Light time calculation in high precision space navigation

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Plan

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Fundamentals of space navigation

Space Navigation is based on a radio exchange of information between the probe, equipped with an antenna and a transponder, and one or more ground stations, equipped with a clock.

$\Longleftrightarrow$

Fundamentals of space navigation

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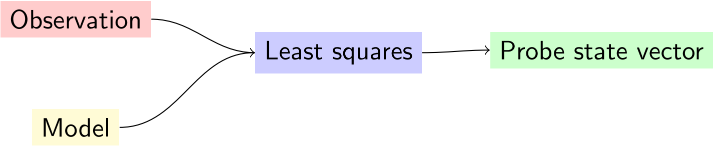
Ephemeris of a space mission is built following this procedure :



The ephemeris is necessary for :

- operational goals - space probe navigation
- scientific goals - measurements in fundamental physics

The standard Moyer's model

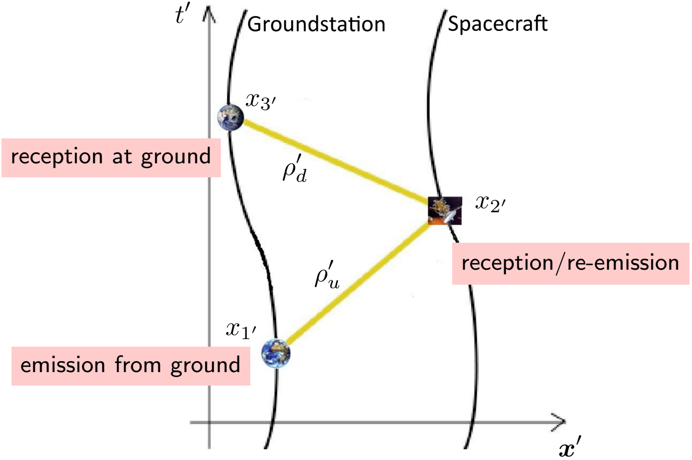


Fig : 2-ways light time

T.D. Moyer, 2000

Observables : Ranging

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Ranging is basically the 2-way light-time + calibrations :

- Relativistic terms (Shapiro, etc...)
- Newtonian flight time

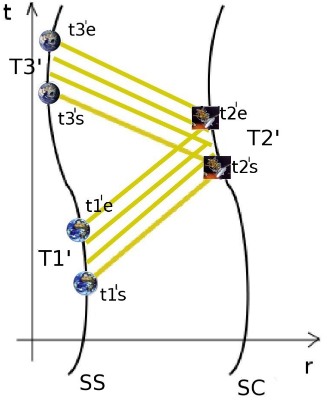
$$\rho' = \frac{R_{1'2'}}{c} + \frac{R_{2'3'}}{c} + \frac{\Delta(\boldsymbol{x}_{1'}^{GS}, \boldsymbol{x}_{2'}^{SC})}{c} + \frac{\Delta(\boldsymbol{x}_{2'}^{SC}, \boldsymbol{x}_{3'}^{GS})}{c} + \delta C + \delta t$$

- Atmospheric delay, Time scale calibration
- Transponder delay ($\approx 2.5\ \mu s$)

where $R_{ab} = |\boldsymbol{x}_b - \boldsymbol{x}_a|$ and c is the speed of light.

Observables : Doppler

Differential Doppler :



Doppler shift is linked to the radial velocity of the observed body :

$$\frac{\Delta\nu'}{\nu_{1'}} = \frac{T_{3'} - T_{1'}}{T_{1'}} = M_2 f_T \dot{\rho}'$$

In $\dot{\rho}'$ constant (or slowly changing) terms $(\delta\rho', \delta t_{2'})$ simplify.

Our model

Our study : rigorous introduction of the transponder delay in the 2-way light-time modeling

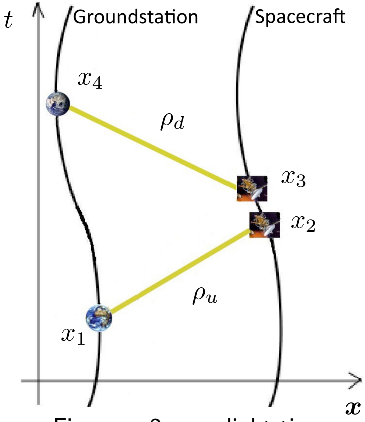


Fig : our 2-ways light time

- Separation of reception and re-emission events
  $t_3 = t_2 + \delta t$
- Modification of the 2-way light-time and of the Ranging/Doppler observables

The Ranging observable in the 2 models

At the Newtonian approximation :

$$\text{Moyer : } \rho' = \frac{R_{1'2'}}{c} + \frac{R_{2'3'}}{c} + \delta C + \delta t$$

$$\text{Alternative : } \rho = \frac{R_{12}}{c} + \frac{R_{34}}{c} + \delta C + \delta t$$

supposing that the reception time $t_4 = t'_3$ is the same for the two models :

$$\begin{aligned} \boldsymbol{x}'_3 &= \boldsymbol{x}_4 & \boldsymbol{x}'_2 &= \boldsymbol{x}_3 \\ \boldsymbol{x}_3 &= \boldsymbol{x}_{2+\delta t} & \boldsymbol{x}'_1 &= \boldsymbol{x}_{1+\Delta\rho} \end{aligned} \quad \Longrightarrow \quad \begin{aligned} R_{34} &= R_{2'3'} \\ R_{12} &\neq R_{1'2'} \end{aligned}$$

we obtain :

$$\Delta\rho = \rho - \rho' = \delta t \frac{(\boldsymbol{v}_1^{GS} - \boldsymbol{v}_2^{SC}) \cdot \boldsymbol{N}_{12}}{c} + \mathcal{O}(c^{-2})$$

Modifications in the Doppler observable

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The Doppler signal is computed by :

$$\frac{\Delta\nu}{\nu_1} = M_2 f_T \dot{\rho}$$

- In Moyer's model, the Doppler observable is independent from the transponder delay.
- In the alternative model, the transponder delay contribution on the Ranging depends on the spacecraft and ground station state : $\Delta\rho$ is not constant.

Since $\Delta\rho$ is not constant, $\Delta\dot{\rho} \neq 0$: the explicit computation of δt modifies the Doppler observable.

Application to a real case

Earth flyby trajectory by Near and Rosetta

- Ephemeris provided by the NASA Naif/Spice toolkit
- This configuration is favorable for $\Delta\rho$ and $\Delta\dot{\rho}$

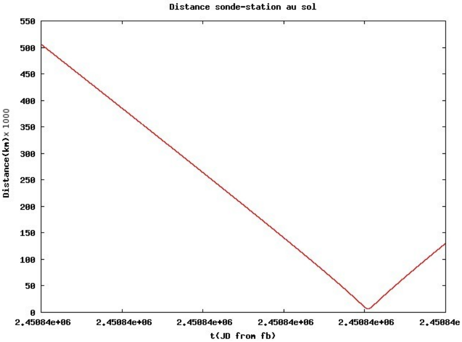

Fig. : typical probe-groundstation distance during an Earth-flyby

Ranging difference between the two models

Computation of

$$\Delta\rho = \rho - \rho' = \delta t \frac{(\boldsymbol{v}_1^{GS} - \boldsymbol{v}_2^{SC}) \cdot N_{12}}{c} + \mathcal{O}(c^{-2})$$

with real spacecraft ephemeris :

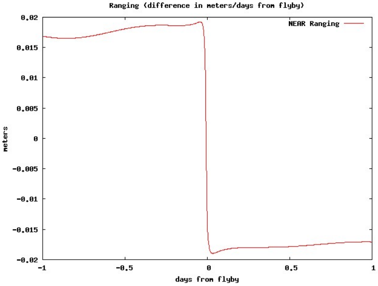

Fig a : NEAR $c\,\Delta\rho$ ($\approx$ 2 cm)

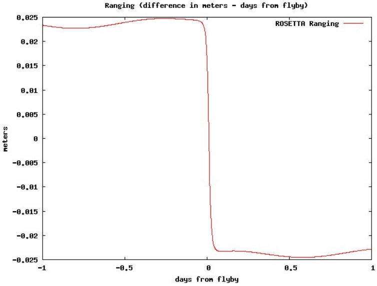

Fig b : ROSETTA $c\,\Delta\rho$ ($\approx$ 2.5 cm)

Doppler difference between the two models

Computation of

$$\Delta \dot{\rho}$$

with real spacecraft ephemeris :

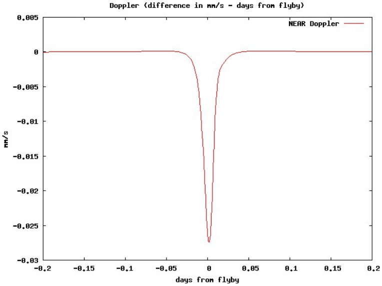

Fig a : NEAR $c\,\Delta\dot{\rho}$ ($\approx$ 0.02 mm/s)

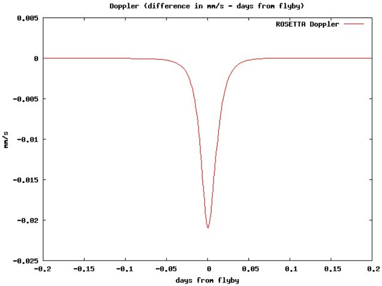

Fig b : ROSETTA $c\,\Delta\dot{\rho}$ ($\approx$ 0.025 mm/s)

High dependency of the introduced terms

High dependency of the Ranging and Doppler difference between the two models, following the spacecraft-ground station configuration.

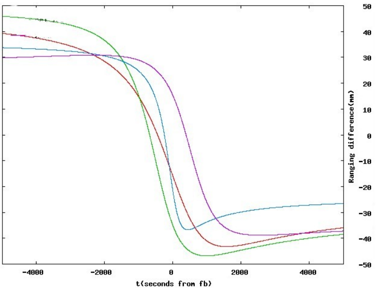

Fig a : Ranging observation of different probes trajectories

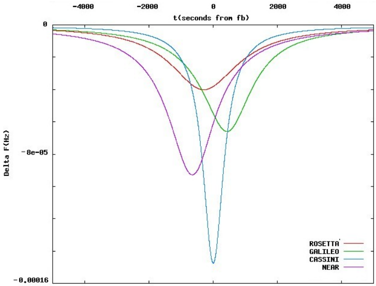

Fig b : Doppler observation of different probes trajectories

Impact of $\Delta\rho$ on the orbit reconstruction

Integration of a NEAR-like flyby orbit and fit to minimize $\Delta\rho$:

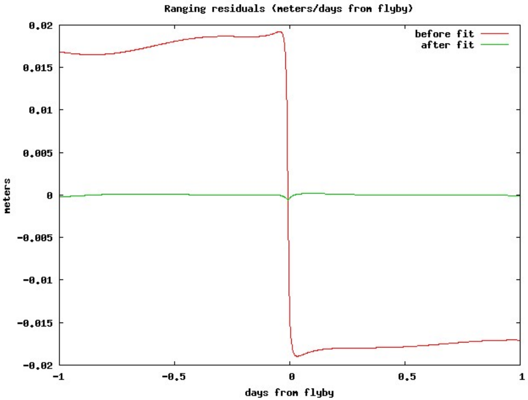

Fig : Pre-post fit residuals

The effects of the transponder delay are almost, even if not completely, absorbed by the fit.

Conclusions

- the transponder delay influence on the light-time calculation is subtler than the addition of a constant term ;
- a rigorous treatment of the transponder delay in the light-time computation does have an influence on the Doppler calculation ;

- the error is of the order of several cm for actual transponders ($\approx 2.5 \mu s$) : the standard approximation is acceptable for most operational goals at present.
- what about future and past observations ?