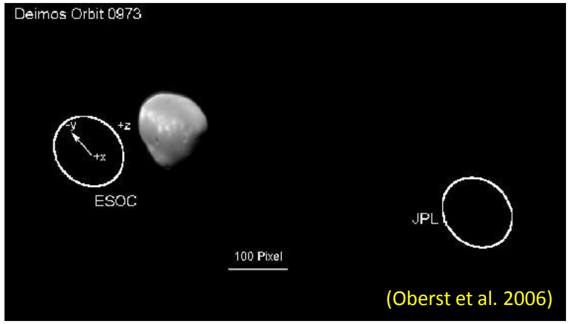
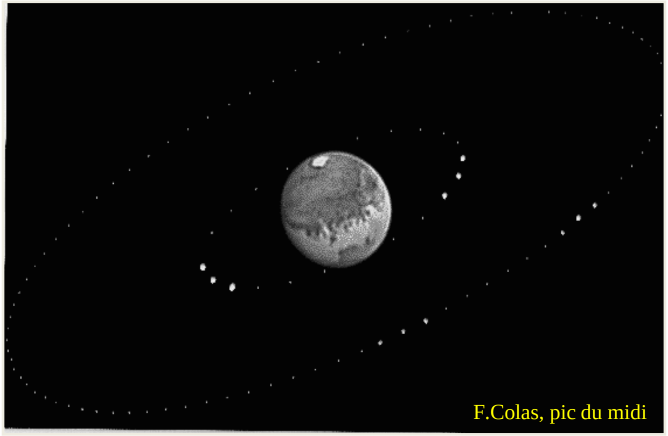
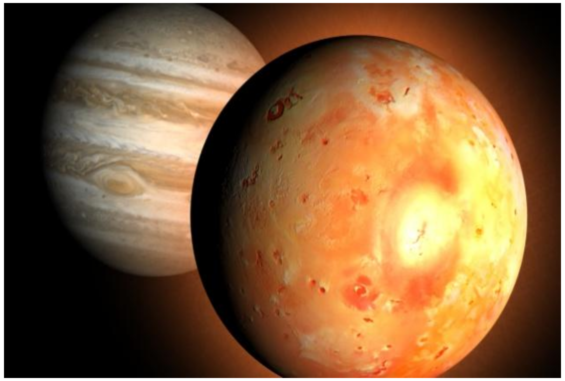
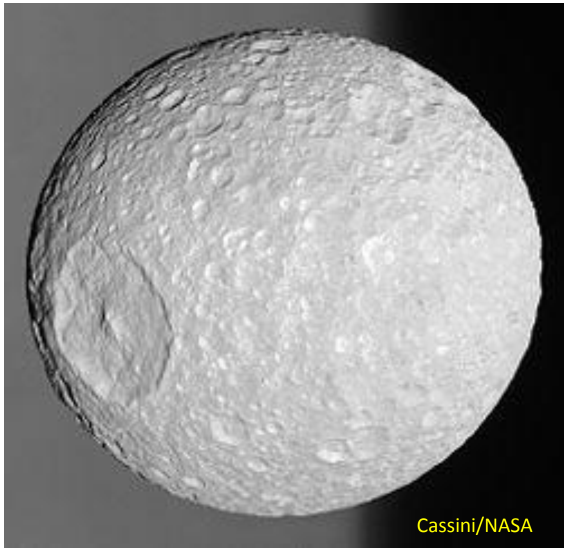
Orbit determination of natural satellites

V. Lainey (IMCCE-Paris Observatory)









Outline:

1- Comparison between spacecraft and natural satellite orbitography

2- Example of the Mars system

3- The ESPaCE network (FP7)

1- Comparison between spacecraft and natural satellite orbitography

1- Comparison between spacecraft and natural satellite orbitography

→ *We follow exactly the same methodology…*

Method in three steps:

1- modeling of the dynamical system

2- gathering the observations

3- fitting the model to the observations

Today, this kind of work is done completly numerically

S/C: GINS, DPODP, GEODYN, …

SAT: NOE, …

1- Comparison between spacecraft and natural satellite orbitography

Step 1: Modeling of the dynamical system

Equations of motion

$$\ddot{\vec{r}}_i = -G(m_0 + m_i)\left(\frac{\vec{r}_i}{r_i^{3}} - \nabla_i U_{\bar{i}\hat{0}} + \nabla_0 U_{\bar{0}\hat{i}}\right) + \sum_{j=1,\, j\neq i}^{N} G m_j \left(\frac{\vec{r}_j - \vec{r}_i}{r_{ij}^{3}} - \frac{\vec{r}_j}{r_j^{3}} - \nabla_j U_{\bar{j}\hat{i}} + \nabla_i U_{\bar{i}\hat{j}} + \nabla_j U_{\bar{j}\hat{0}} - \nabla_0 U_{\bar{0}\hat{j}}\right)$$

$$+ \frac{(m_0 + m_i)}{m_i m_0}\left(\vec{F}_{\bar{i}\hat{0}}^{\,T} - \vec{F}_{\bar{0}\hat{i}}^{\,T}\right) - \frac{1}{m_0} \sum_{j=1,\, j\neq i}^{N} \left(\vec{F}_{\bar{j}\hat{0}}^{\,T} - \vec{F}_{\bar{0}\hat{j}}^{\,T}\right)$$

1- Comparison between spacecraft and natural satellite orbitography

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N-body problem

1- Comparison between spacecraft and natural satellite orbitography

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N-body problem

Extended gravity fields

1- Comparison between spacecraft and natural satellite orbitography

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N-body problem

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N-body problem
Extended gravity fields
Tidal effects
+ Few relativistic terms

1- Comparison between spacecraft and natural satellite orbitography

Step 1: Modeling of the dynamical system

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N-body problem
Extended gravity fields
Tidal effects
+ Few relativistic terms

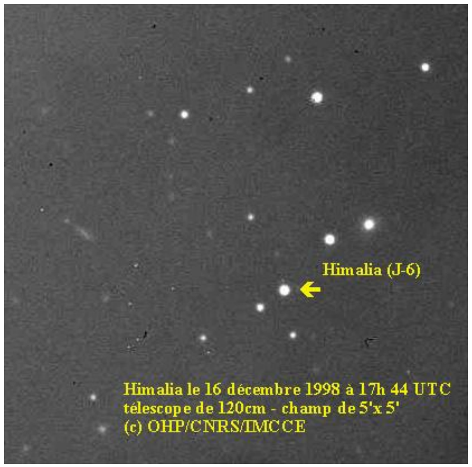
Variational equations

$$\frac{\partial}{\partial c_l}\left(\frac{d^2\vec{r}_i}{dt^2}\right) = \frac{1}{m_i}\left[\sum_j\left(\frac{\partial\vec{F}_i}{\partial\vec{r}_j}\frac{\partial\vec{r}_j}{\partial c_l} + \frac{\partial\vec{F}_i}{\partial\dot{\vec{r}}_j}\frac{\partial\dot{\vec{r}}_j}{\partial c_l}\right) + \frac{\partial\vec{F}_i}{\partial c_l}\right]$$

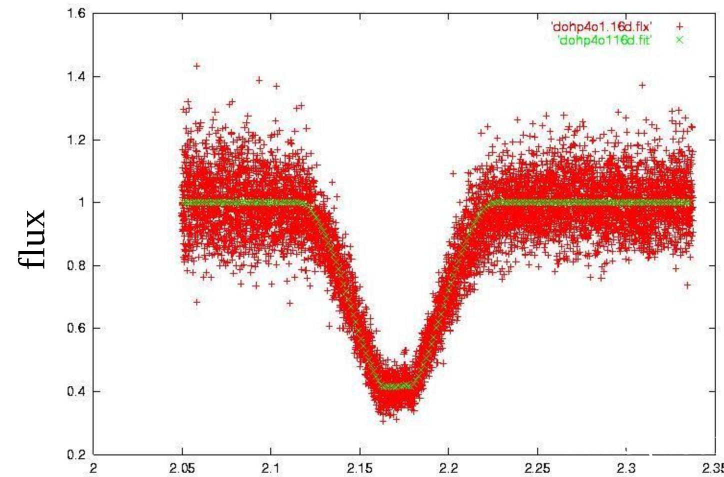
1- Comparison between spacecraft and natural satellite orbitography

Step 2: gathering the observations

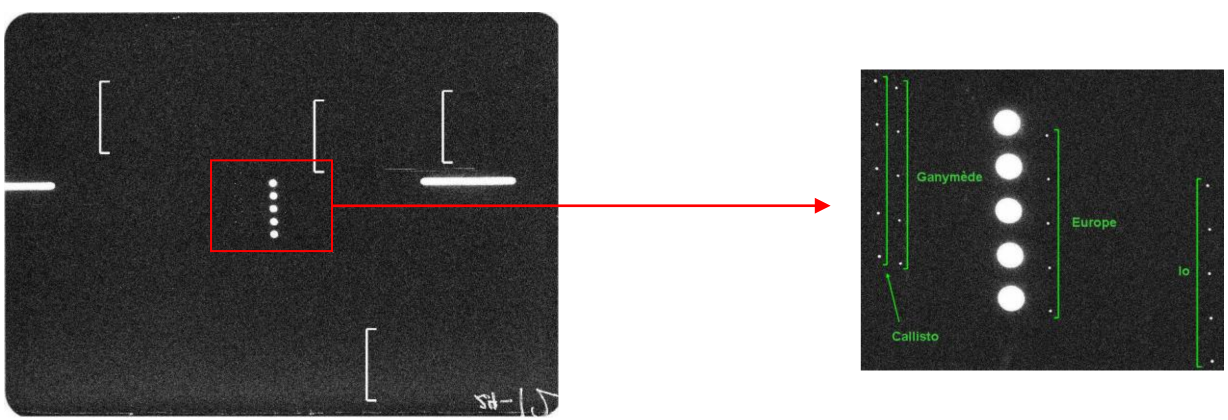
Direct astrometric measurement



Undirect astrometric measurement (photometry)



Astrometric remeasurement (benefit from modern scanning machine)



Step 3: Fitting the model to the observations

Step 2 (observations)

Step 1 (Integration of the equations of motion + variational equations)

Step 3 (fitting the model)
Approximation by a linear system
=> least squares method

$$\left(O - C \right) = \sum_{l=1}^{N} A_i(c_l) \, . \, \Delta c_l + O((\Delta c_l)^2)$$

unknown

1- Comparison between spacecraft and natural satellite orbitography

Two important points:

1 - S/C have polar orbit while SAT have equatorial orbits

→ SAT and S/C are sensitive to different harmonics of the primary's gravity field

2 - S/C data are regularly splitted into arcs (wheel off loading, drag pressure…) while SAT= 1 arc

→ S/C will be useful for short term dynamics (gravity fields, mutual perturbations…) while SAT will be useful for long term dynamics (tidal effects…)

SAT and S/C dynamics are complementary!!

2- Example of the Mars system

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Former works:

Sinclair, 1972

Shor, 1975

Sinclair, 1989

Jacobson et al. 1989

Chapront-Touzé, 1990

Morley, 1990

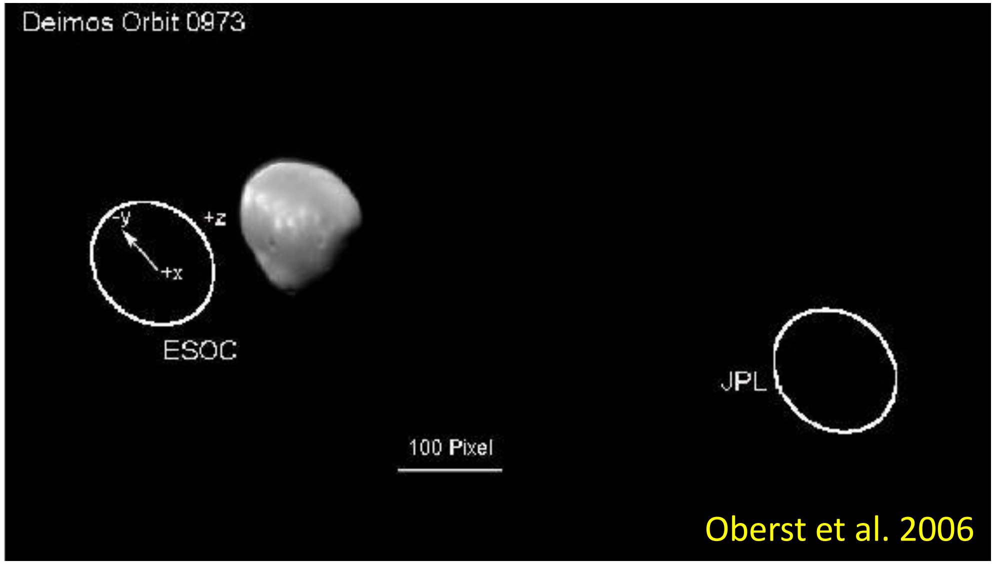
Emelianov et al. 1993

→All these models were analytic

→Tidal effects modelled by a t^2 term in the longitude.

2- Example of the Mars system

Since the 90s the Martian moon ephemerides had drifted…



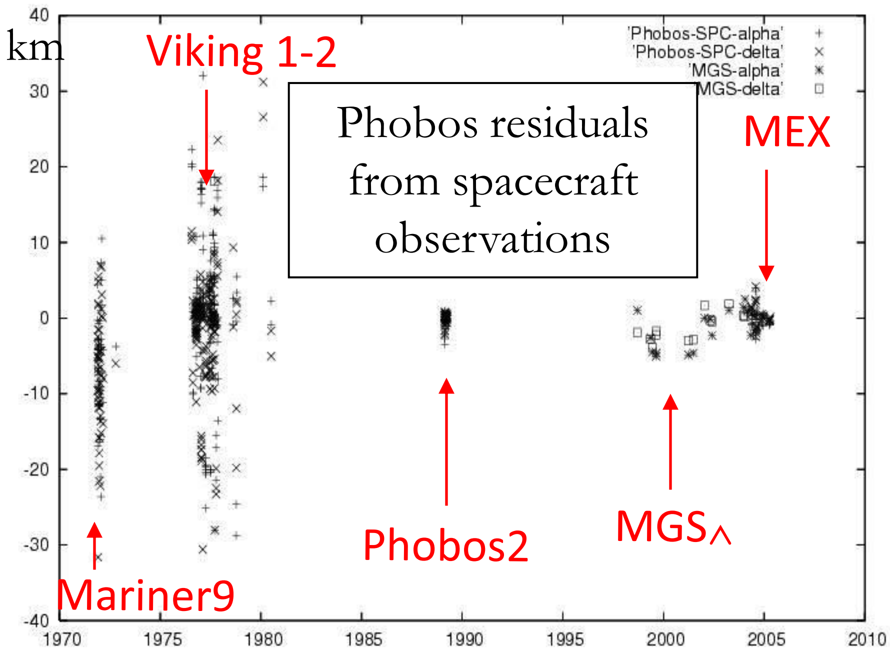
New ephemerides have been developed at JPL and IMCCE/ROB these last years to garantee a good accuracy of martian moon position in the context of MEX and MRO.

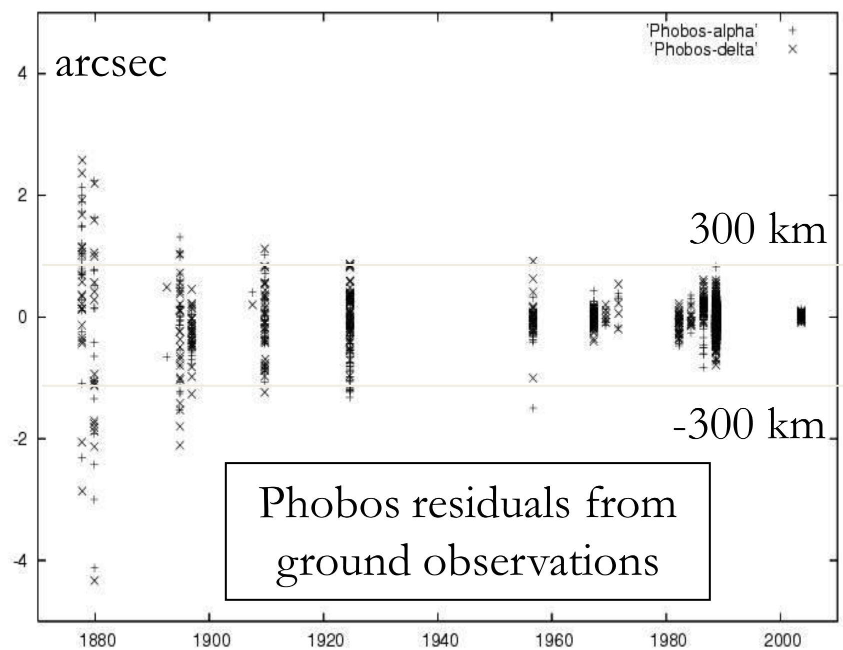
(both ephemerides are based on numerical integration)

See Lainey et al. (2007); Jacobson (2010)

<u>2- Example of the Mars system</u>

Astrometric post-fit residuals for Phobos et Deimos after fit of initial state vectors, Mars dissipation factor Q and Phobos' oblate parameters c_{20}, c_{22}.





Lainey, Dehant and Pätzold (2007)

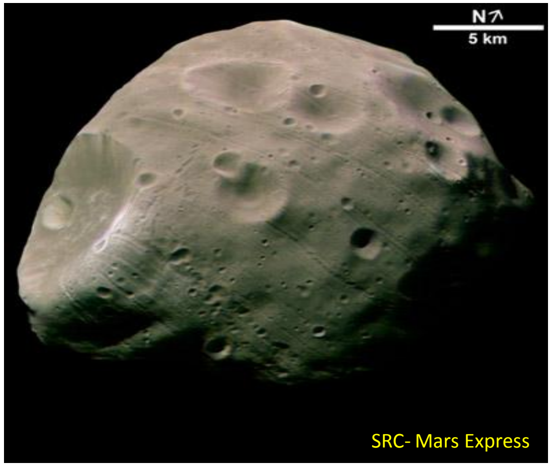
<u>NB:</u> Just one « arc » was used!!

2- Example of the Mars system

Two current challenges concerning the astrometry of Mars moons:

(precision of measurements=500 metres; more than 30,000 revolutions over 30 years)

1- Influence of Phobos' J_2+6c_{22} (several kilometres)

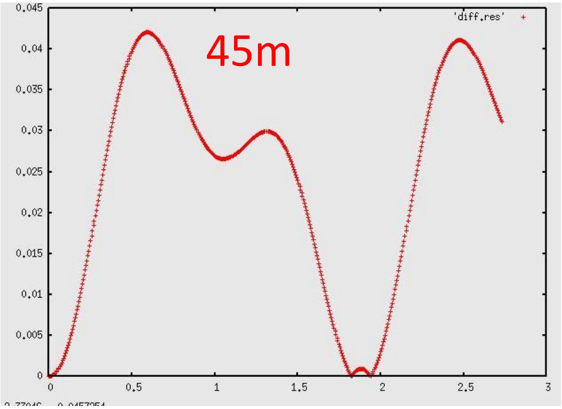


Jacobson (2010) solution in agreement with Willner (2010)

→Suggests that Phobos is almost homogeneous

2- Seasonal variations of Mars J_2

→ Signal close to the limit of current accuracy…



3- The ESPaCE network

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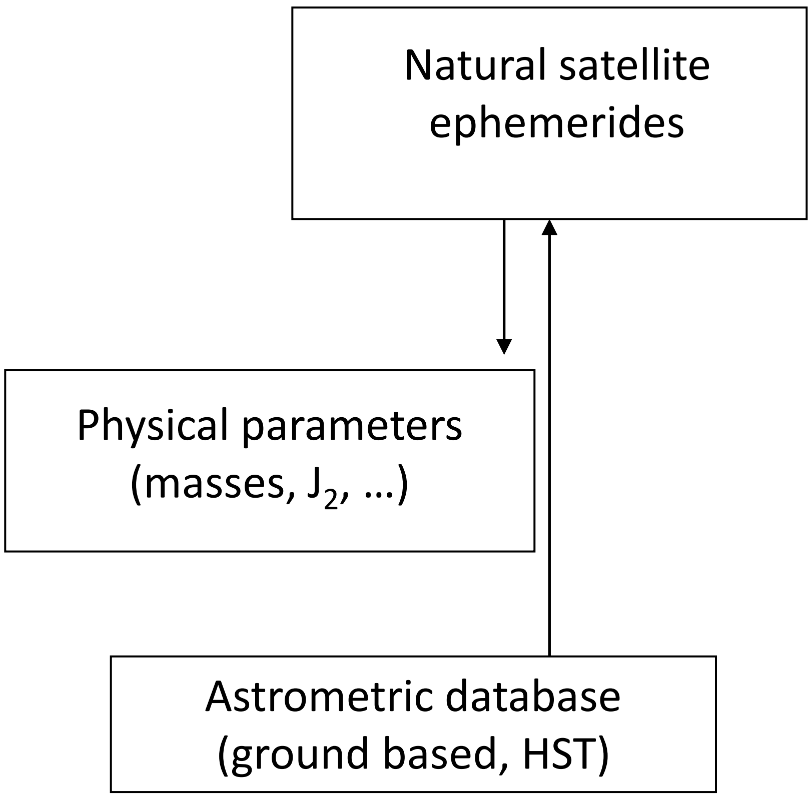

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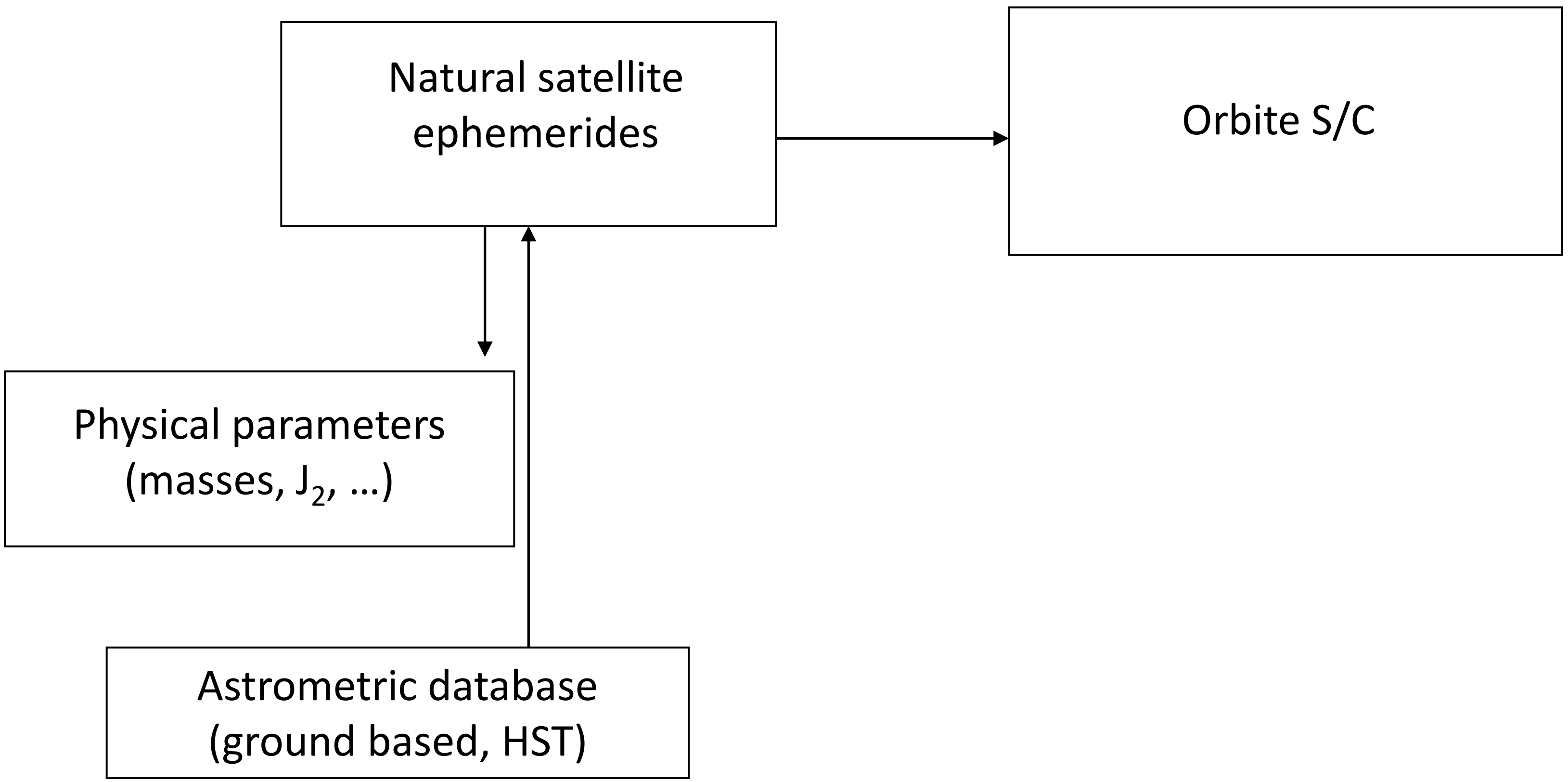

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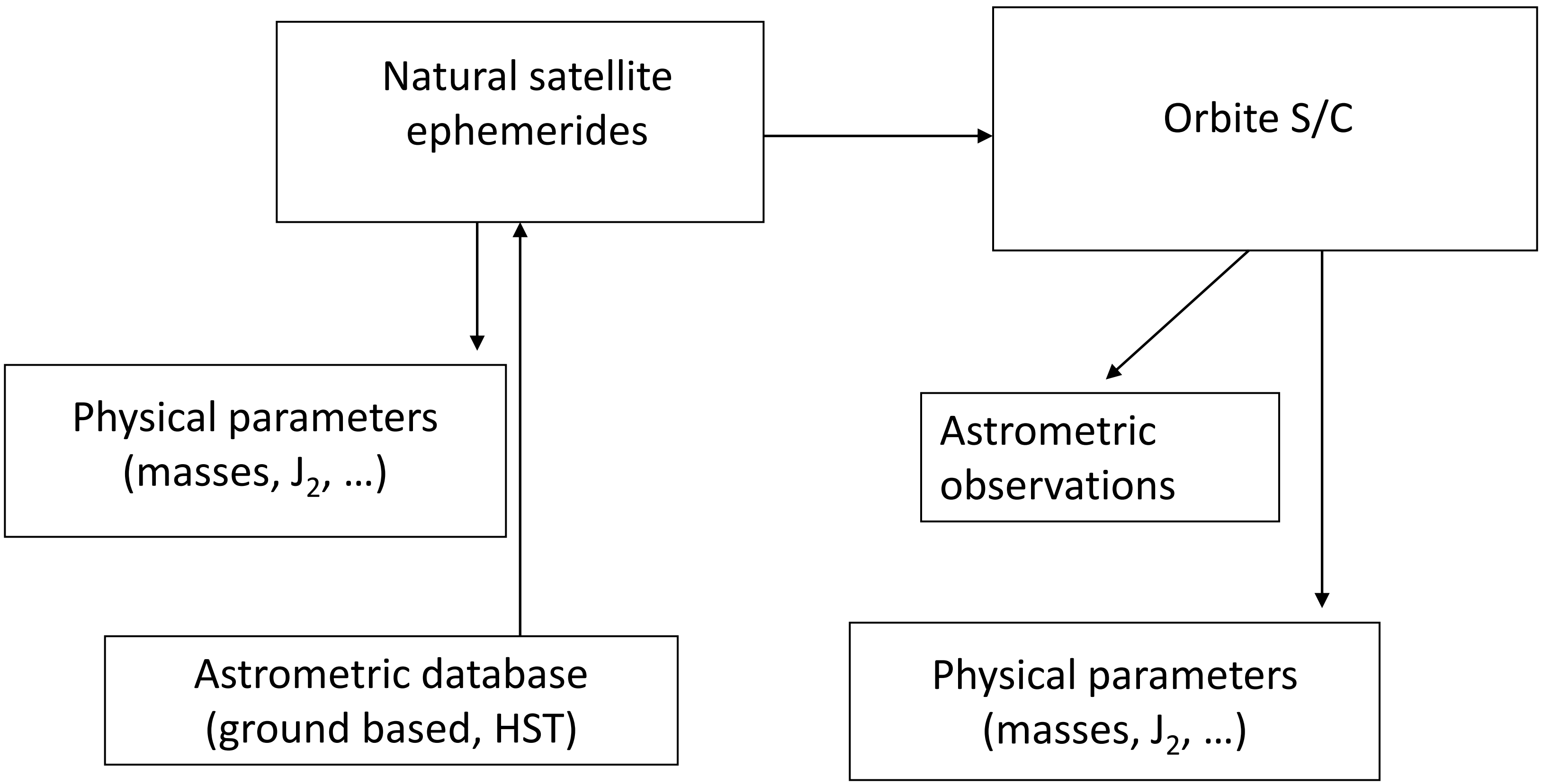

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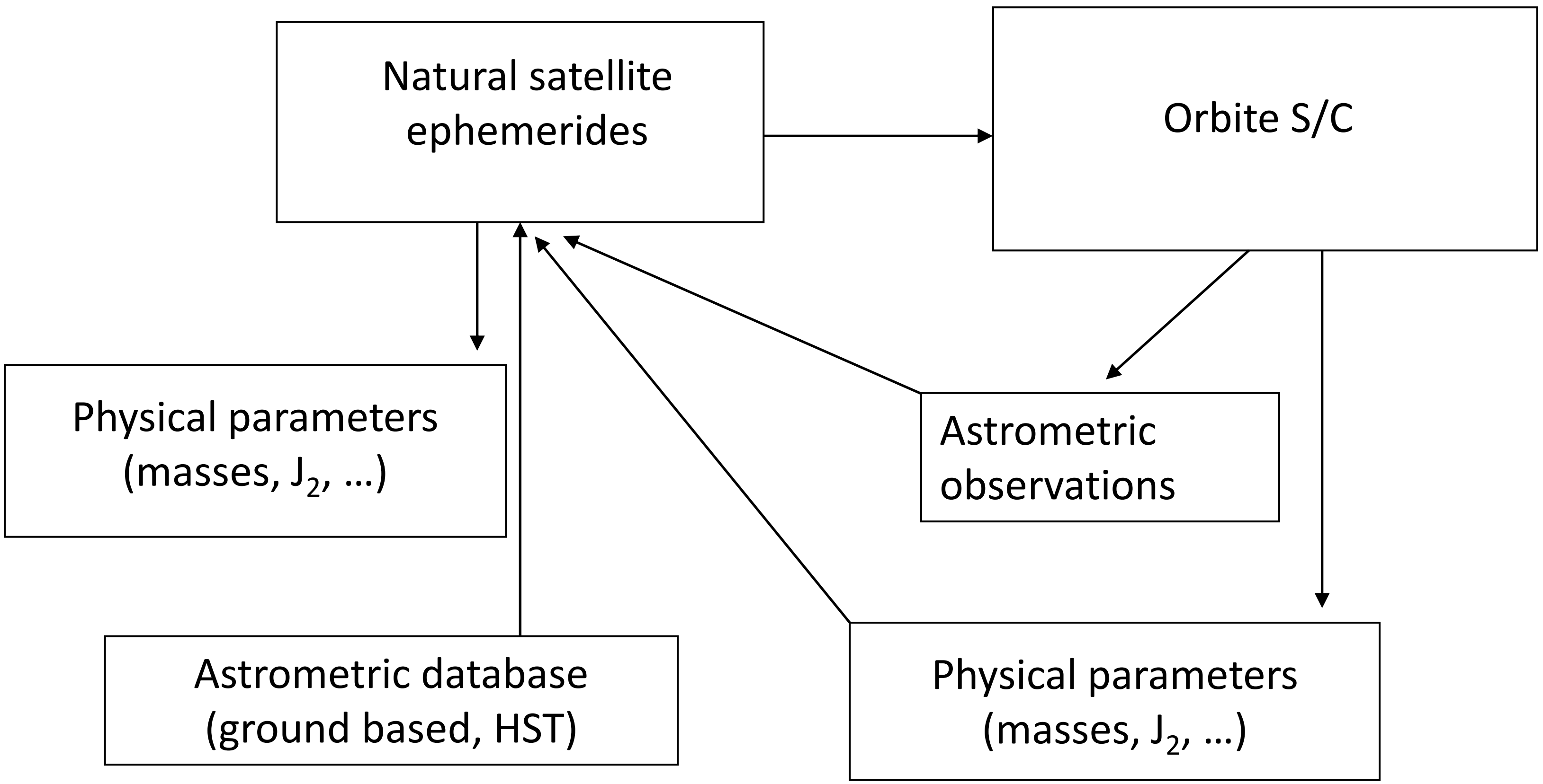
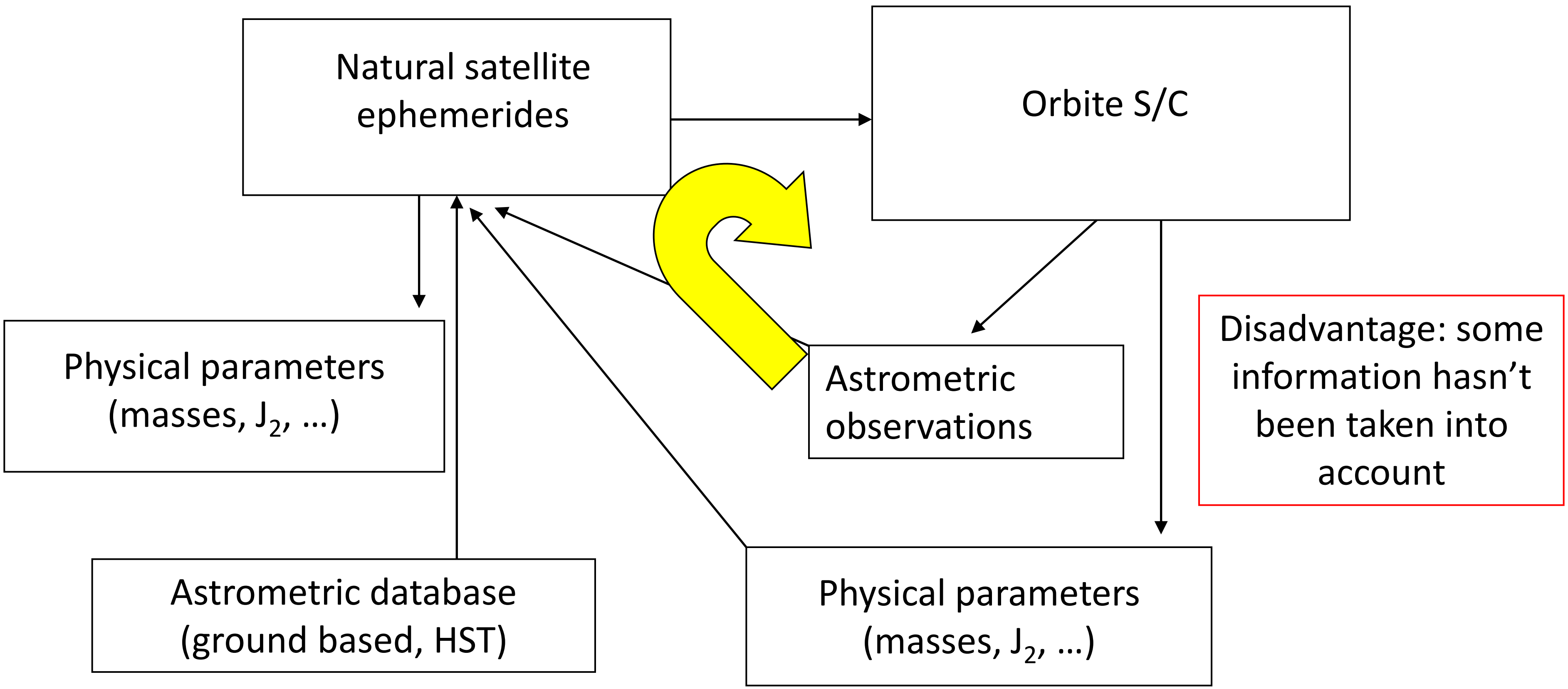


3- The ESPaCE network

Iterative methode with independent fits



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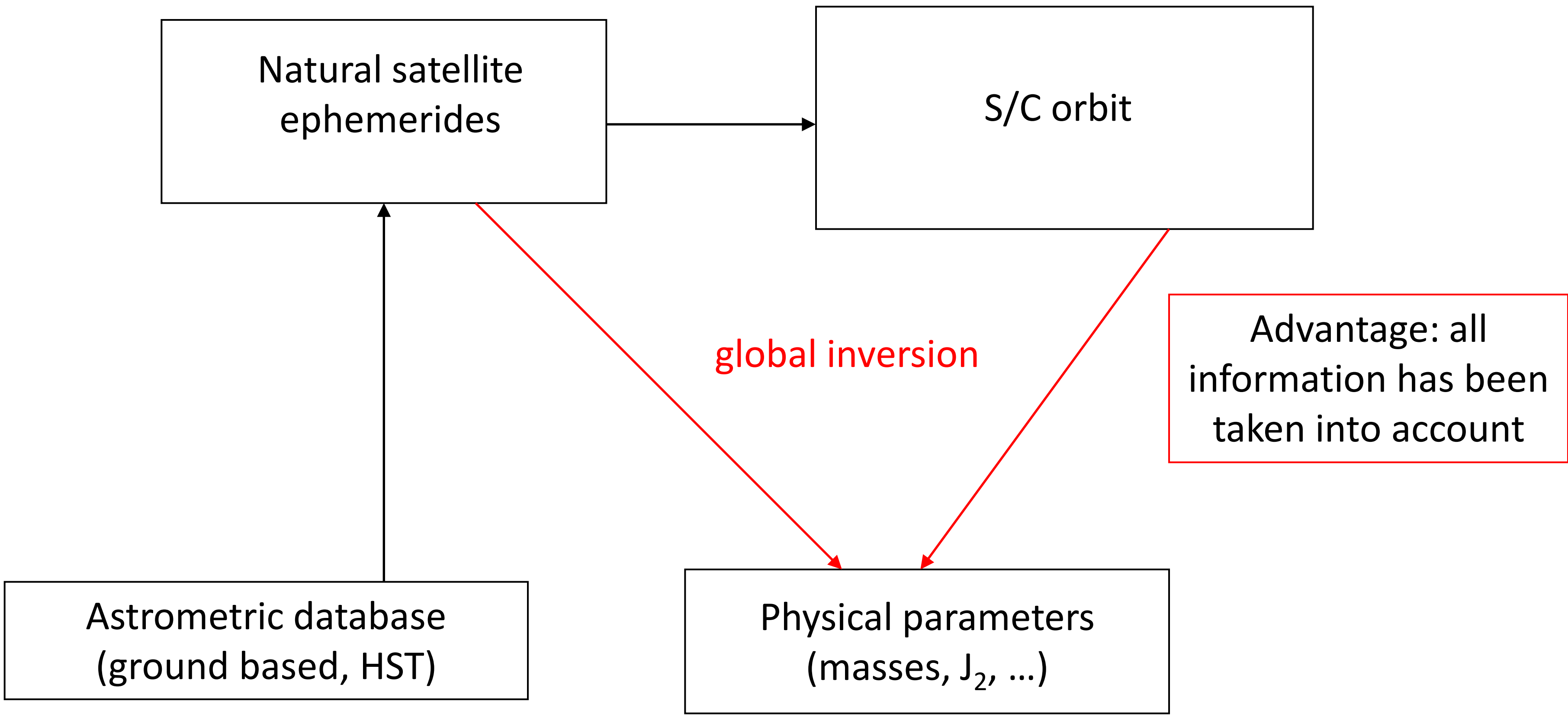
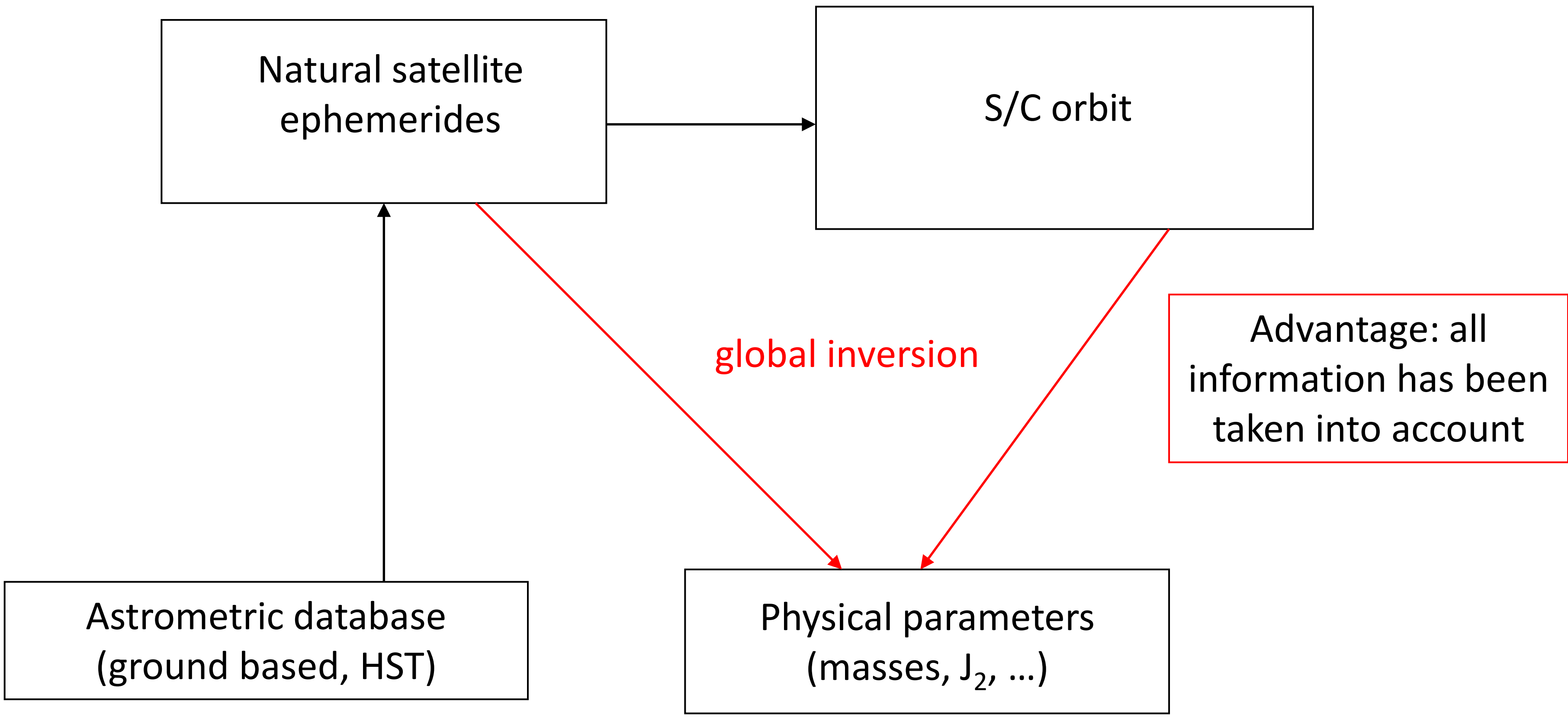


3- The ESPaCE network

Method used only at JPL so far! (flyby of Miranda, flybys of Phobos…)



3- The ESPaCE network

An expertise rising in Europe: ESPACE (FP7) network (IMCCE, ROB, DLR, CNES, TUB, TUD, JIVE)

- Development of new orbit fitting techniques
- Production of HIGH accurate orbits for S/C and SAT
- Will help Europe to be at the US level in ephemeris developments
- Will be an important experience when treating next generation of European space mission (JUICE, …)

Can SAT and S/C community get even closer?

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