# Bayesian Orbit Computation

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Orbital uncertainty is an important factor in many applications but a rigorous estimate for it can be challenging to obtain.

# When is uncertainty information required?

- linking astrometric data sets to specific objects (AKA cross-correlation, identification)
- planning of follow-up observations
- recovery of lost objects
- collision-probability estimation
- object classification
- in preparation of initial conditions for orbital integrations that are carried out to study the dynamics of a specific real object

observation (R.A. & Dec.)

=

theoretical prediction

+

systematic noise

+

random noise

observation

=

theoretical prediction

+

random noise

observation

=

theoretical prediction

+

random noise

The theoretical prediction is a nonlinear function of the orbital elements. The equations are usually linearized, but the validity of the Gaussian approximation was not questioned until...

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# Asteroid Orbit Determination Using Bayesian Probabilities

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Bayesian (AKA statistical) inversion fundamentally means that the parameters to be solved for (e.g., orbital elements $P$ ) are treated probabilistically and their posterior probability density function is defined as

$$p(P \mid \Phi) = \frac{p(P)\,p(\Phi \mid P)}{p(\Phi)}$$

or just

$$p(P \mid \Phi) \propto p(P)\,p(\Phi \mid P) = p(P)\exp\left[-\frac{1}{2}\chi^2\right]$$

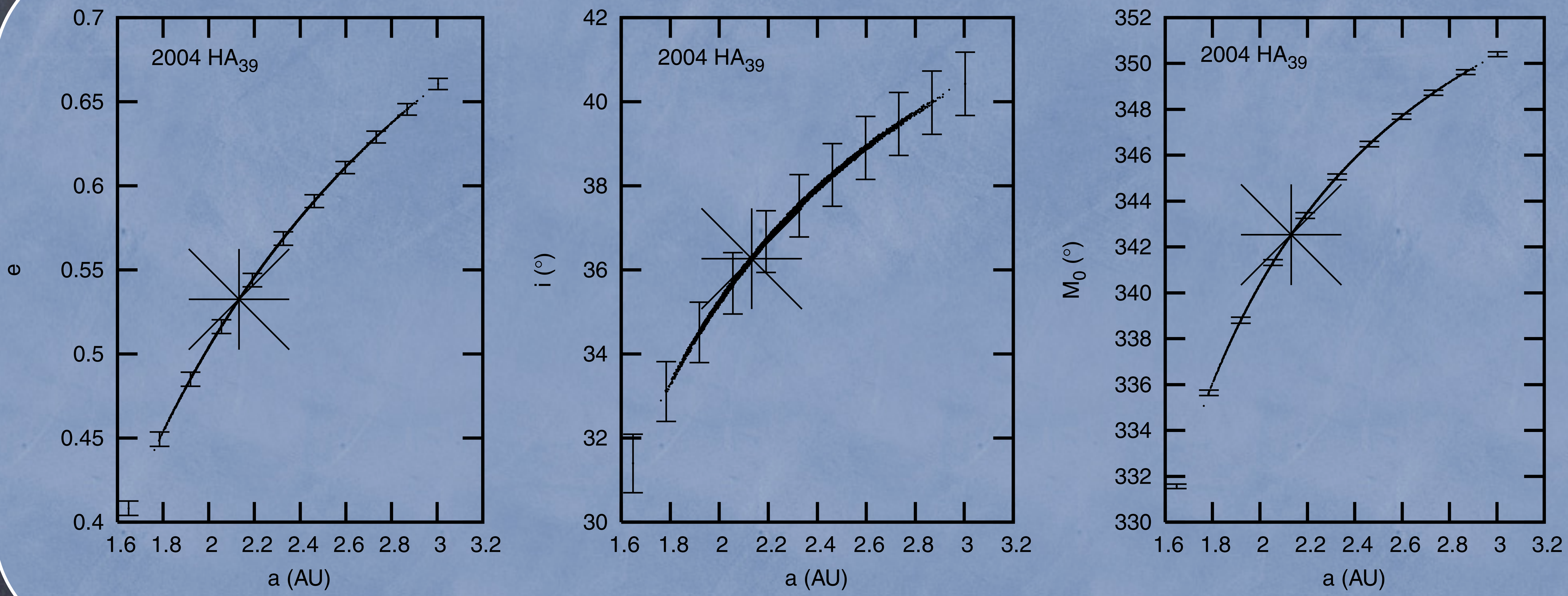
# Statistical Ranging

Virtanen et al. 2001, MCMC version Oszkiewicz et al. 2009

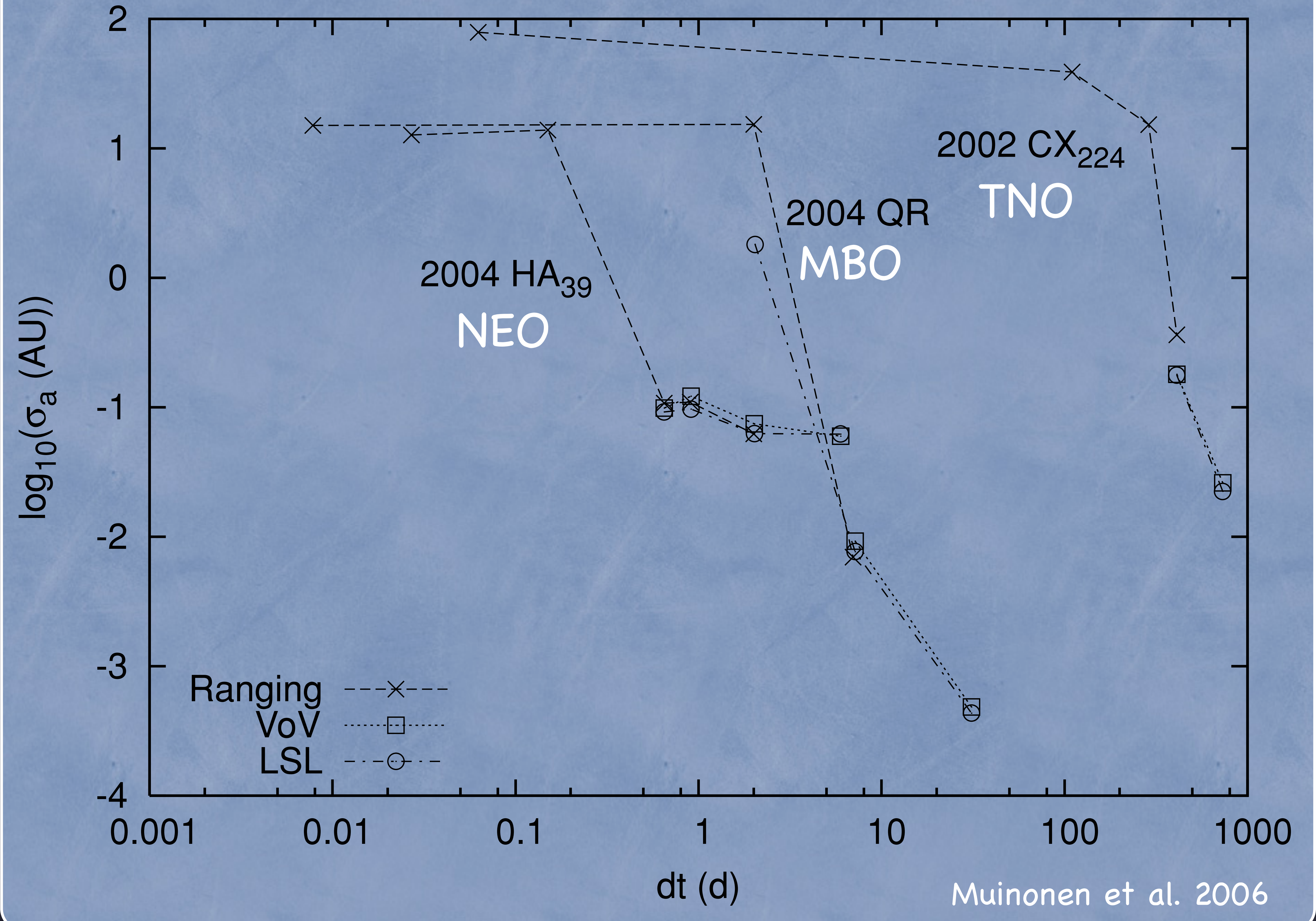
$$\alpha_1 + \Delta\alpha_1$$
$$\delta_1 + \Delta\delta_1$$
$$\rho_1$$
$$\alpha_2 + \Delta\alpha_2$$
$$\delta_2 + \Delta\delta_2$$
$$\Delta\rho = \rho_2 - \rho_1$$

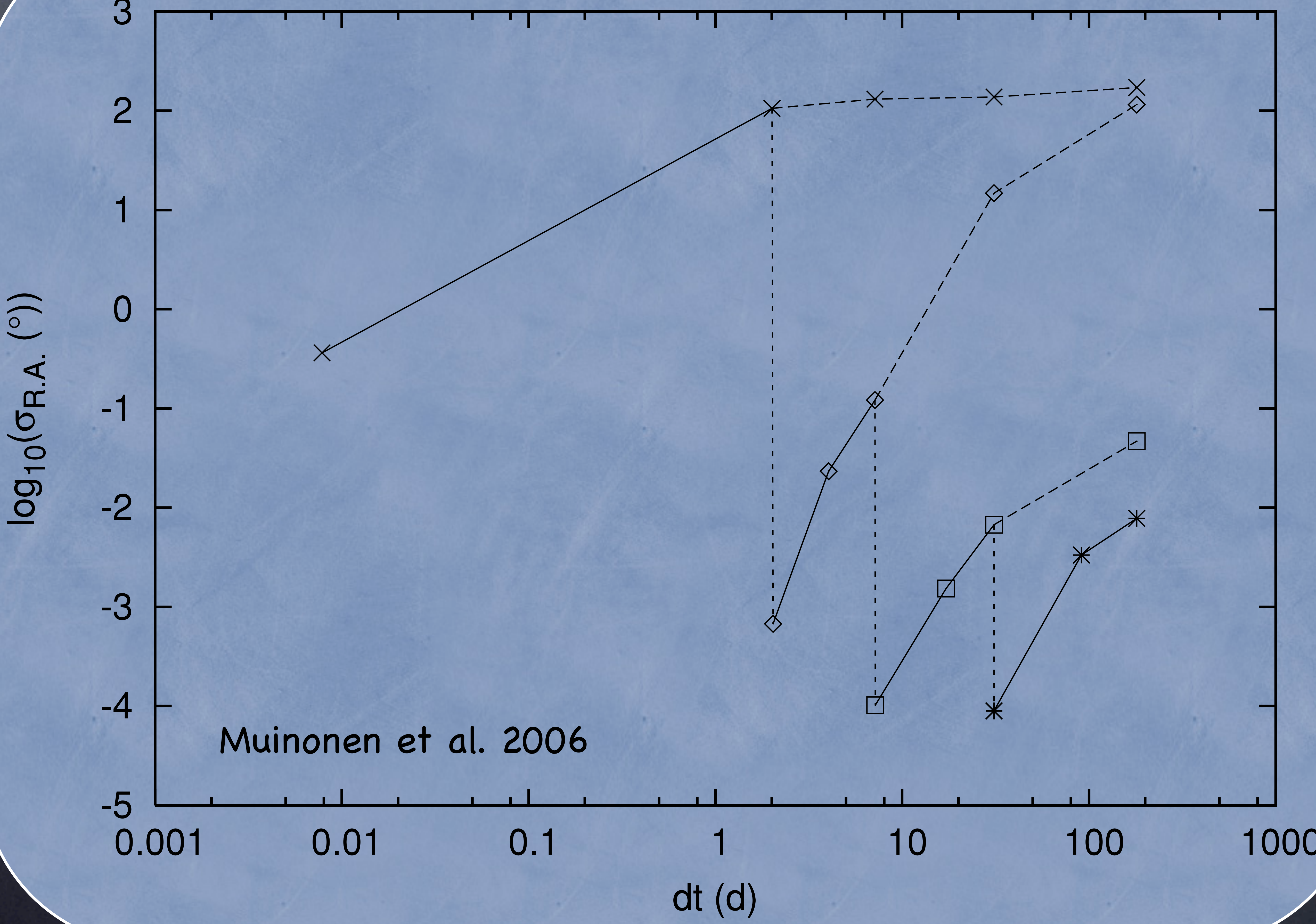
Criterion for acceptance:

$$\Delta\chi^2 < \Delta\chi^2_{lim}$$
$$\Phi_{ij} - \varphi_{ij}(P) < n\sigma_{ij} \quad (n \geq 3)$$

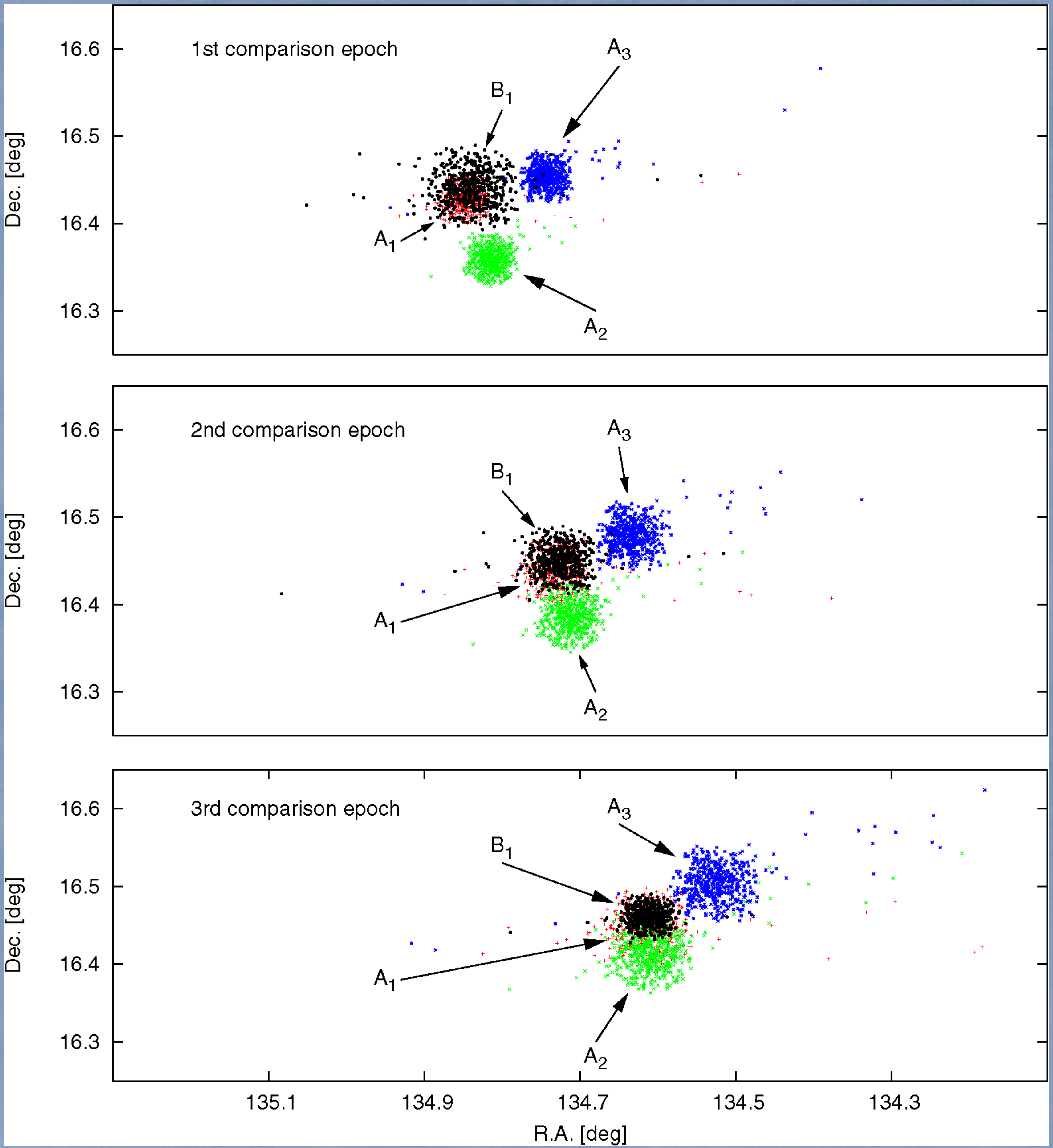


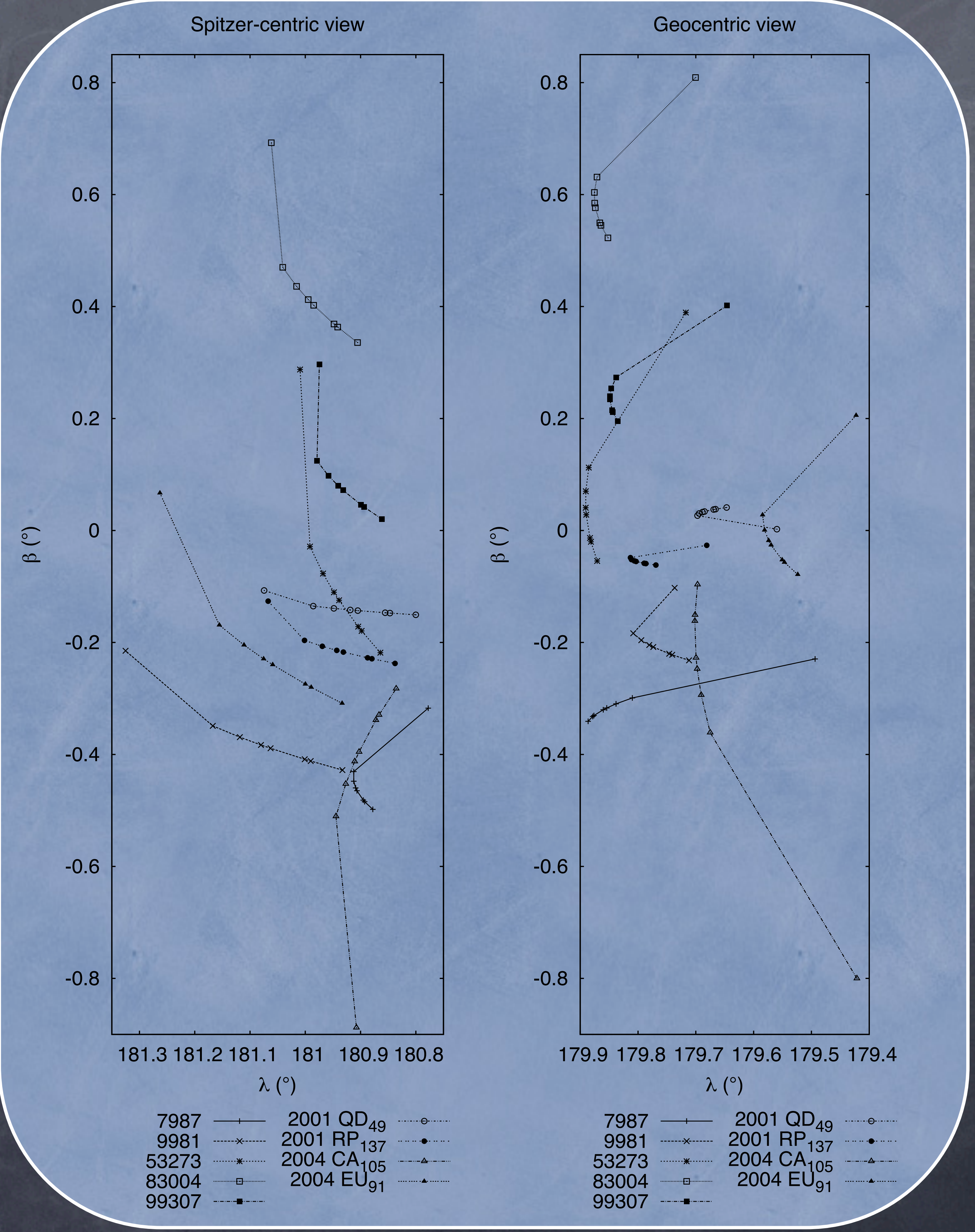

Muinonen et al. 2006


2
1
0
-1
-2
-3
-4
0.001
0.01
0.1
1
10
100
1000
log10(σa (AU))
dt (d)
2002 CX224
TNO
2004 QR
MBO
2004 HA39
NEO
Ranging
VoV
LSL
Muinonen et al. 2006



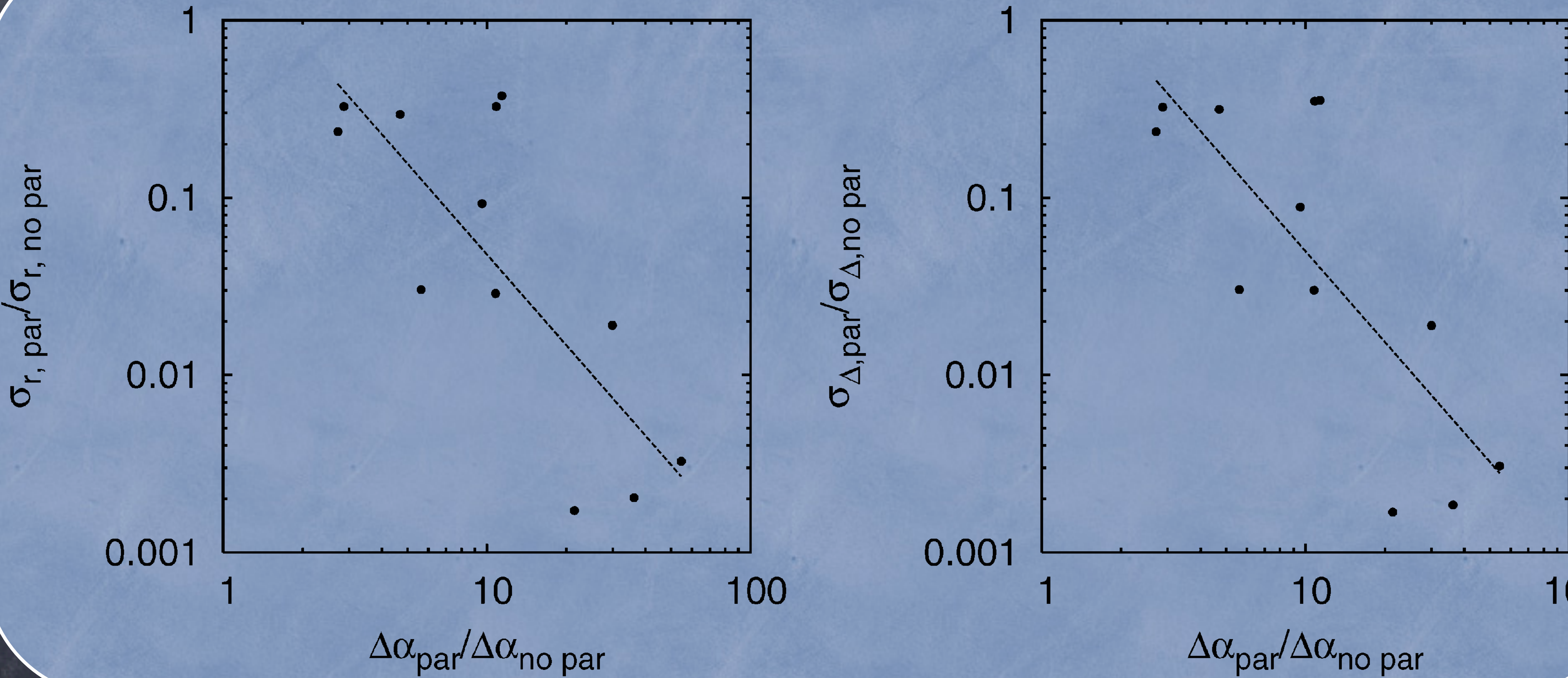
3
2
1
0
-1
-2
-3
-4
-5
log10(σR.A. (°))
Muinonen et al. 2006
0.001
0.01
0.1
1
10
100
1000
dt (d)


1st comparison epoch
2nd comparison epoch
3rd comparison epoch
A1
A2
A3
B1
Dec. [deg]
16.6
16.5
16.4
16.3
135.1
134.9
134.7
134.5
134.3
R.A. [deg]



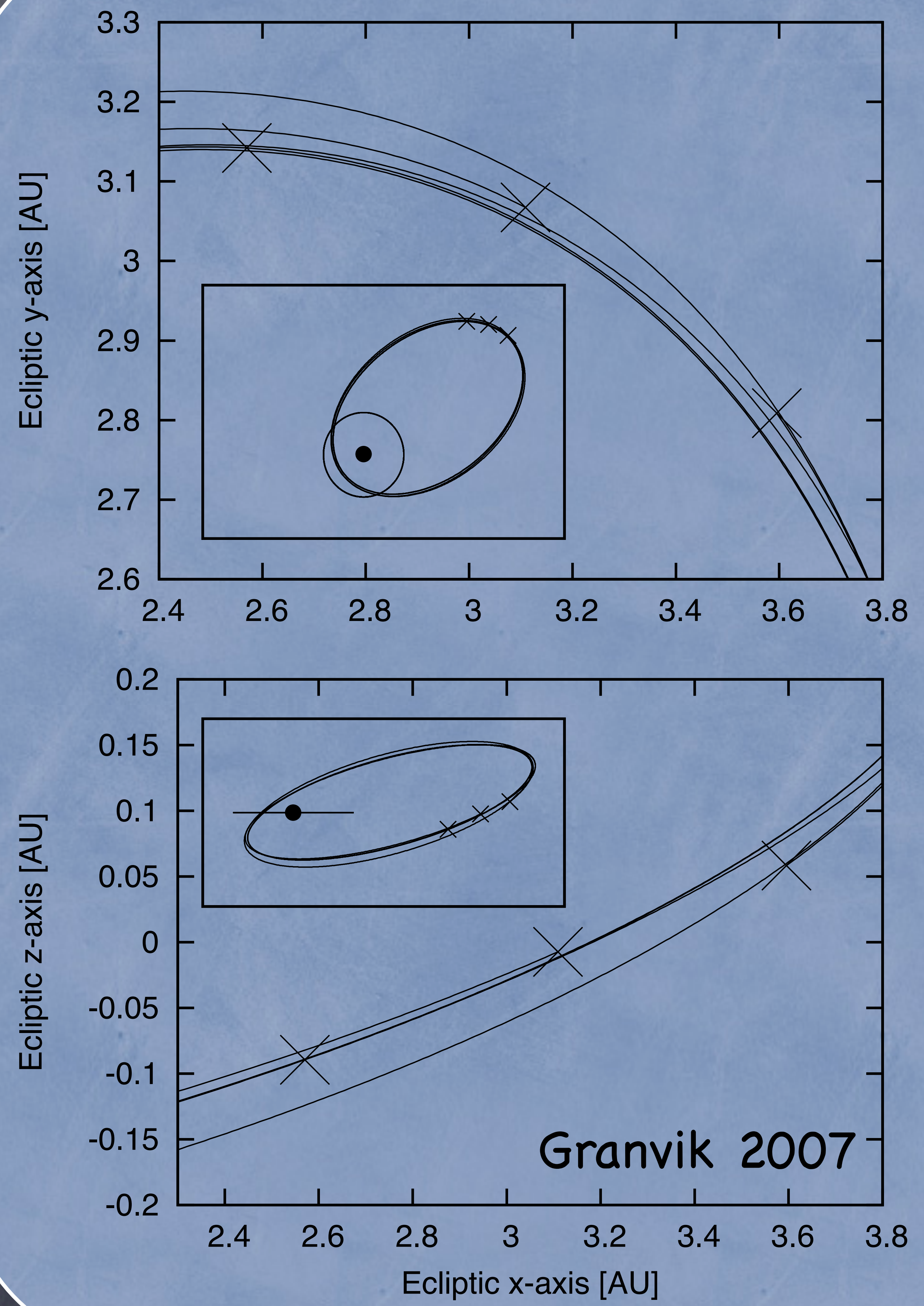

Granvik et al. 2007

# Parallax on MBOs




Granvik et al. 2007

Linkages between 3 single-night sets of NEO astrometry over 21 years.



# Markov Chain Monte Carlo

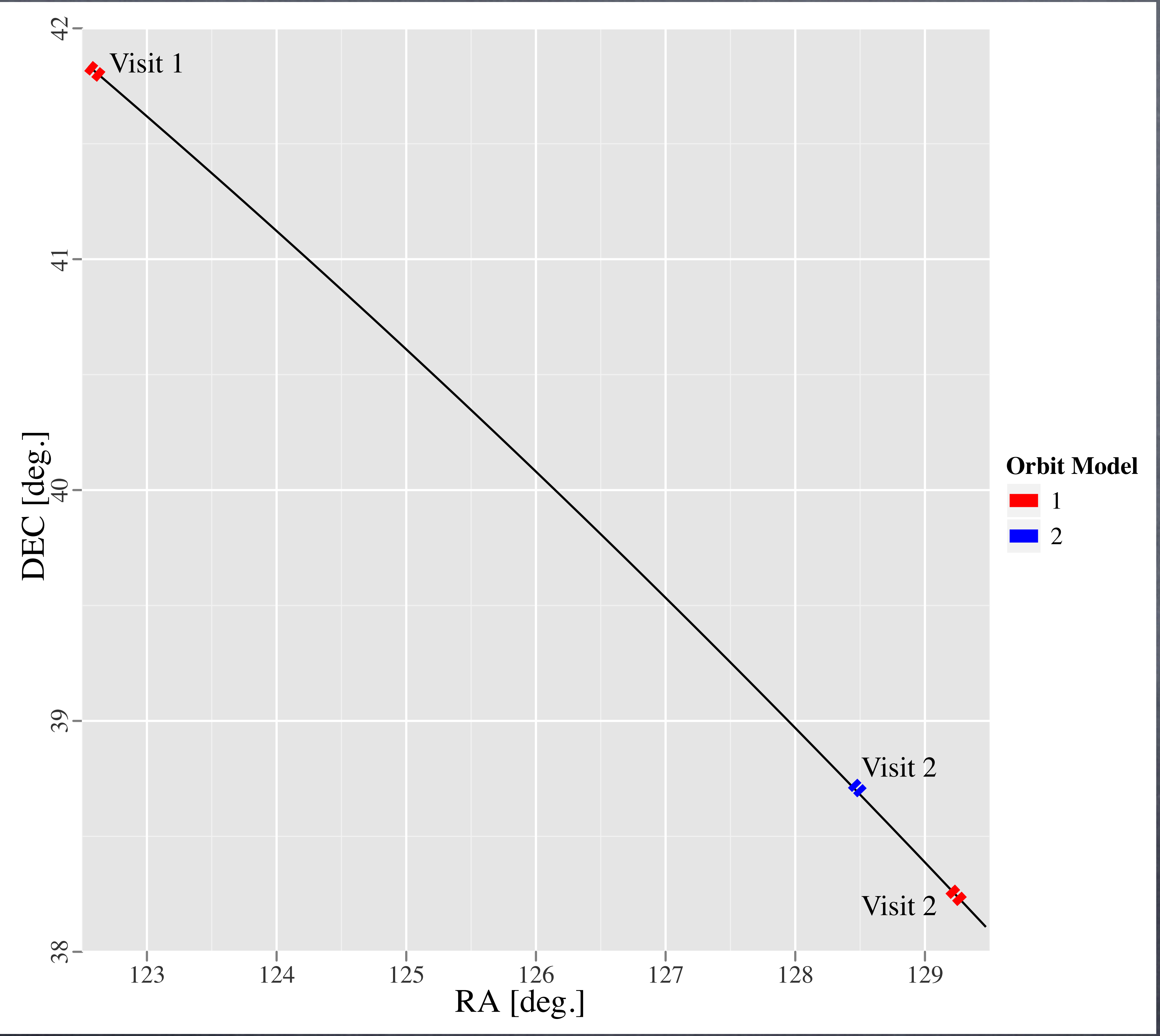
A Markov Chain is a sequence of random numbers following an arbitrarily complicated distribution.

# Metropolis-Hastings

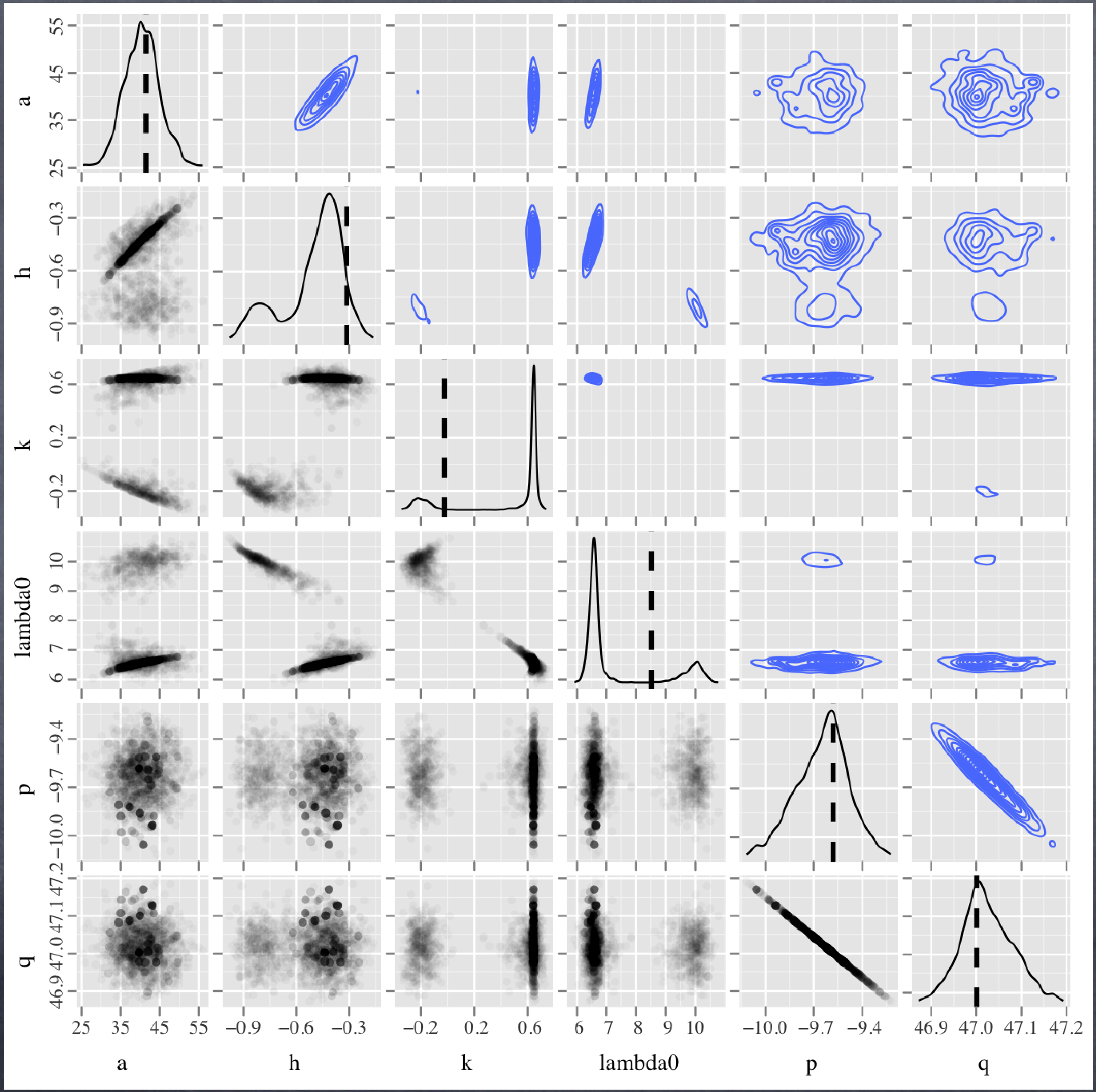
- Let $q\,(P_1;P_2)$ be the proposal density for orbit $P_1$ which is used to generate orbit $P_2$.
- A trial orbit $P'$ is accepted after comparison with the last accepted orbit $P$ if a random value $\alpha$ belonging to U(0,1) satisfies

$$\alpha < \frac{p(P')\,q(P;P')}{p(P)\,q(P';P)}$$

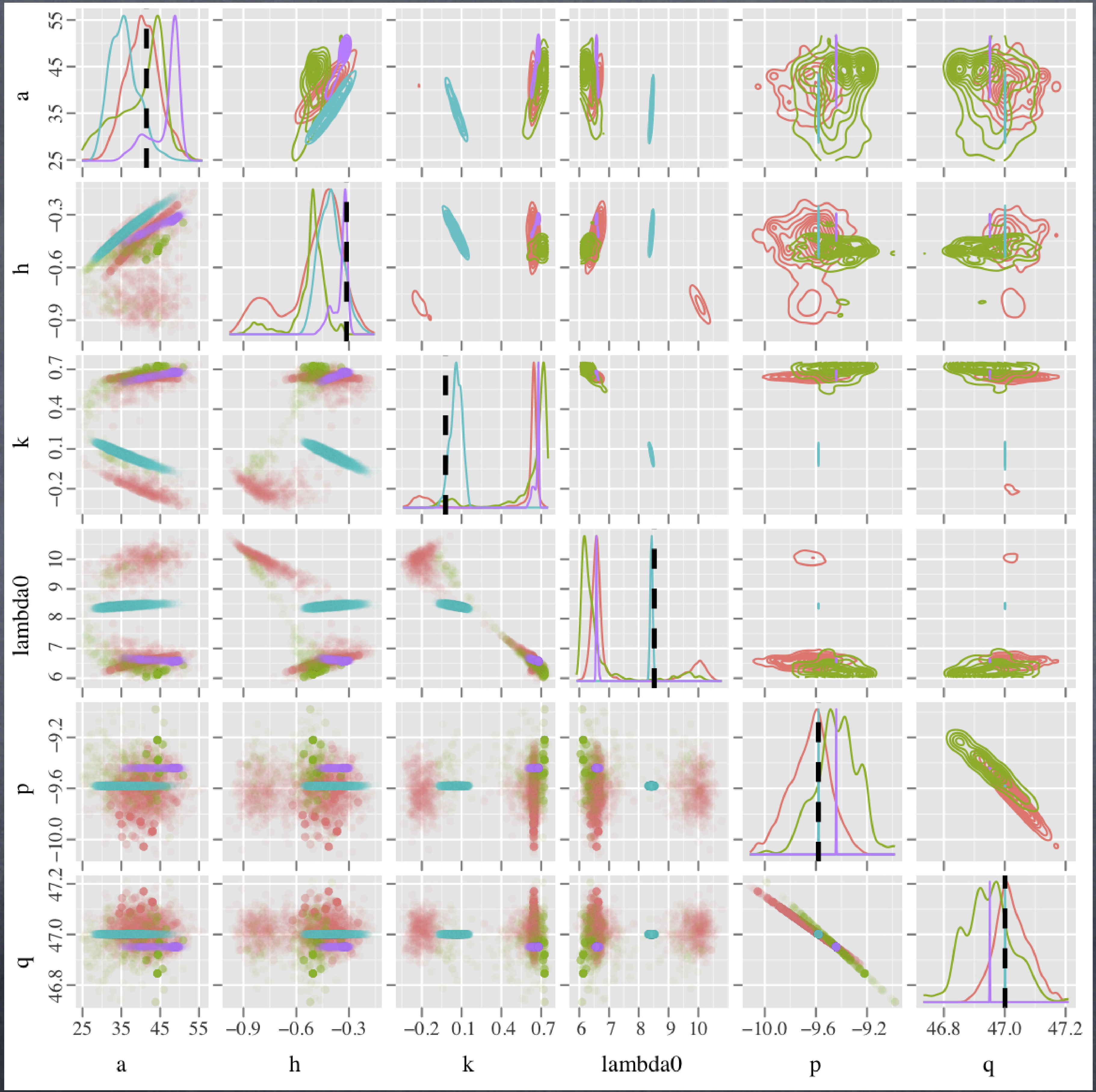
- A symmetric proposal density is preferred as $q\,(P_1;P_2) = q\,(P_2;P_1)$ and the $q$ terms cancel.




Schneider (2011)




Schneider (2011)




Schneider (2011)

# OpenOrb

- open-source orbit-computation package
- includes all the orbital inversion methods discussed and much more...
- object-oriented Fortran95 + Python bindings
- used by Pan-STARRS, LSST, NEOSSat, etc in addition to individual researchers
- GNU General Public License v3
- http://code.google.com/p/oorb/

The Bayesian formalism and the statistical inverse theory is a viable means to compute rigorous estimates for the orbital uncertainty.