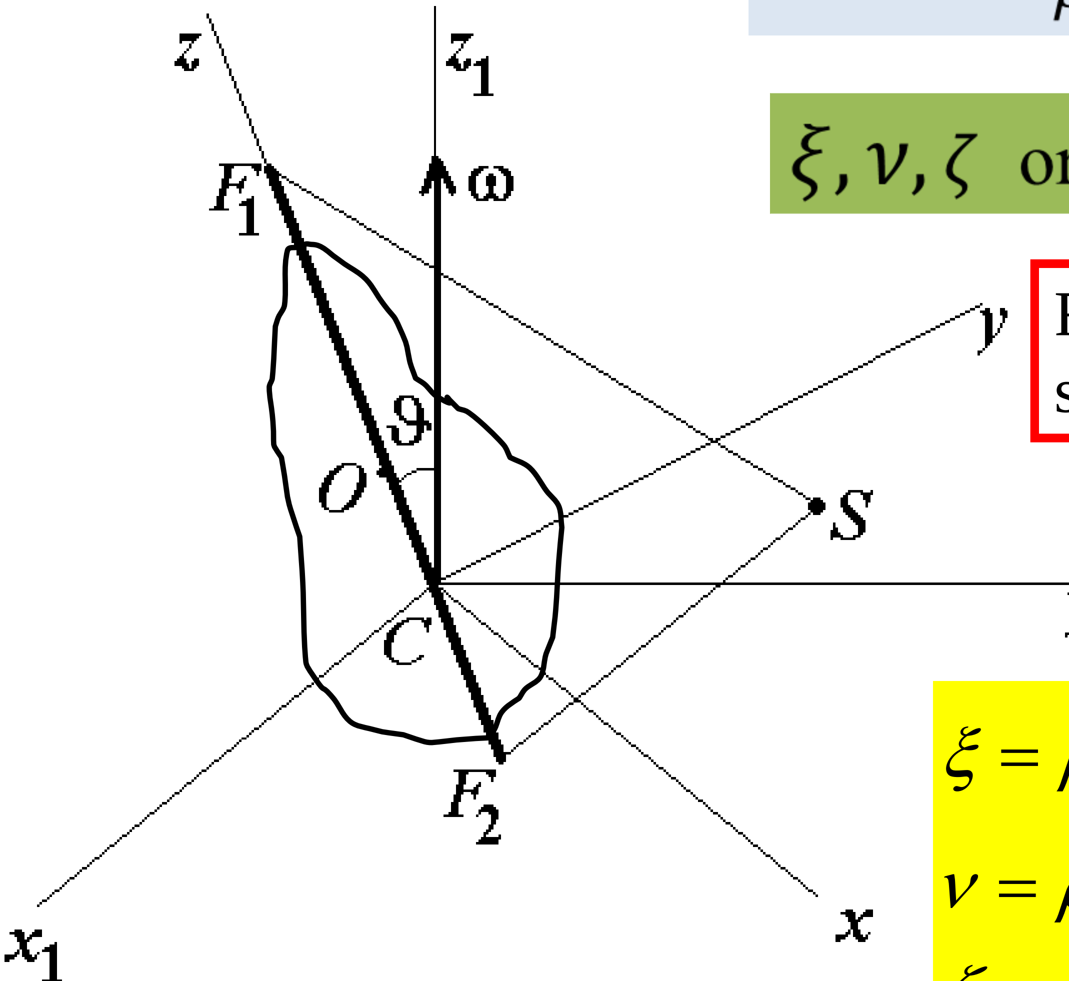
ϑ is the angle of nutation

Dimensionless variables

The cable length $\to 2$, $\omega \to 1$,

$$x \to \xi, \quad y \to \nu, \quad z \to \zeta$$

$$\rho = \sqrt{\xi^2 + \nu^2}$$



ξ, ν, ζ or ρ, φ, ζ or φ, γ

For motions on internal surface of ellipsoid

$$\frac{\rho^2}{1-e^2} + (\zeta - ed)^2 = 1$$

$$\xi = \rho\cos\varphi = \sqrt{1-e^2}\sin\gamma\cos\varphi$$

$$\nu = \rho\sin\varphi = \sqrt{1-e^2}\sin\gamma\sin\varphi$$

$$\zeta = de + \cos\gamma$$

Motion equations

$$\rho\ddot{\varphi} + 2\dot{\rho}(\dot{\varphi} + \cos\vartheta) - 2\dot{\zeta}\sin\varphi\sin\vartheta + \cos\varphi\sin\vartheta\,(\rho\sin\varphi\sin\vartheta + \zeta\cos\vartheta) = 0$$

This equation does not depend on $\tilde{\Pi}$ and λ !

$$\ddot{\zeta} - 2\frac{d(\rho\cos\varphi)}{d\tau}\sin\vartheta - (\zeta\sin\vartheta - \rho\cos\vartheta\sin\varphi)\sin\vartheta + \frac{\partial\tilde{\Pi}}{\partial\zeta} = \lambda\frac{\partial F}{\partial\zeta} = 2\lambda(\zeta - de)$$

$$\ddot{\rho} - \rho\dot{\varphi}(\dot{\varphi} + 2\cos\varphi) + 2\dot{\zeta}\sin\vartheta\cos\varphi + \zeta\sin\varphi\sin\vartheta\cos\vartheta - \\ -\rho(1 - \sin^2\varphi\sin^2\vartheta) + \frac{\partial\tilde{\Pi}}{\partial\rho} = \lambda\frac{\partial F}{\partial\rho} = \frac{2\rho\lambda}{1 - e^2}$$

where $\tilde{\Pi}$ is dimensionless gravitational potential of the asteroid

λ is Lagrange's multiplier,

$\lambda<0$ correspond to motions along internal surface of the ellipsoid, (the 'constrained' motion, the cable is tensed)

$\lambda=0$ correspond to motions inside the ellipsoid (the 'free' motion or 'the station has lost the constraint', the cable is weakened)

$\lambda>0$ is impossible

Lagrangian, Jacobi's integral

$$L = T_2 + T_1 + T_0 - \Pi(\rho, \zeta)$$

$$J = T_2 - T_0 + \Pi(\rho, \zeta) = \text{const}$$

Integrable cases

1) $\vartheta = 0$ There is a cyclic variable φ

2) $\vartheta = \pi/2; \quad y = 0$

Equilibria types

$$\cos\varphi \sin\vartheta\,(\rho \sin\varphi \sin\vartheta + \zeta \cos\vartheta) = 0$$

Coplanar equilibria $\varphi = \pm\pi/2$

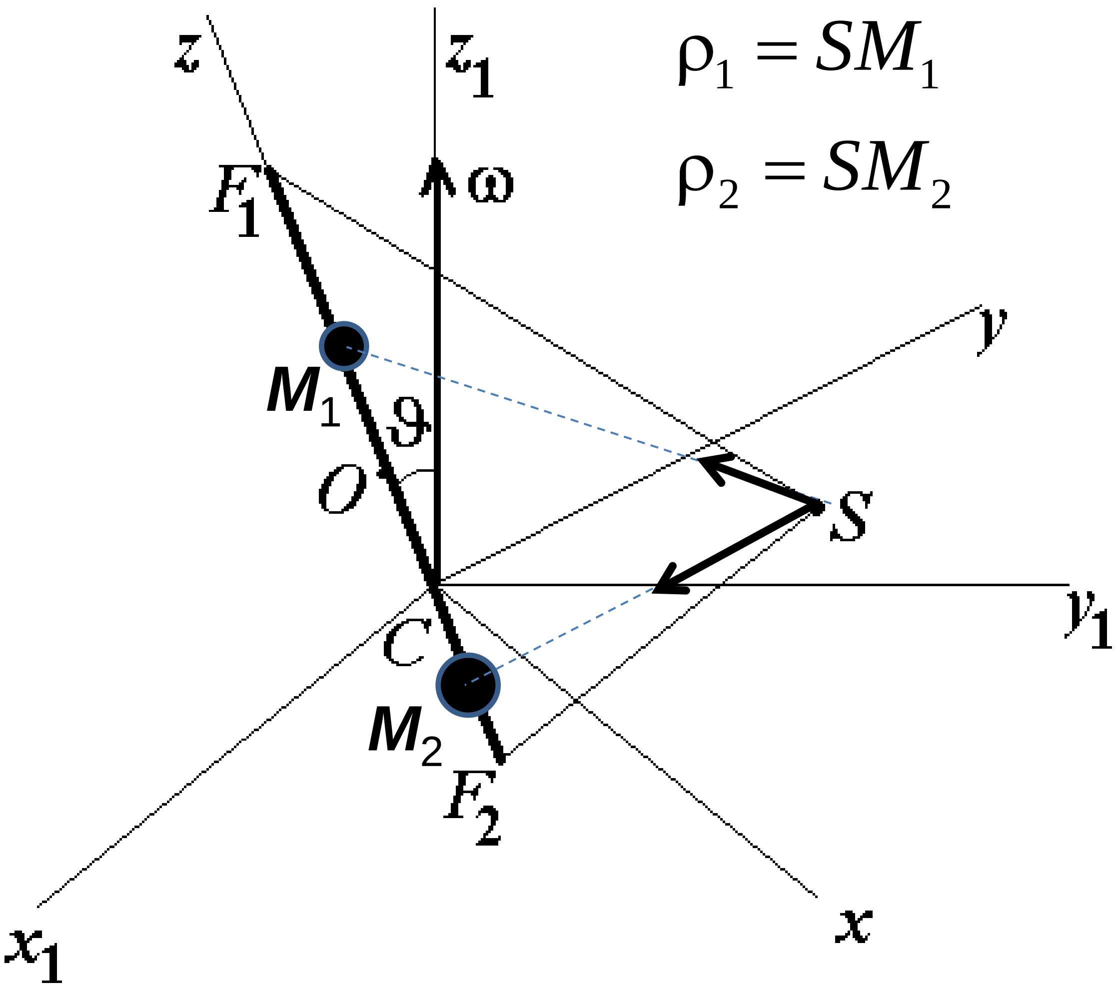
They belong to the plane composed by Cz and Cz_1

Triangular equilibria $\rho\sin\varphi + z\cot\vartheta = 0$

They belong to the plane Cx_1y_1

The two-particles model

$$\widetilde{\Pi} = -\alpha k^3 e^3 \left(\frac{\mu}{\rho_1} + \frac{1-\mu}{\rho_2} \right)$$



$$\rho_1 = SM_1$$

$$\rho_2 = SM_2$$

Additional parameters

$$k = M_1M_2 / OF_1$$

$$\mu = \frac{m_1}{m_1 + m_2}$$

$$\alpha = \frac{G(m_1 + m_2)}{\omega^2 |M_1M_2|^3}$$

$$|d| < k < 2$$

$$0 < \mu \le 1/2$$

$$\alpha > 0$$

Integrable case $\vartheta=0$

The cyclic integral $\rho^2(\dot{\varphi}+1)=c$

Reduced motion equations

$$\ddot{\zeta}-2\lambda(\zeta-de)+\frac{\partial\widetilde{\Pi}}{\partial\zeta}=0\,, \qquad \ddot{\rho}-\frac{c^2}{\rho^3}-\frac{2\lambda\rho}{1-e^2}+\frac{\partial\widetilde{\Pi}}{\partial\rho}=0$$

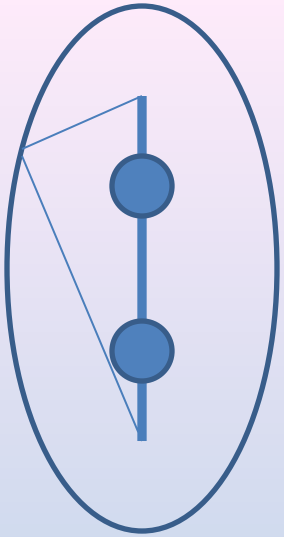
New variable for motion on the ellipsoid surface

$$\rho=\sqrt{1-e^2}\sin\gamma,\;\; \zeta=de+\cos\gamma, \quad 0<\gamma<\pi$$

Jacobi's integral

$$\tfrac{1}{2}(1-e^2\cos\gamma)\dot{\gamma}^2+\widetilde{\Pi}+\frac{c^2}{2(1-e^2)\sin^2\gamma}=h=\text{const}$$

$$\lambda\le 0 \quad\Leftrightarrow\quad \dot{\gamma}^2-\frac{\sin\gamma}{\sqrt{1-e^2}}\frac{\partial\widetilde{\Pi}}{\partial\rho}-\cos\gamma\frac{\partial\widetilde{\Pi}}{\partial\zeta}+\left(\frac{c}{(1-e^2)\sin\gamma}\right)^2\ge 0$$

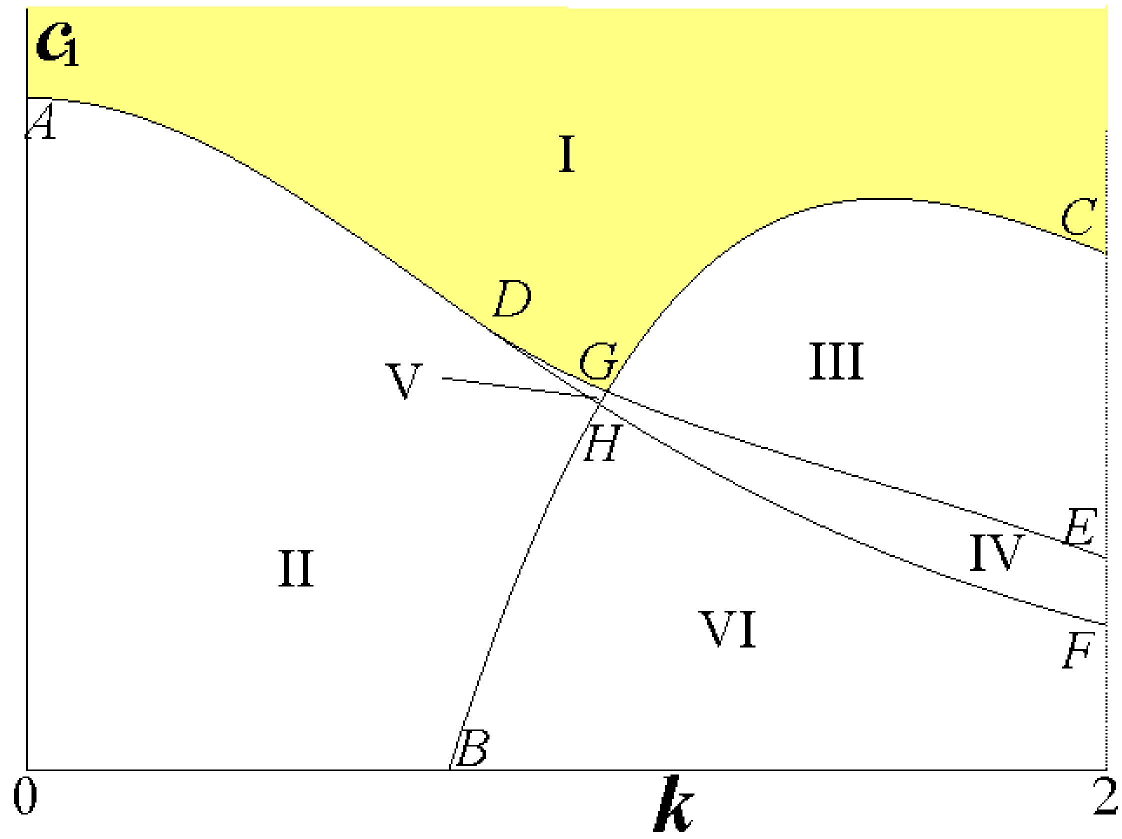


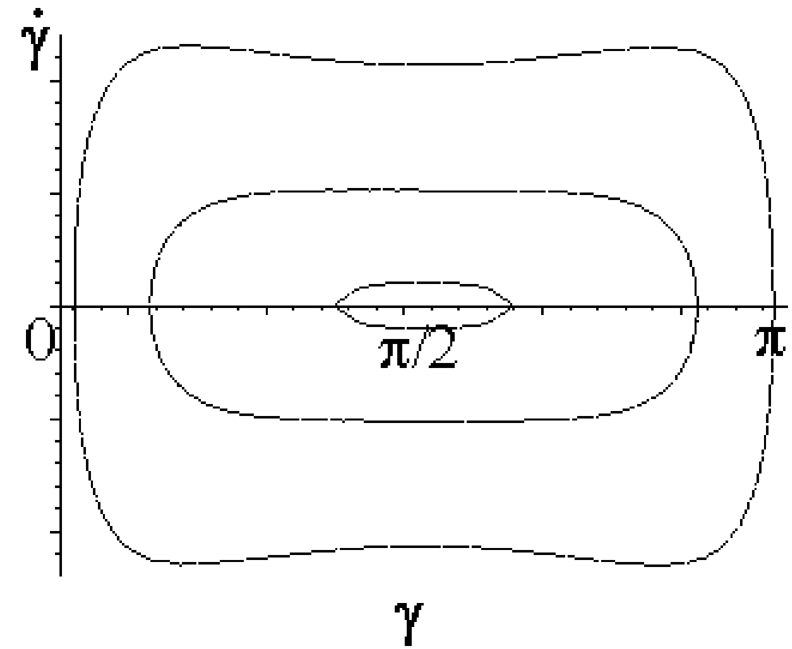
Integrable case $\vartheta=0$

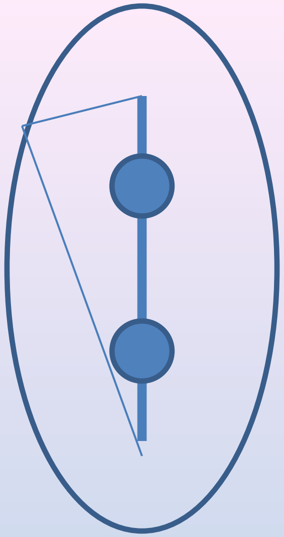
The two-particles model

The full-symmetric case $d=0$, $\mu=1/2$

$$c_1 = c^2/(\alpha k^3)$$







Integrable case $\vartheta=0$

The two-particles model

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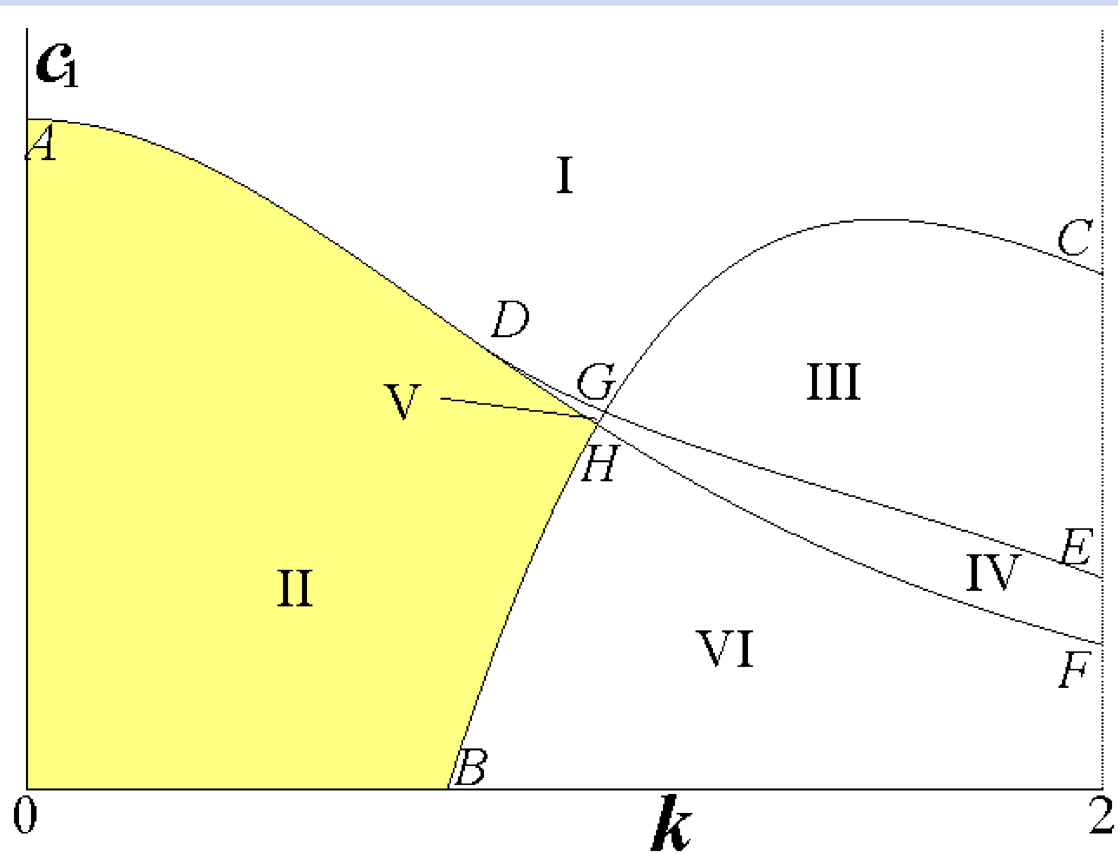

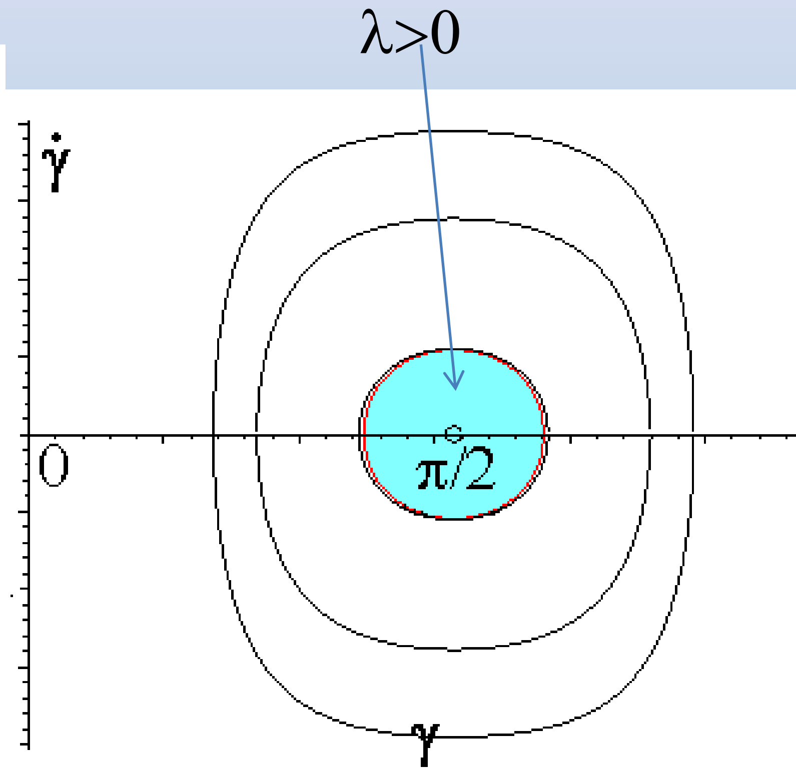

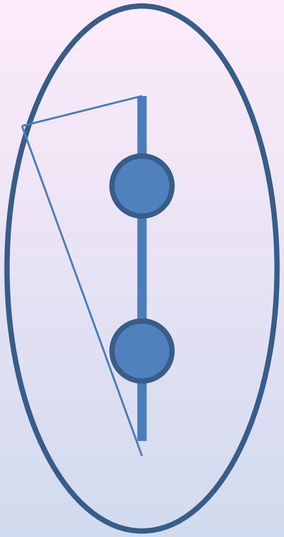

Integrable case $\vartheta=0$

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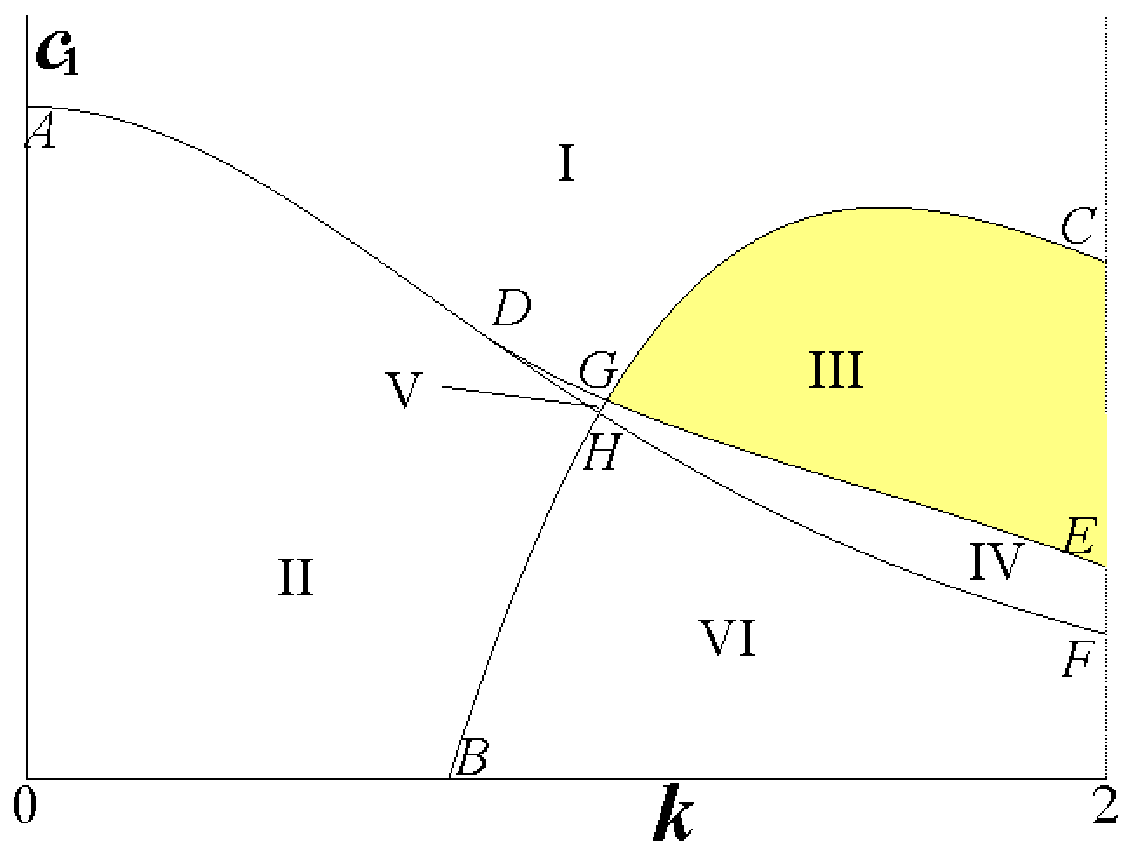
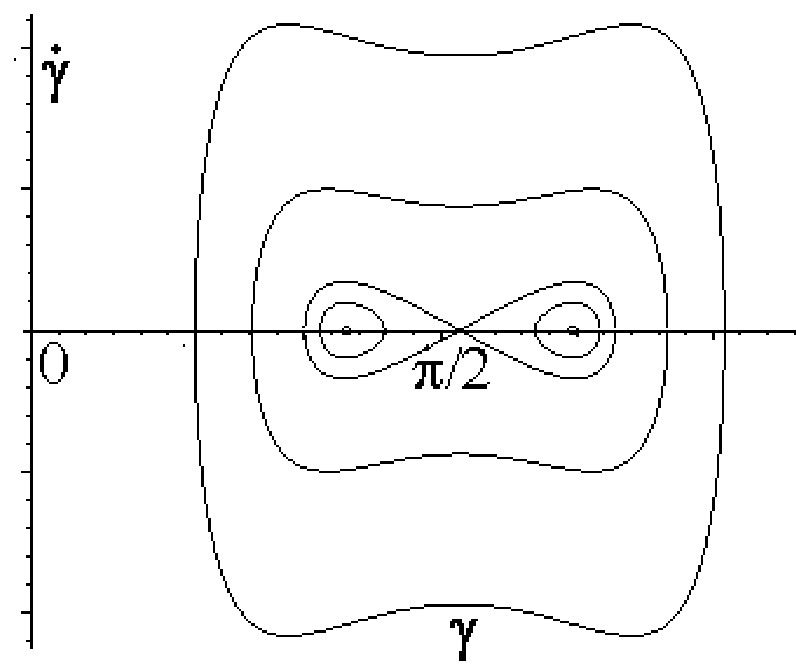
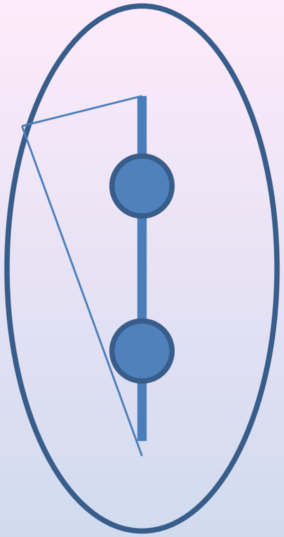




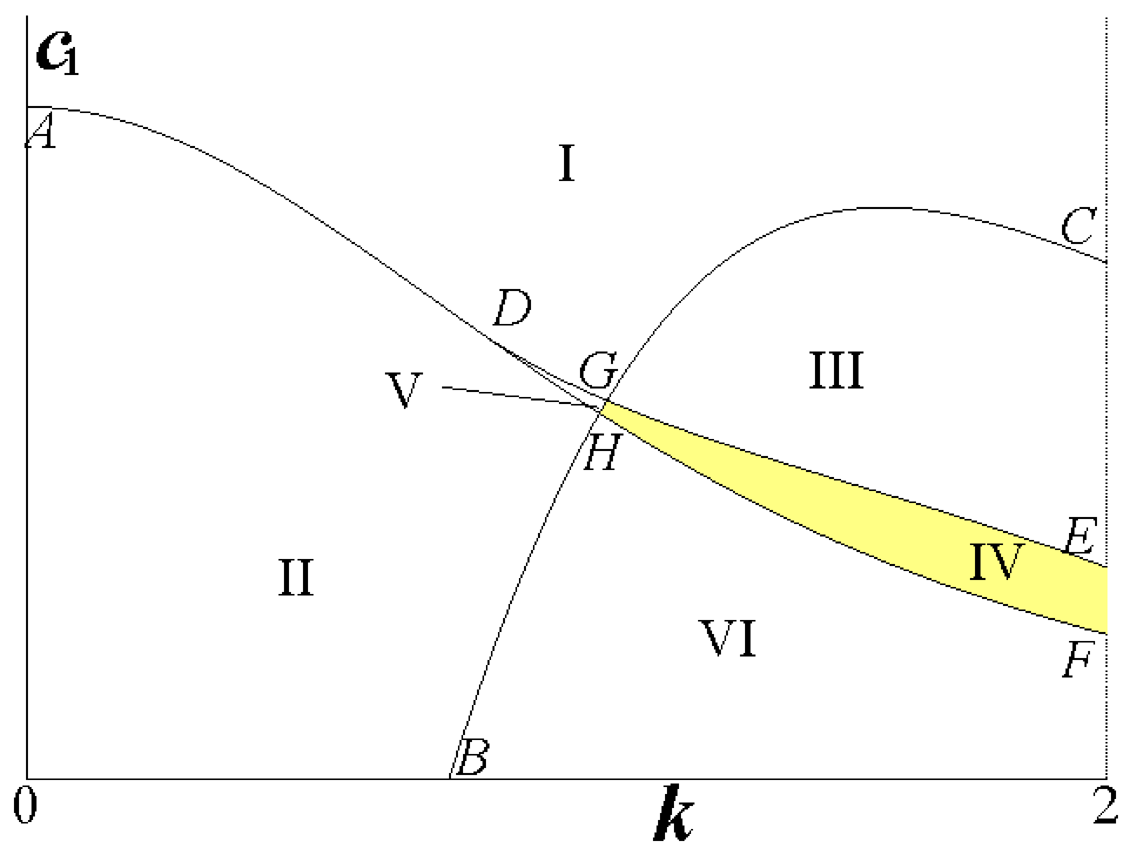


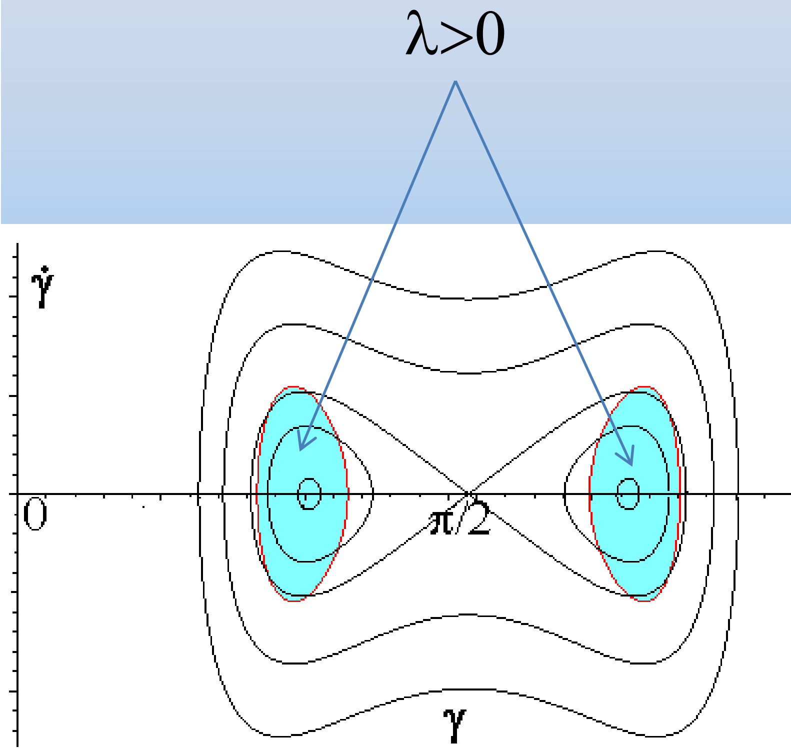
Integrable case $\vartheta=0$

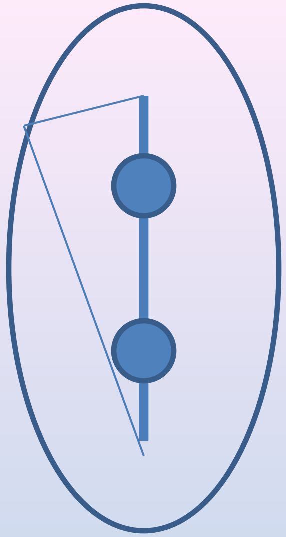
The two-particles model

The full-symmetric case $d=0$, $\mu=1/2$

$$c_1 = c^2/(\alpha k^3)$$





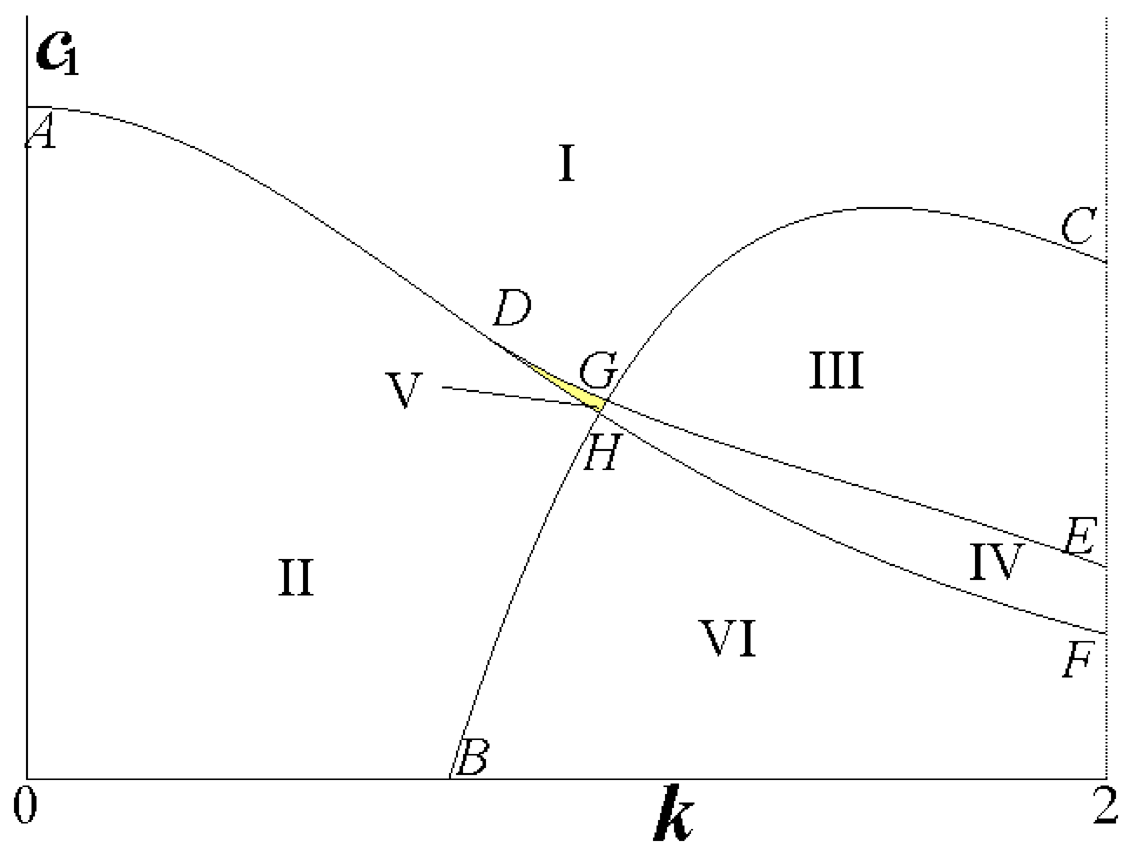


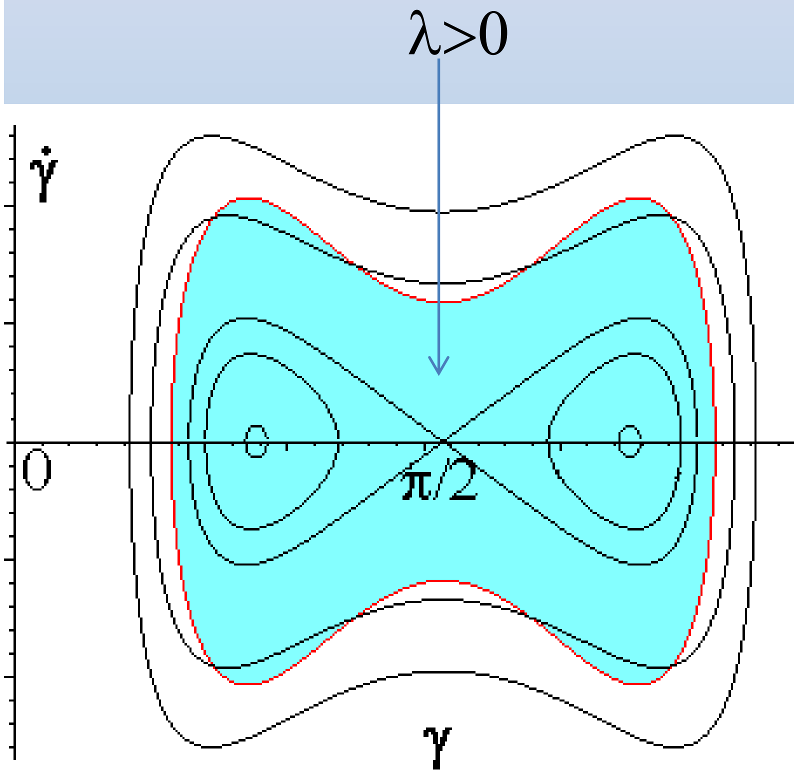
Integrable case ϑ=0

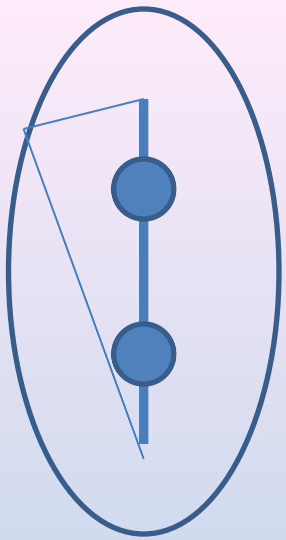
The two-particles model

The full-symmetric case d=0, μ=1/2

$$c_1 = c^2/(\alpha k^3)$$





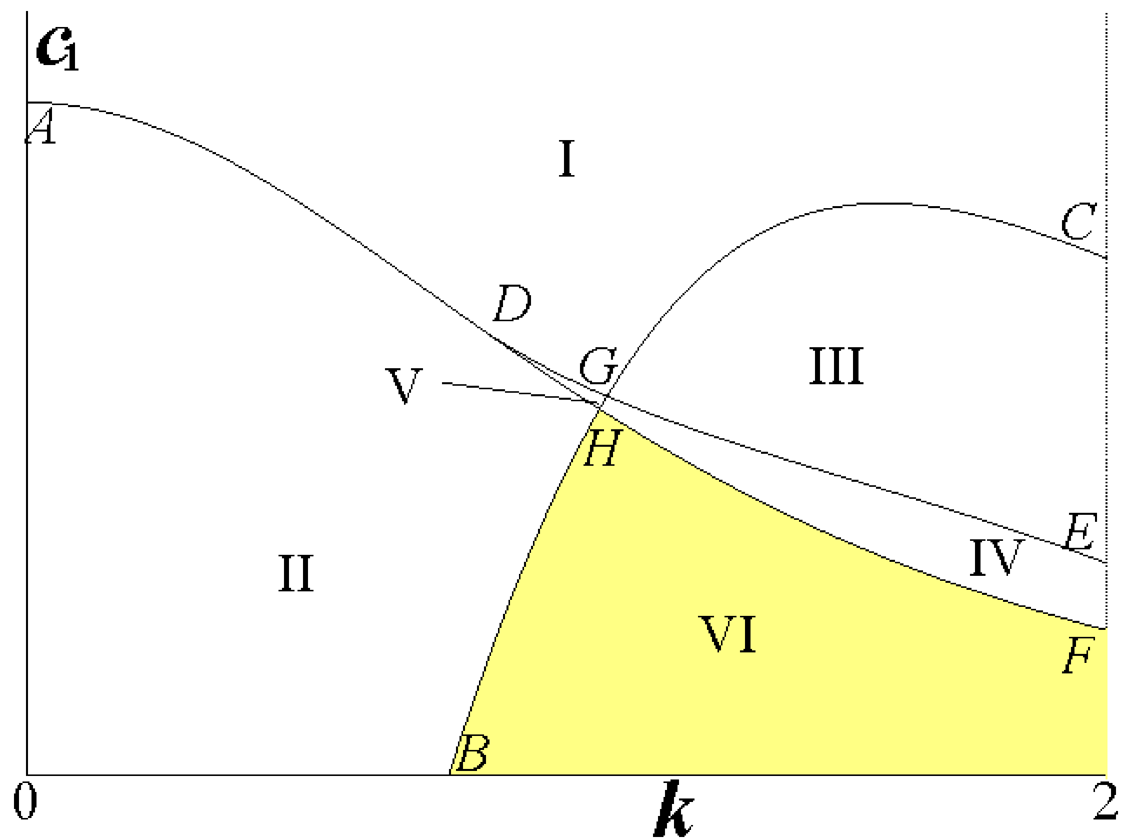


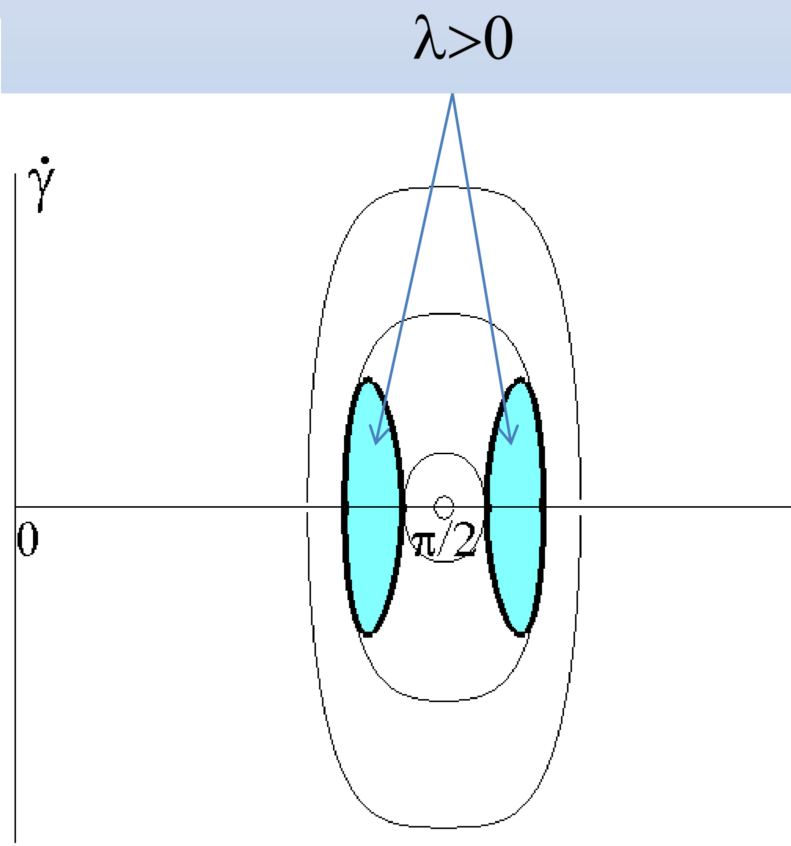
Integrable case $\vartheta=0$

The two-particles model

The full-symmetric case $d=0$, $\mu=1/2$

$$c_1 = c^2/(\alpha k^3)$$





Integrable case $\vartheta=\pi/2$, $y=0$

Motion equations

$$\ddot{\xi} + 2\dot{\zeta} - \xi + \frac{\partial\widetilde{\Pi}}{\partial\xi} = \frac{2\lambda\xi}{1-e^2} \qquad \ddot{\zeta} - 2\dot{\xi} - \zeta + \frac{\partial\widetilde{\Pi}}{\partial\zeta} = 2\lambda(\zeta - ed)$$

New variable for motion along the ellipse

$$\rho = \sqrt{1-e^2}\sin\gamma, \;\; \zeta = de + \cos\gamma, \qquad 0 \le \gamma < 2\pi$$

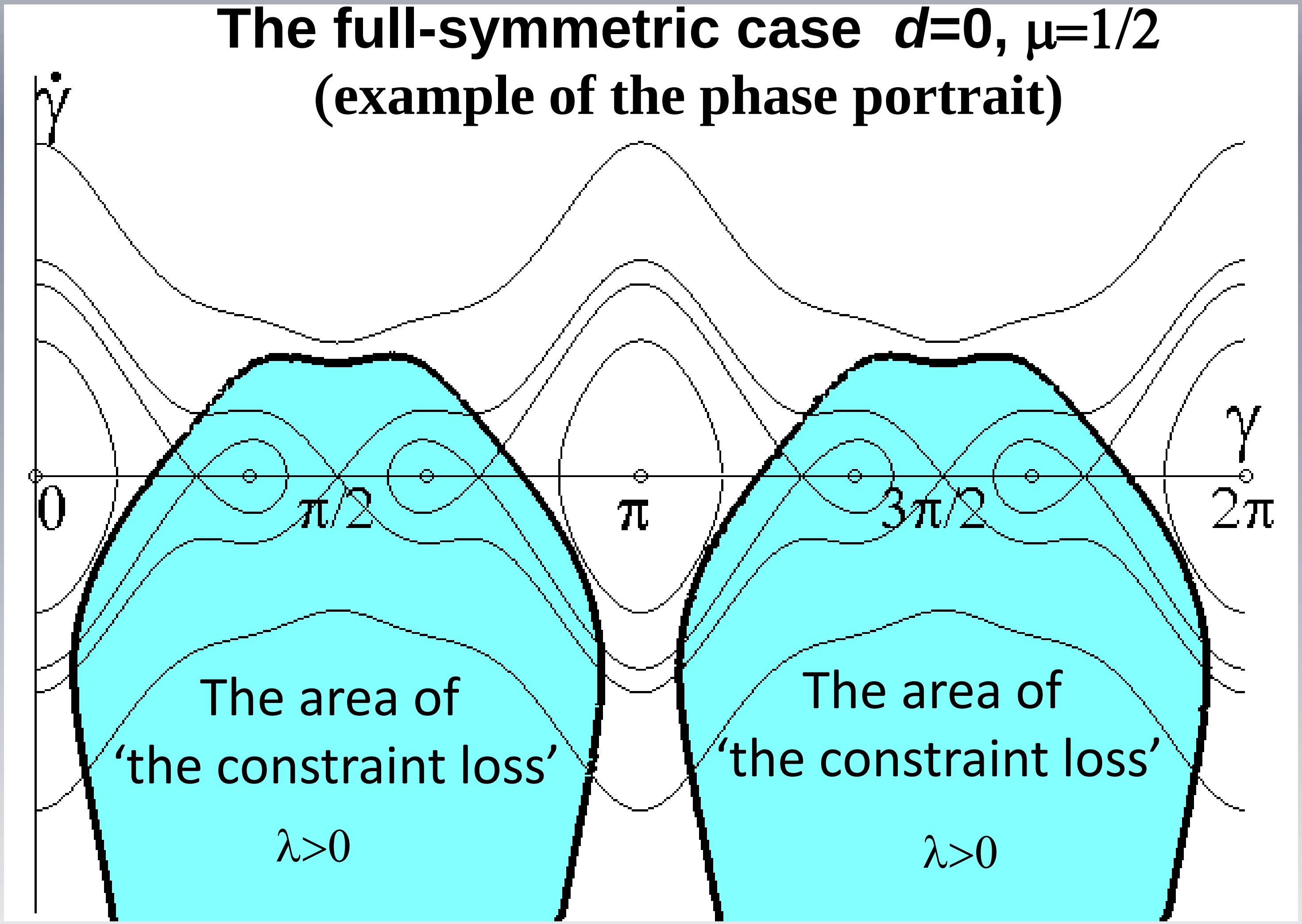
Jacobi's integral

$$\frac{1}{2}(1 - e^2\cos^2\gamma)\dot{\gamma}^2 + \widetilde{\Pi} - de\cos\gamma - \frac{1}{2}e^2\cos^2\gamma = h = \text{const}$$

$\lambda \le 0 \Leftrightarrow$

$$\dot{\gamma}^2 + \frac{2(1-e^2\cos^2\gamma)}{\sqrt{1-e^2}}\dot{\gamma} + de\cos\gamma + 1 - \frac{\sin\gamma}{\sqrt{1-e^2}}\frac{\partial\widetilde{\Pi}}{\partial\xi} - \cos\gamma\frac{\partial\widetilde{\Pi}}{\partial\zeta} \ge 0$$

Integrable case $\vartheta=\pi/2$, $y=0$



The system phase portrait is complex but only equilibria in the ellipse vertexes really exist

Triangular equilibria

$$\frac{\partial\tilde{\Pi}}{\partial\zeta} + 2\lambda d - (1+2\lambda)\zeta = 0$$

$$\frac{\partial\tilde{\Pi}}{\partial\rho} - \left(1 + \frac{2\lambda}{1-e^2}\right)\rho = 0$$

The cable doesn't weaken if

$$\lambda \leq 0 \quad \Leftrightarrow \quad \frac{2\zeta - ke(1+2\mu)}{e\zeta + d(1-e^2)} \leq 0$$

Let the two-particle model be applicable to the asteroid. In this case using A.P.Ivanov theorem it can be shown that all triangular equilibria, excepting triangular libration points of V.V.Beletsky GRC3BP, are **stable** if *motions along the cable are forbidden*

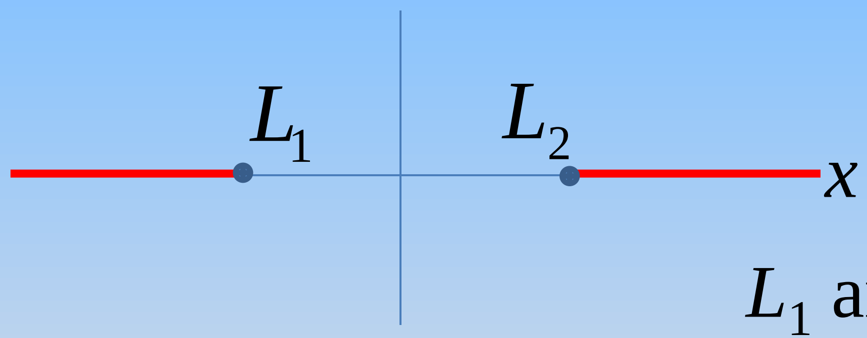
Fixing an asteroid, i.e. fixing values of k, μ, α, ϑ, d we construct sets of triangular equilibria for the varied cable length. Following situations are possible

Triangular equilibria for the two-particles model

The 'full-symmetric case' $\mu = 1/2, \qquad d = 0$

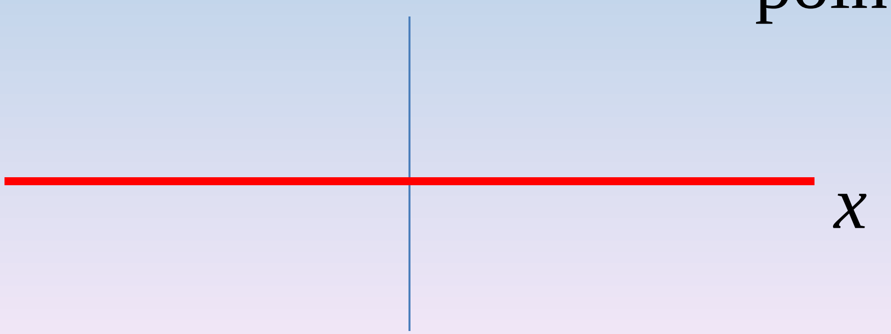
2 equilibria $z = 0; \quad \rho = \sqrt{1 - e^2}; \quad \varphi = 0, \pi; \quad r_1 = r_2 \geq \alpha^{1/3} e k$

are ***unstable*** for any values of parameters



$\alpha > 1/8$

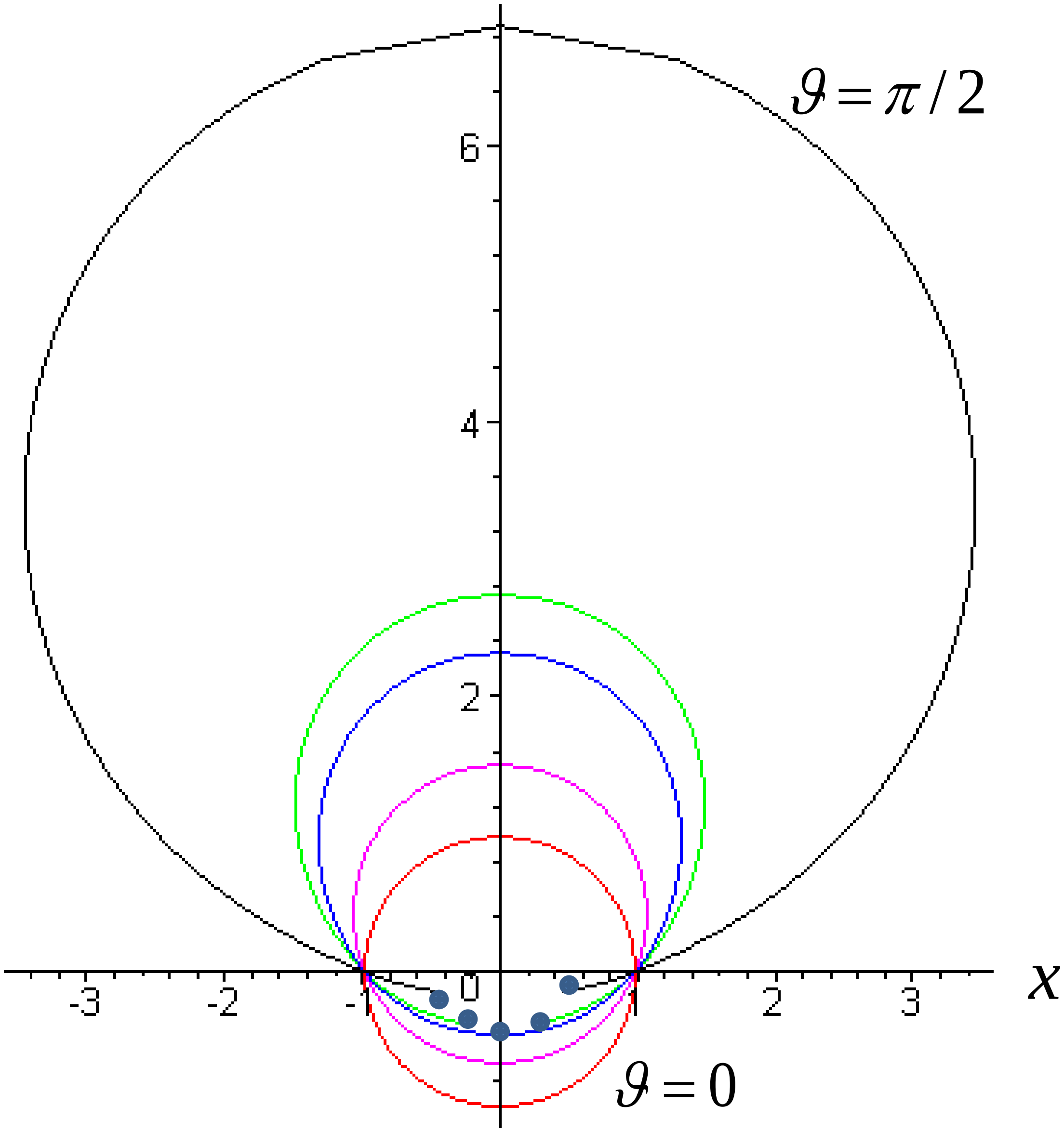
L_1 and L_2 are triangular libration points of GR3BP



$\alpha \leq 1/8$

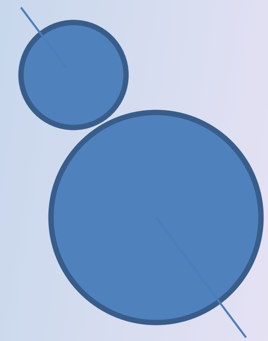
Triangular equilibria for the two-particles model

Common cases $\mu < 1/2, \quad d > 0 \quad \alpha > 1/8$



2 or 4 equilibria for any admissible e

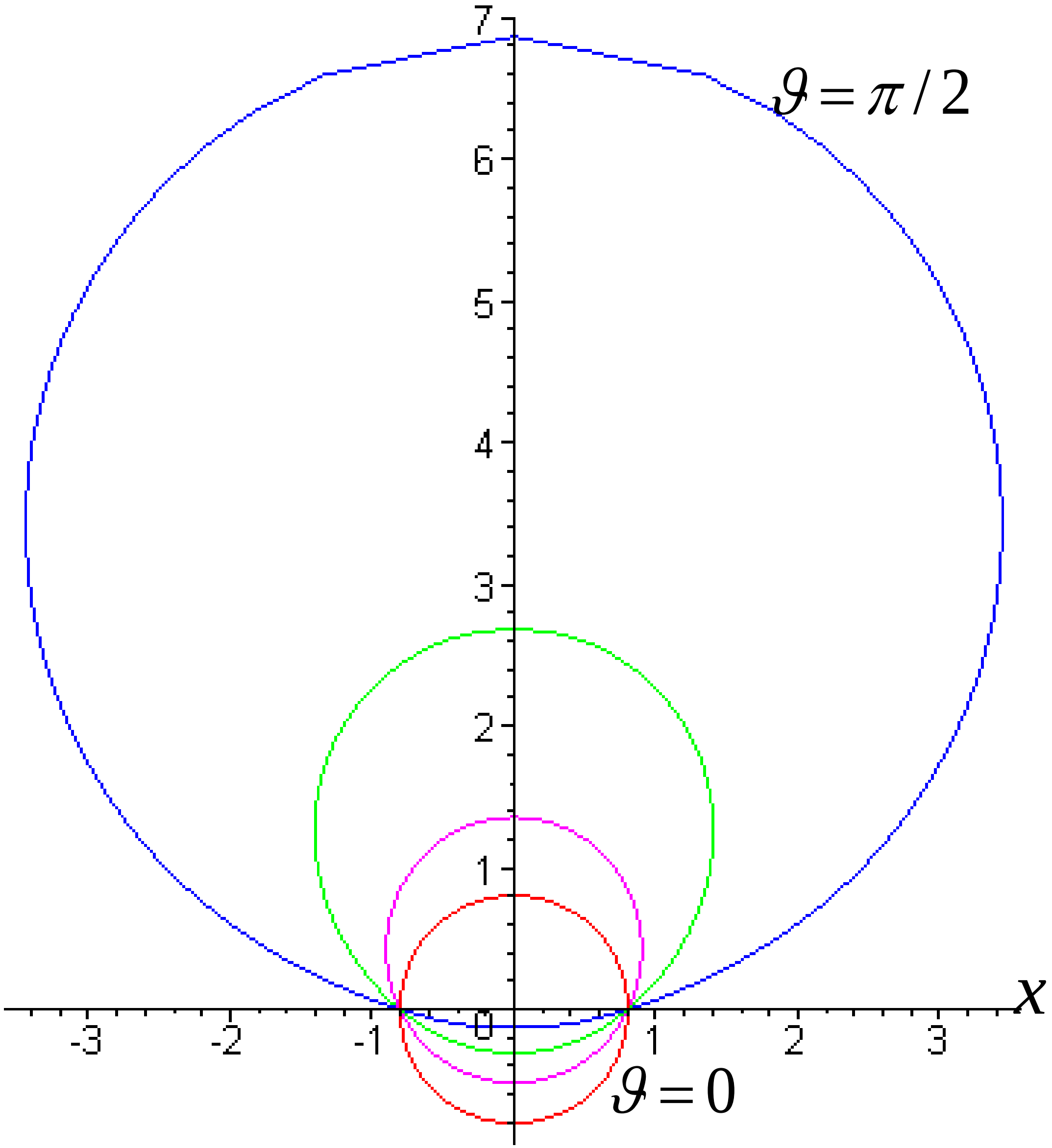
Each color corresponds to a fixed value of ϑ



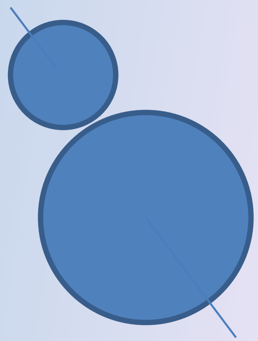
Fat blue points are the triangular libration points of Beletsky's problem

Triangular equilibria for the two-particles model

Common cases $\mu < 1/2,\quad d > 0\quad \alpha < 1/8$



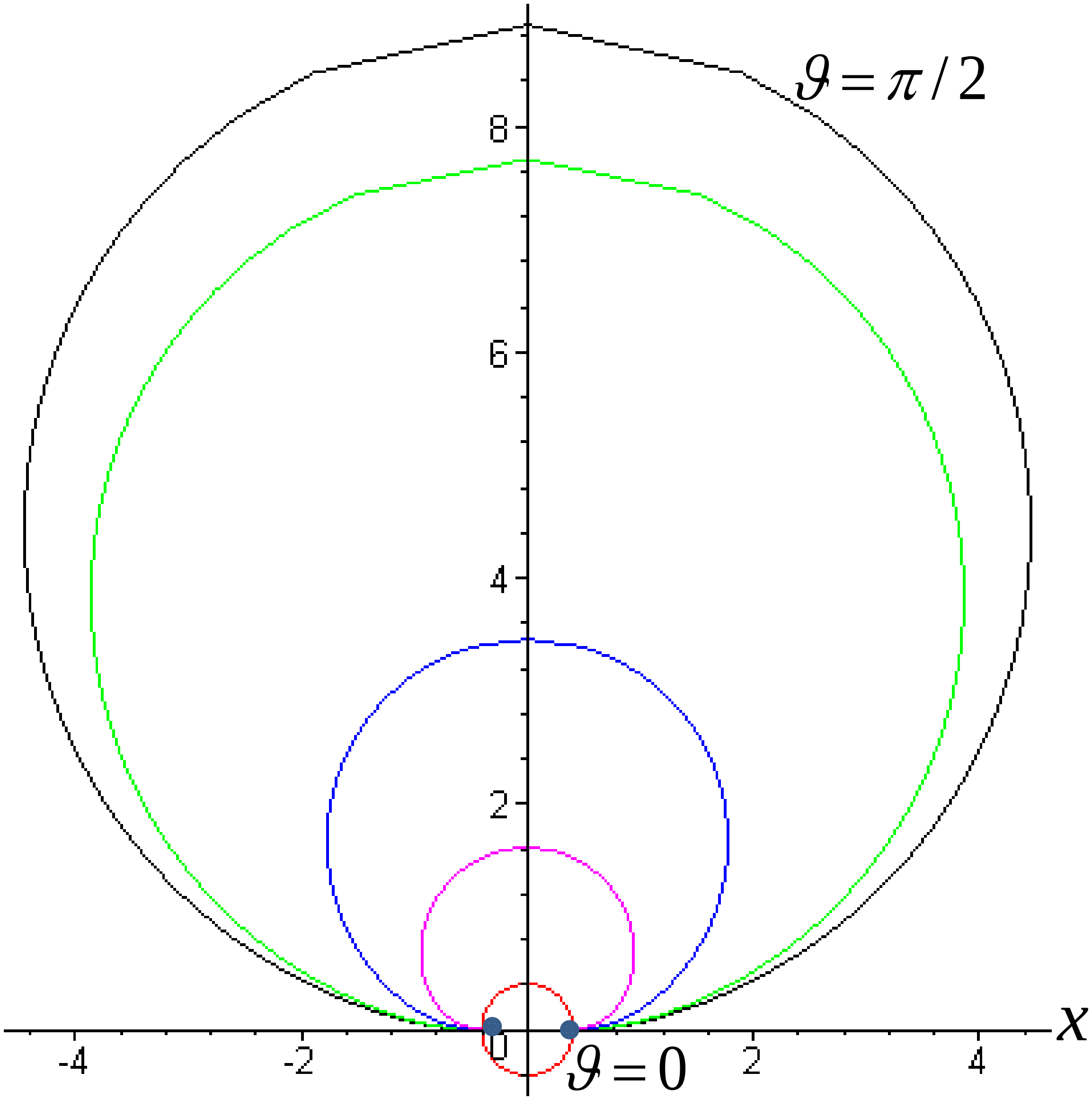
4 equilibria for any admissible *e*



Each color corresponds to a fixed value of ϑ

Triangular equilibria for the two-particles model

The symmetric dumbbell case $\mu = 1/2, \quad d > 0 \quad \alpha > 1/8$



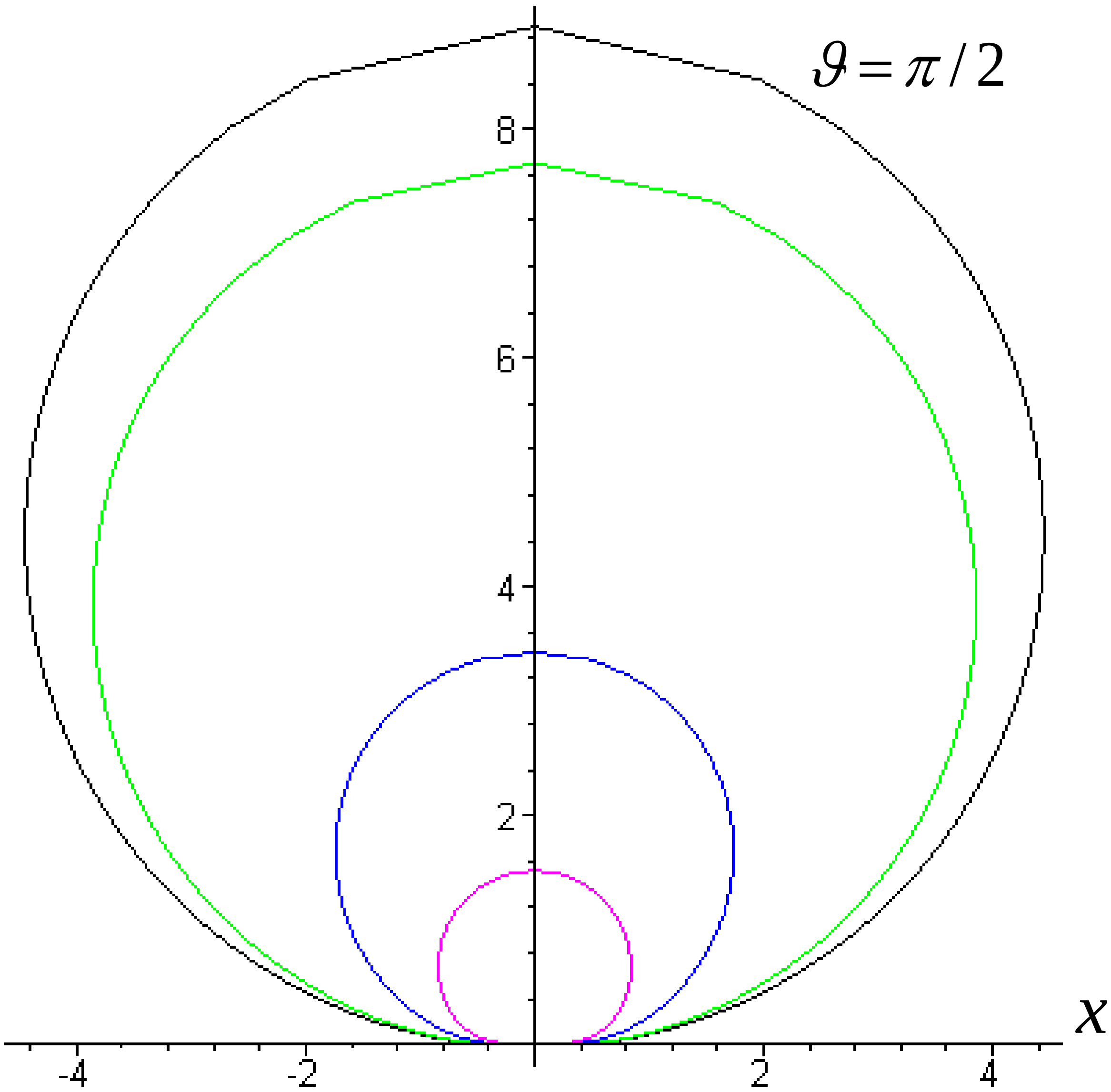
2 equilibria for any admissible *e*

Each color corresponds to a fixed value of ϑ

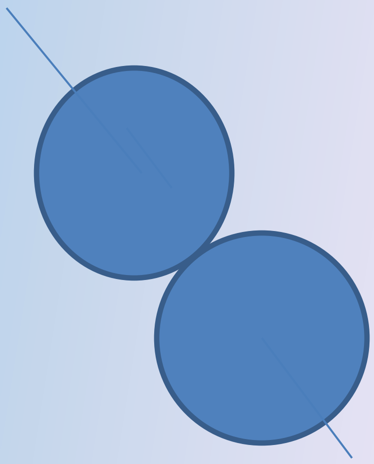
Red is the ring of triangular libration points

Triangular equilibria for the two-particles model

The symmetric dumbbell case $\mu = 1/2, \quad d > 0 \quad \alpha < 1/8$



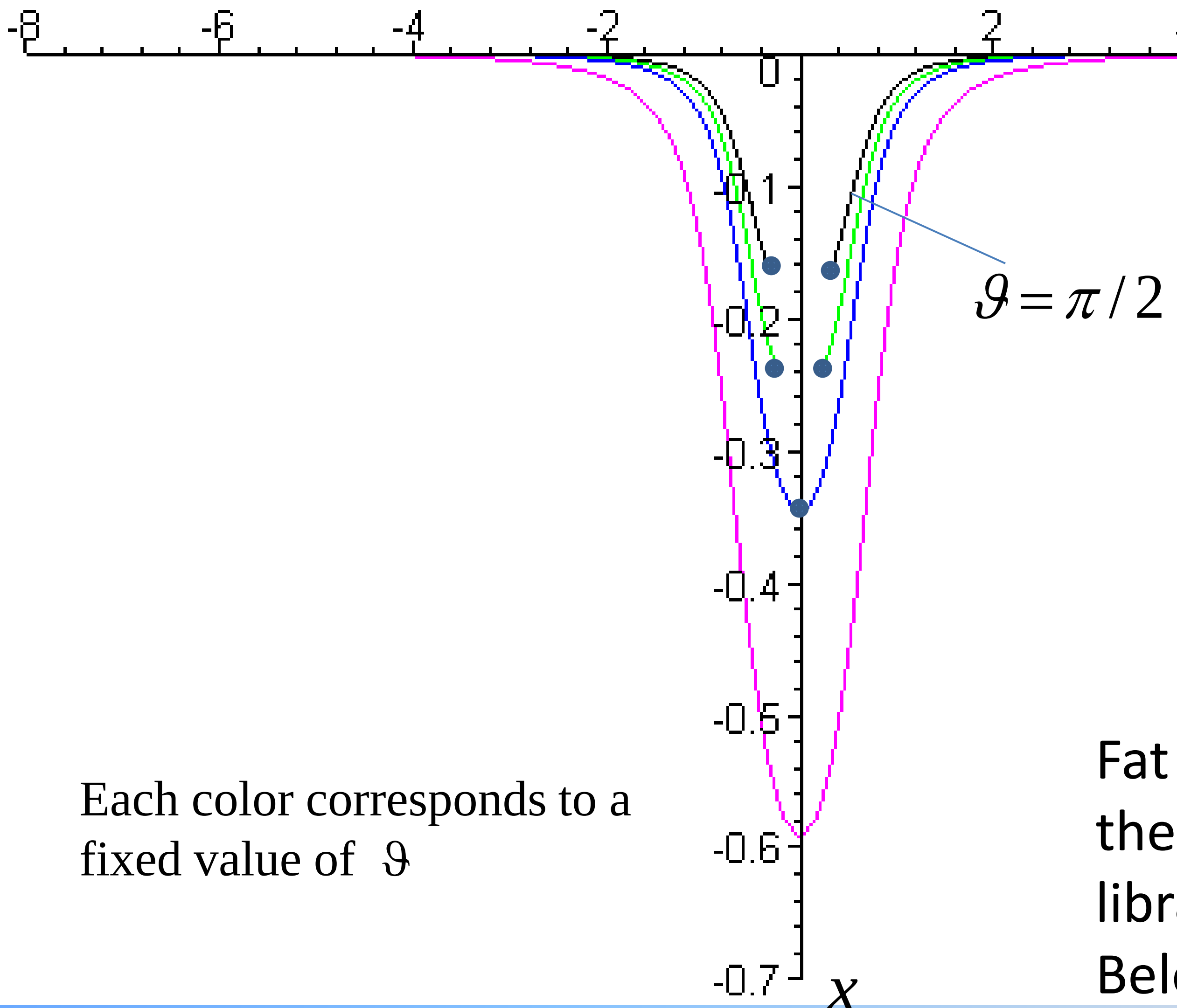
2 equilibria for any admissible *e*



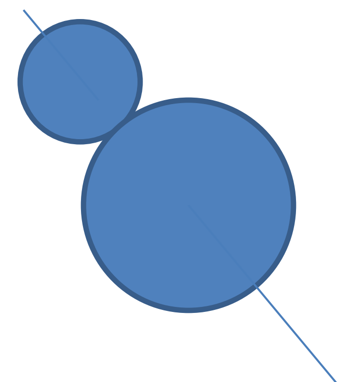
Each color corresponds to a fixed value of ϑ

Triangular equilibria for the two-particles model

The symmetric elevator case $\mu < 1/2, \quad d = 0 \quad \alpha > 1/8$



2 equilibria for any admissible *e*

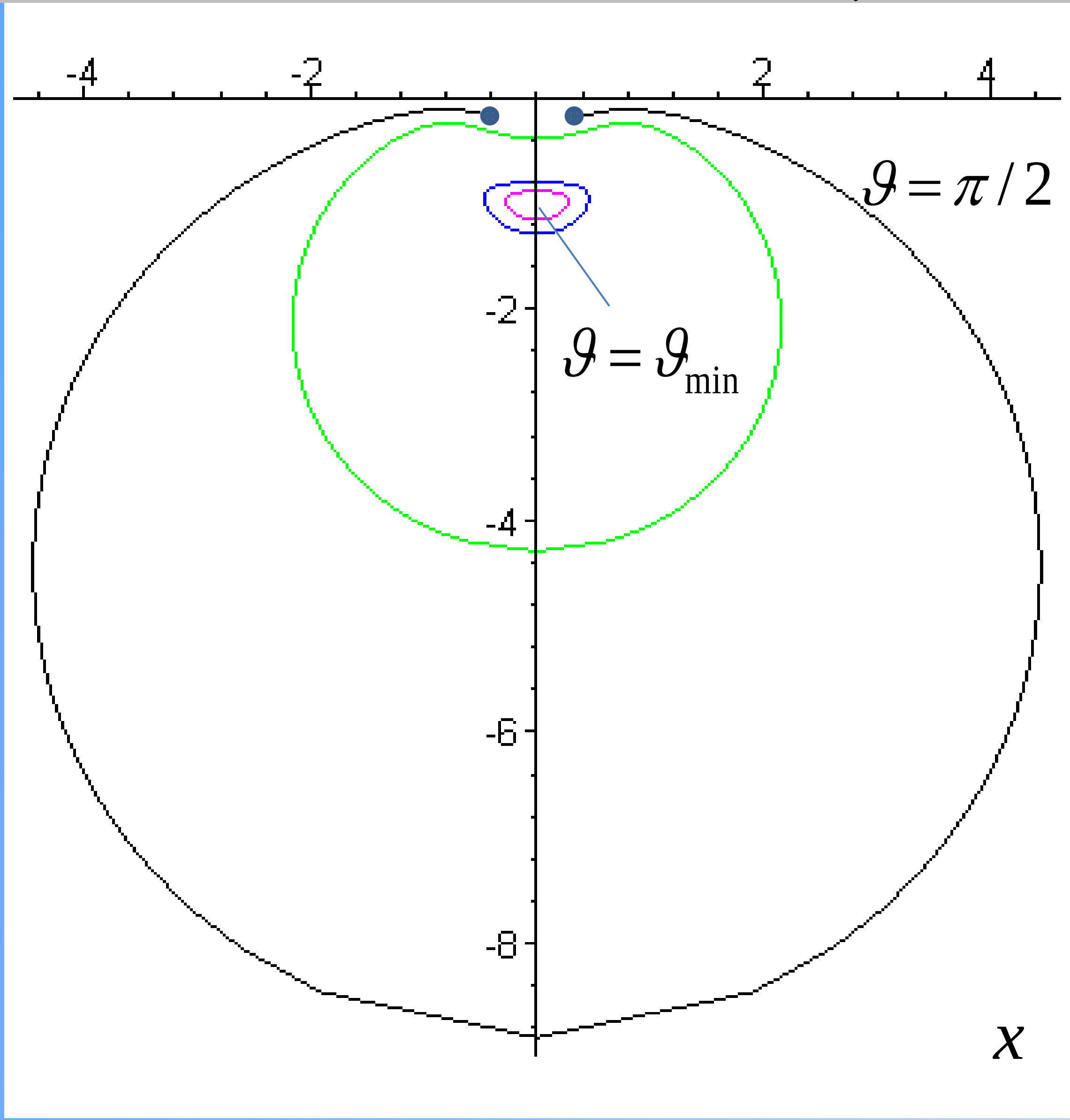


Each color corresponds to a fixed value of ϑ

Fat blue points are the triangular libration points of Beletsky’s problem

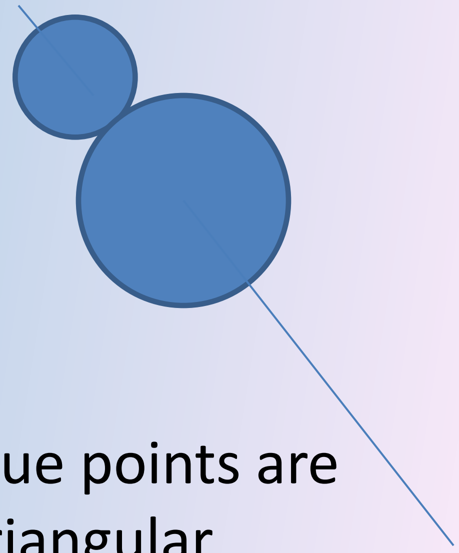
Triangular equilibria for the two-particles model

'The big tower' case $\mu < 1/2, \quad d < 0 \quad \alpha > 1/8$



2 equilibria for any admissible *e*

Each color corresponds to a fixed value of ϑ



Fat blue points are the triangular libration points of Beletsky's problem

Instead of **conclusions** there are new **problems**

1) Studying of integrability and searching for non-trivial integrable cases
2) Complete studying of equilibria stability
3) Studying of perturbed system

4) Previous problems for other models of gravitation

5) Formulation of the problem for three-axial asteroids
6) Previous problems for the formulated problem

7)….∞) etc. etc.

I am often asked when I think the first space elevator might be built. My answer has always been: about 50 years after everyone has stopped laughing. Maybe I should now revise it to 25 years. (*Arthur C. Clarke*)

The classical space elevator length with the opposite mass is more than 36000 km

The asteroid elevator length is about tens or hundreds km

What of these projects to realize easier?

Discussions with prof. S.Ya.Stepanov was the reason to formulate this problem.

The author is grateful to prof. V.V.Beletsky, prof. A.A.Burov, prof. Yu.F.Golubev and prof. I.I.Kosenko for notes, comments and useful discussions.

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Merci beaucoup pour votre attention

Thank you for your attention