ON CAPTURE IN RESTRICTED FOUR BODY PROBLEM

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Introduction

A number of irregular moons of the Jovian planets have recently been discovered. Most adequate way of their origin is capture, but detailed mechanism is unknown. A few possibilities are discussed: collisions, gas drag, tidal destruction of a binary asteroid.

The capture process in restricted four body problem (RFBP) is researched. The interaction with regular satellite can be studied by this way as well as binary asteroid destruction in Hill sphere of planet. The energetic criteria of ballistic capture are studied and some numerical experiments are developed.

Let to consider restricted four body problem (RFBP). Main notations are: m_1 – main body or reduced mass in barycentre (Sun), m_2 – secondary mass (Planet), m_3 – infinitesimal test particle, m_4 – mass, orbiting m_2 (Satellite), f – is a gravity constant, V_i – respective velocities,

Δ_{ij}- distances between points. Hamiltonian for this problem is:

$$H = 1/2\sum m_i V_i^2 - 1/2 f \sum_{j=0}^{n-1}\sum_{i=0}^{n-1}\frac{m_j m_i}{\Delta_{ij}}$$

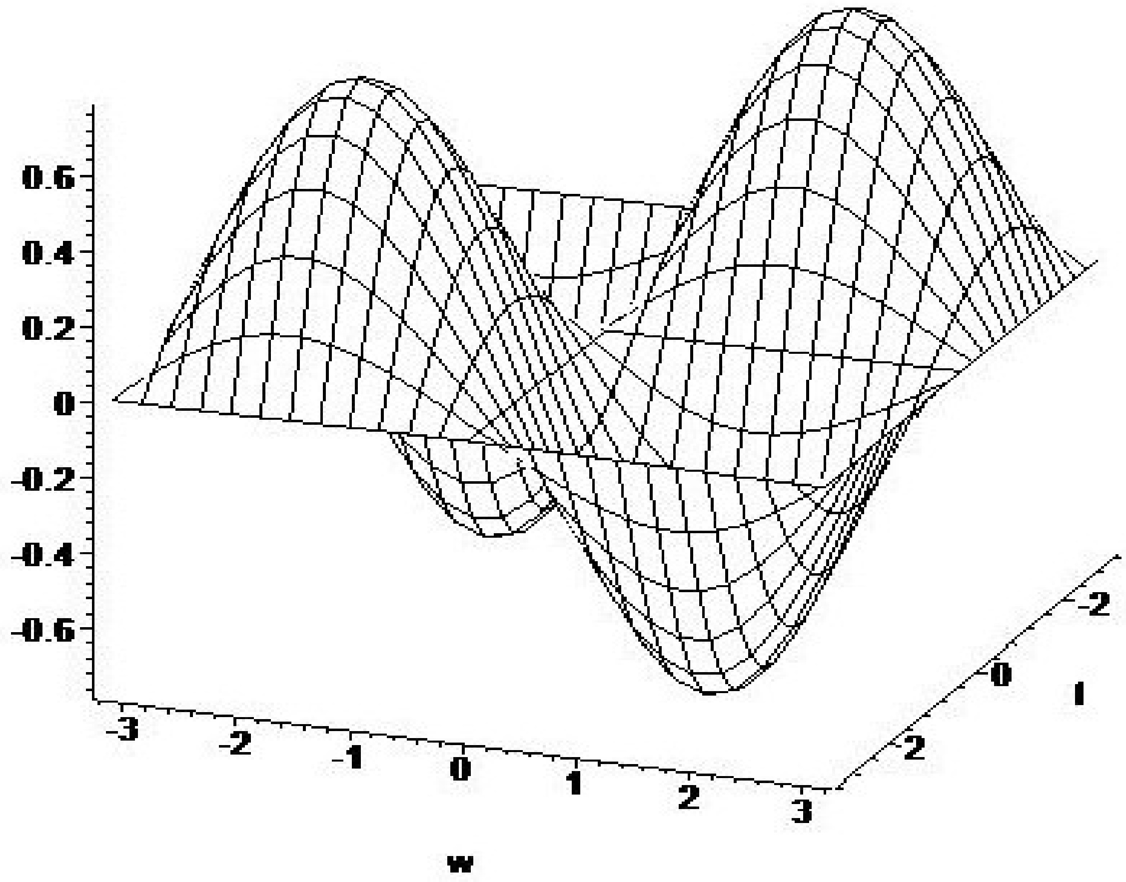


Fig.1.Three body problem case. Relative energy on pericenter longitude w and true anomaly f dependence.

Let $m_1=1, V_1=0$.

$$H = 1/2 m_2 V_2^2 + 1/2 m_3 V_3^2 + 1/2 m_4 V_4^2 - f \frac{m_2}{\Delta_{12}} - f \frac{m_3}{\Delta_{13}} - f \frac{m_2 m_3}{\Delta_{23}} - f \frac{m_4}{\Delta_{14}} - f \frac{m_2 m_4}{\Delta_{24}} - f \frac{m_3 m_4}{\Delta_{34}}$$

Denote

V_{23} - velocity of m_3 relative the second primary,

V_{24} - velocity of satellite relative the second primary:

$$V_3^2 = V_2^2 + V_{23}^2 - 2 V_2 V_{23} \cos(V_2, V_{23})$$

$$V_4^2 = V_2^2 + V_{24}^2 - 2 V_2 V_{24} \cos(V_2, V_{24})$$

Denote h_{23} - energy of m_3 relative the second primary.

$$h_{23} = 1/2 m_3 V_{23}^2 - f \frac{m_2 m_3}{\Delta_{23}}$$

Definition. (Belbruno) P_3 is ballistically captured at P_2 at time t if the two body Kepler energy of P_3 with respect in P_2-centred inertial coordinates:

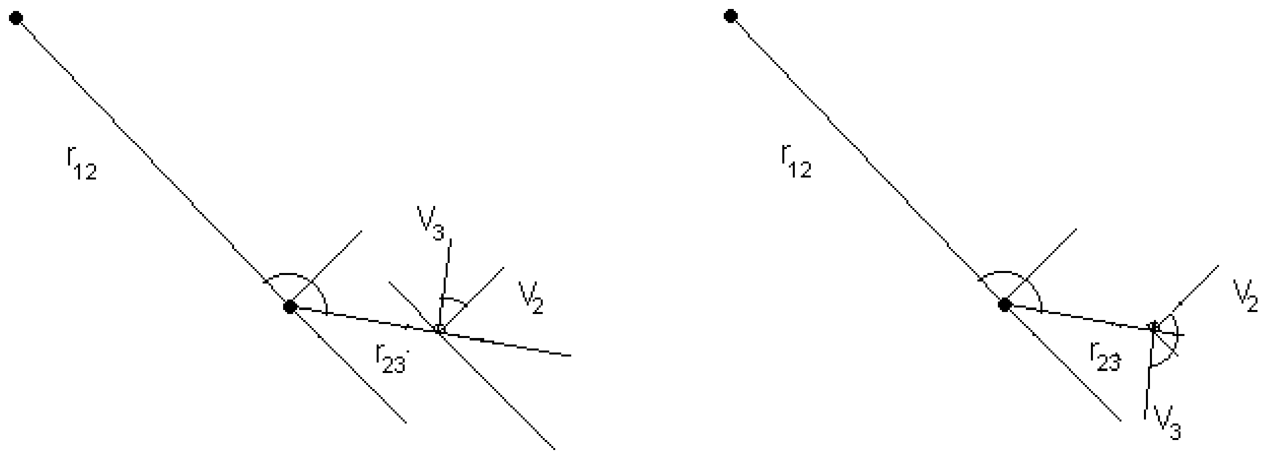
$$h_{23}(\mathbf{X}, \dot{\mathbf{X}}) = \frac{1}{2}\dot{\mathbf{X}}^2 - \frac{\mu}{|\mathbf{X}|} \le 0 \qquad 0 \le \mu < 1/2$$

for a solution $\phi(t) = (\mathbf{X}, \dot{\mathbf{X}})$ of the elliptic restricted problem relative to P_2, $|\mathbf{X}|>0$

$$\cos\beta \approx \pm\cos(V_2, V_{24}) \qquad \cos\alpha \approx \pm\cos(V_2, V_{23})$$

Sign + is valid for retrograde, sign – for prograde orbits (Fig.).

$$\Delta_{13} = \sqrt{r_{12}^2 + r_{23}^2 - 2r_{12}r_{23}\cos(\alpha)} \qquad \Delta_{14} = \sqrt{r_{12}^2 + r_{24}^2 - 2r_{12}r_{24}\cos(\beta)}$$



At least, orbits of m_1, m_2 and m_4 are not change during encounter. Let m_2 move on fixed circular orbit around m_1 and m_4 move on fixed circular orbit around m_2 :

$$H' = H - \left(-m_2 + m_4 + m_3\right) V_2^2 / 2 - 1/2 m_4 V_{24}^2 + f \frac{m_2 m_4}{r_{24}} \approx const$$

Finally:

$$h_{23} = H' \pm m_3 V_2 \sqrt{\frac{m_2}{r_{23}}(1 + e_{23})} \cos\alpha_b + \frac{m_3}{\sqrt{r_{12}^2 + r_{23}^2 - 2 r_{23} r_{12} \cos\alpha_b}} \pm m_4 V_2 \sqrt{\frac{m_2}{r_{24}}} \cos\beta_b +$$

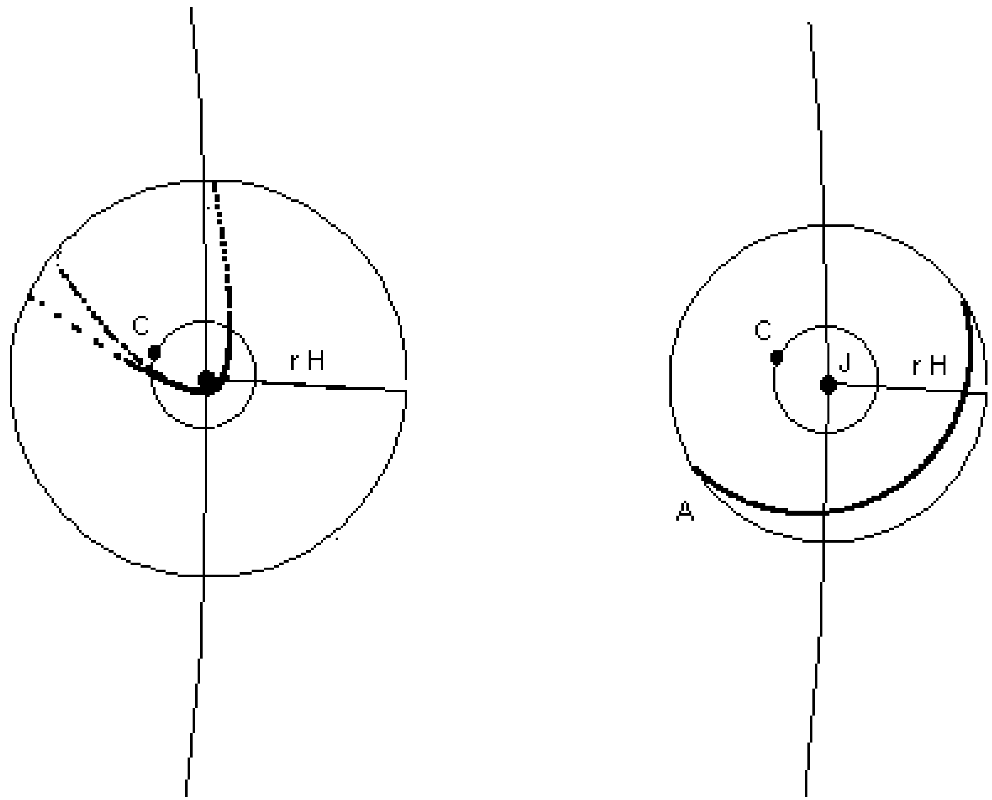
$$+ \frac{m_4}{\sqrt{r_{12}^2 + r_{24}^2 - 2 r_{24} r_{12} \cos\beta_b}} + \frac{m_4 m_3}{\sqrt{r_{24}^2 + r_{23}^2 - 2 r_{23} r_{24} \cos\gamma}}$$

After using Legendre expansion:

$$h_{23} = H' + m_3\left[\pm\sqrt{\frac{m_2}{r_{23}} + \frac{r_{23}}{r_{12}}}\right]\cos\alpha_f + m_3/2\left[\frac{r_{23}}{r_{12}}\right]^2\left(3\cos^2\alpha_f - 1\right) +$$

$$+ m_4\left[\pm\sqrt{\frac{m_2}{r_{24}}} + \left[\frac{r_{24}}{r_{12}}\right]\right]\cos\beta_f + m_4/2\left[\frac{r_{24}}{r_{12}}\right]^2\left(3\cos^2\beta_f - 1\right) + \frac{m_4 m_3}{\Delta_{34}}$$

Application to Jupiter System



Two kinds of capture orbits

Second case. The perturbing satellite is far from test particle m_3. We can neglect last term and terms, proportional m_3. The necessary conditions for capture are:

$$h_{23b} = m_3\left[\sqrt{\frac{m_2}{r_{23}}(1+e_{23b})}\cos\alpha_b\right] + m_3\frac{r_{23}}{r_{12}}(\cos\alpha_b) + m_4\left[\sqrt{\frac{m_2}{r_{24}}} + \frac{r_{24}}{r_{12}}\right](\cos\beta_b) > 0$$

$$h_{23f} = m_3\left[\sqrt{\frac{m_2}{r_{23}}(1+e_{23f})}\cos\alpha_f\right] + m_3\frac{r_{23}}{r_{12}}(\cos\alpha_f) + m_4\left[\sqrt{\frac{m_2}{r_{24}}} + \frac{r_{24}}{r_{12}}\right](\cos\beta_f) < 0$$

For Jupiter and Callisto:

$$h_{23b} = \left[\sqrt{\frac{0.001}{0.113}(1+e_{23b})}\cos\alpha_b\right] + 0.113(\cos\alpha_b) + \frac{m_4}{m_3}\left[\sqrt{\frac{0.001}{0.012}} + 0.012\right](\cos\beta_b) > 0$$

$$h_{23f} = \left[\sqrt{\frac{0.001}{0.113}(1+e_{23f})}\cos\alpha_f\right] + 0.113(\cos\alpha_f) + \frac{m_4}{m_3}\left[\sqrt{\frac{0.001}{0.012}} + 0.012\right](\cos\beta_f) < 0$$

Rough estimation, based on Kepler's law $\beta \approx 27\alpha$

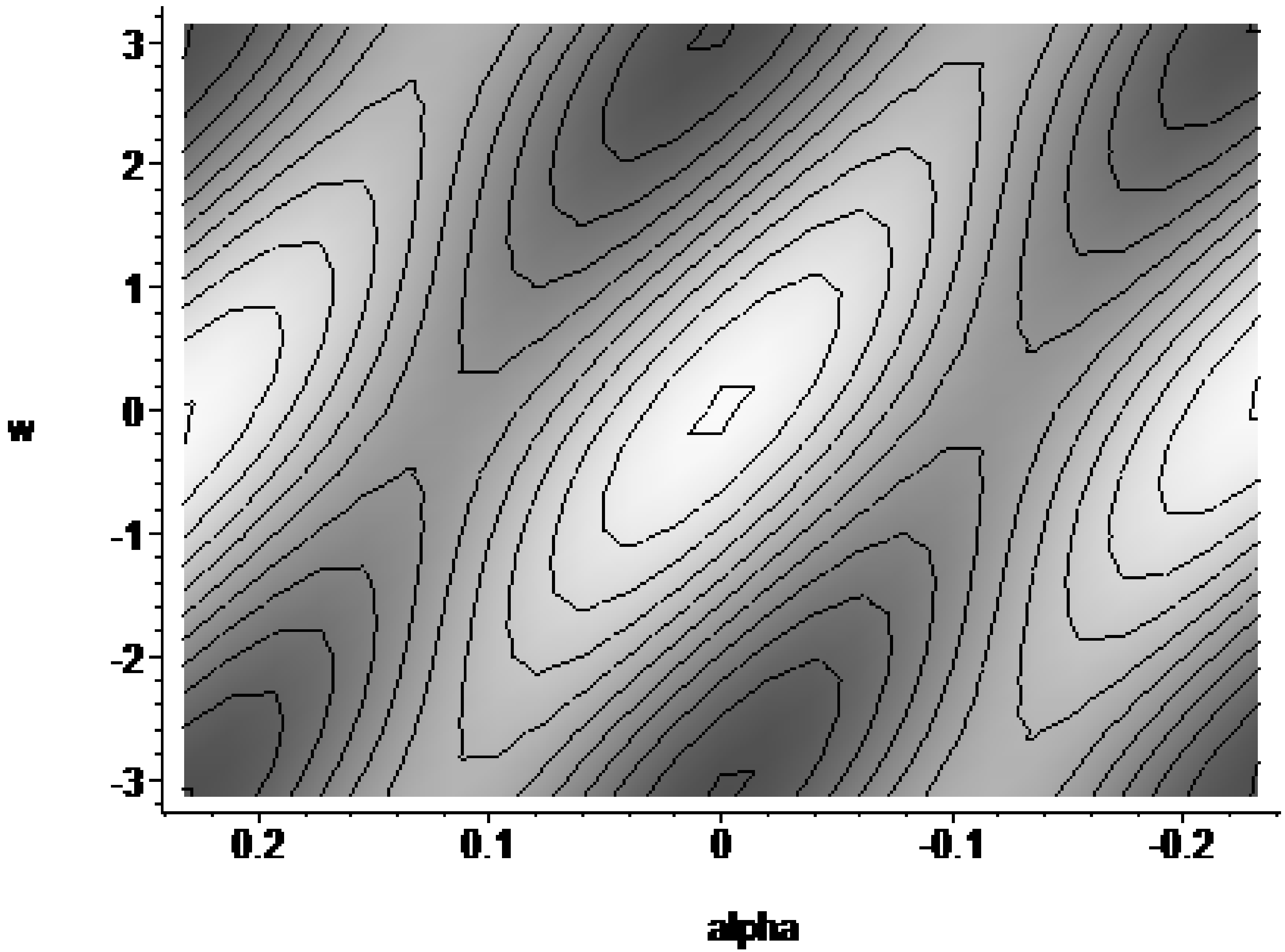
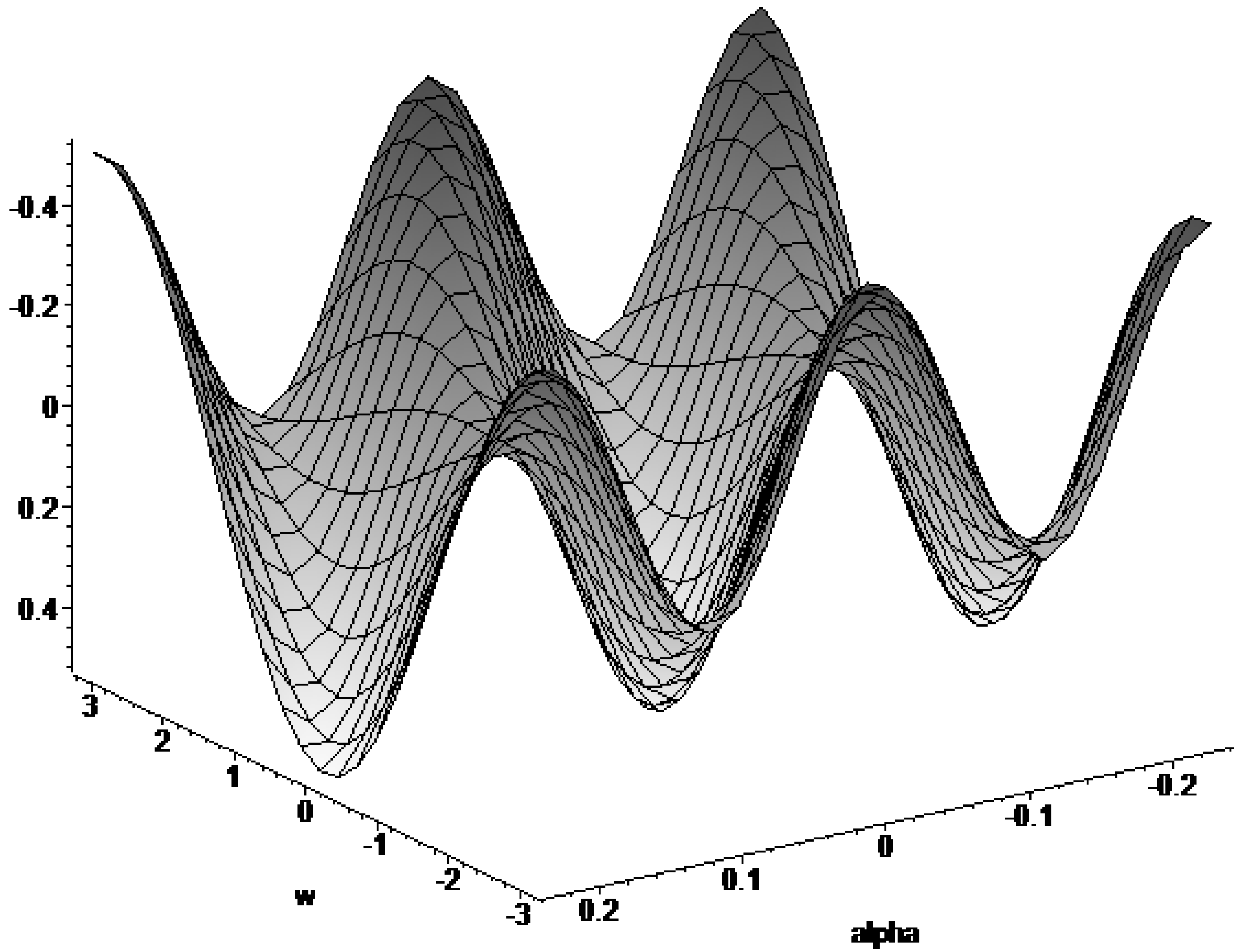


Fig. Energy of capture h_{23} on pericentre longitude w and true anomaly alpha dependence. Dark area respect negative energy.



Energy of capture h_{23} on pericentre longitude w and true anomaly alpha dependence. 3D view.

In energetic calculation, eccentricity change cannot be properly estimated. On the other hand, at action sphere boundary, e<1 not guarantee capture. So we put e=0.5 fixed (fig.). Similar character of h(f) dependence hold in wide range of e 0.1<e<1.95.

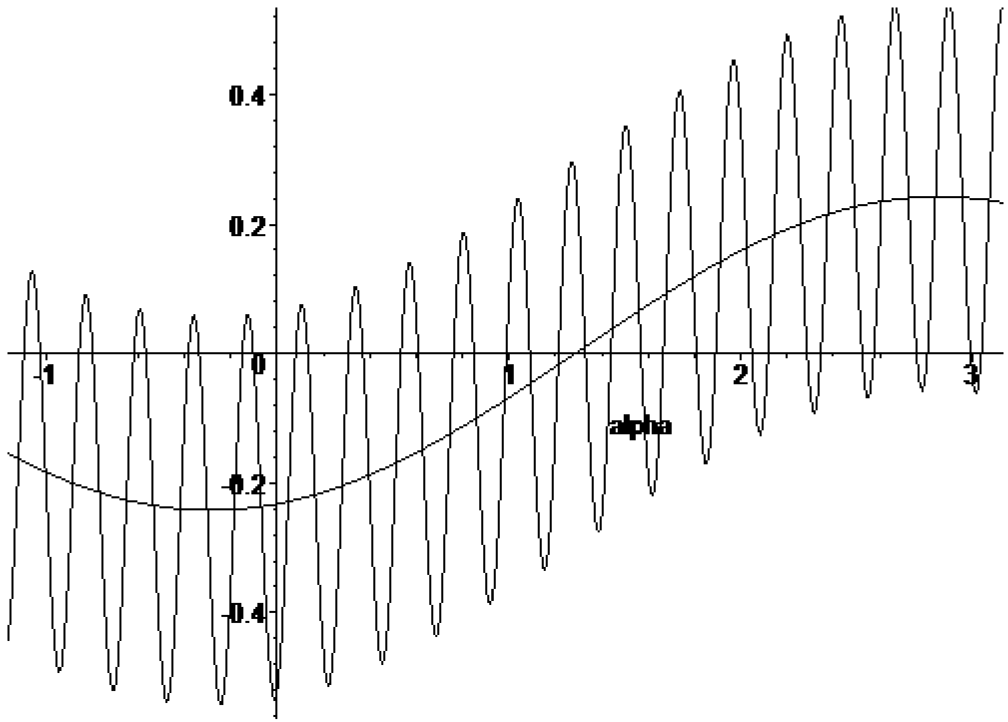


Fig. Compare with 3-body probllem.

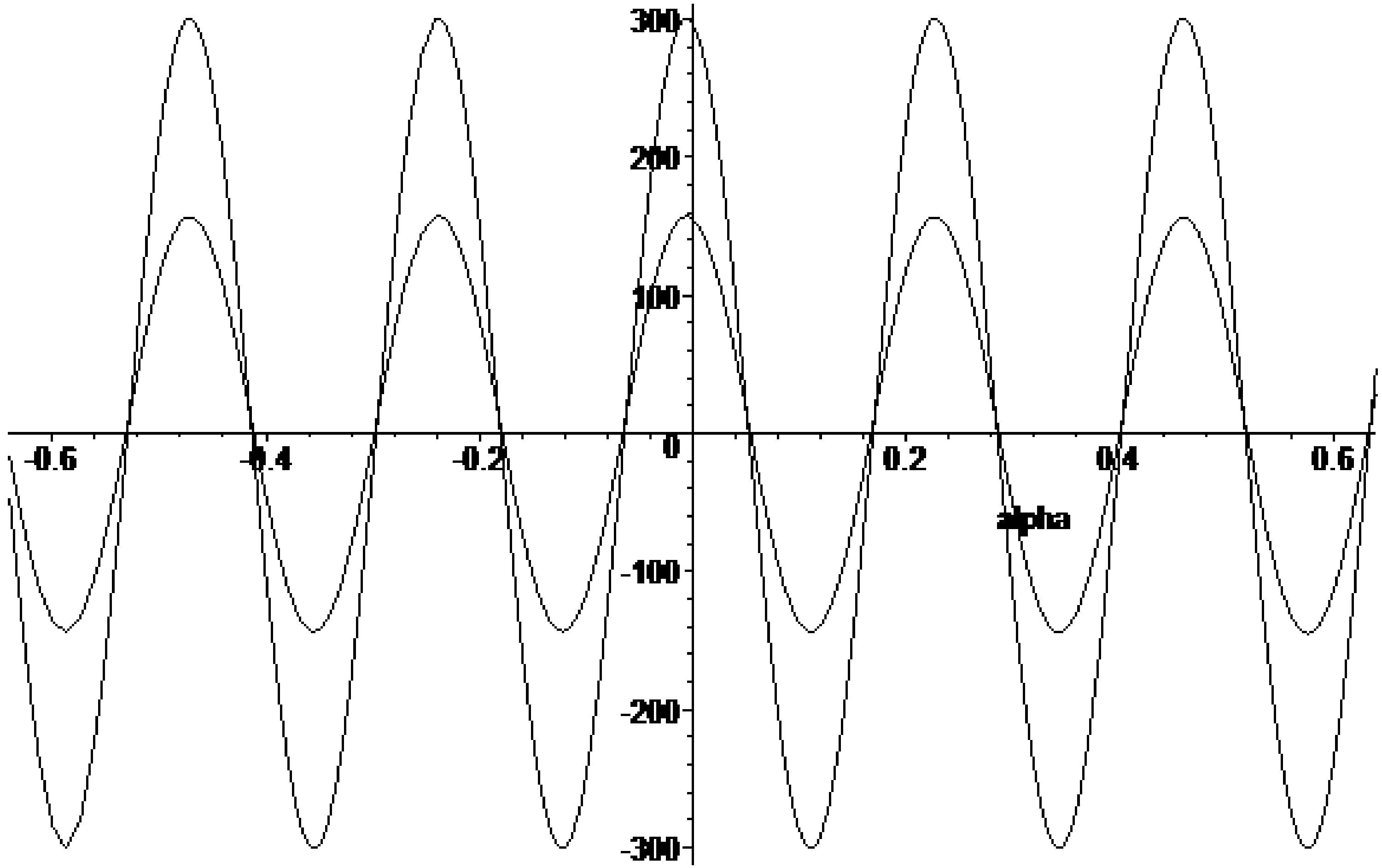


Fig. Test particle relative energy (*h23*) of on phase *f* dependence.
Solid line: *m4/m3*=1, dashed line: *m4/m3*=1000. Prograde orbit.

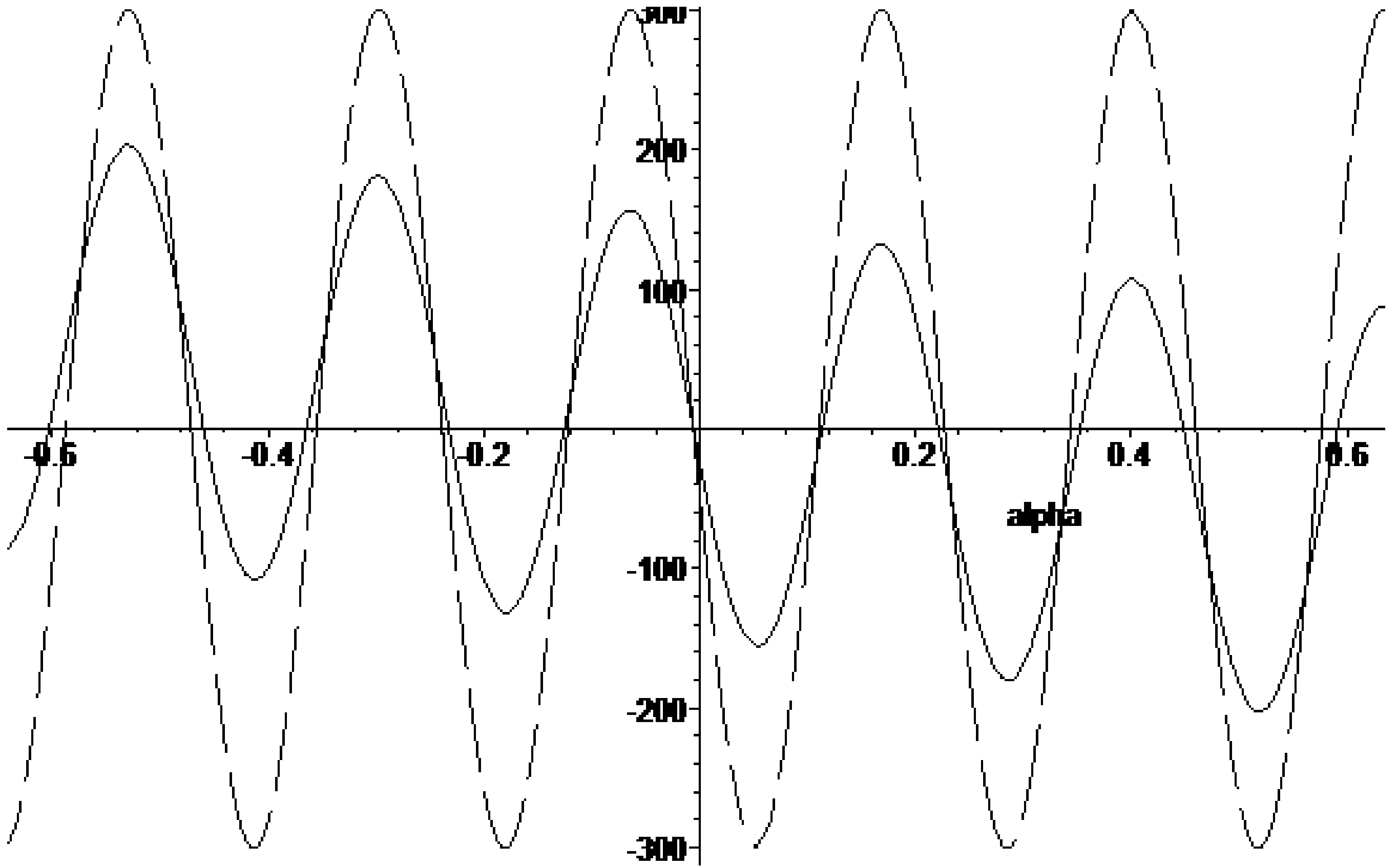


Fig. Test particle relative energy (*h23*) of on phase *f* dependence.
Solid line: *m4/m3*=1, dashed line: *m4/m3*=1000. Retrograde orbit.

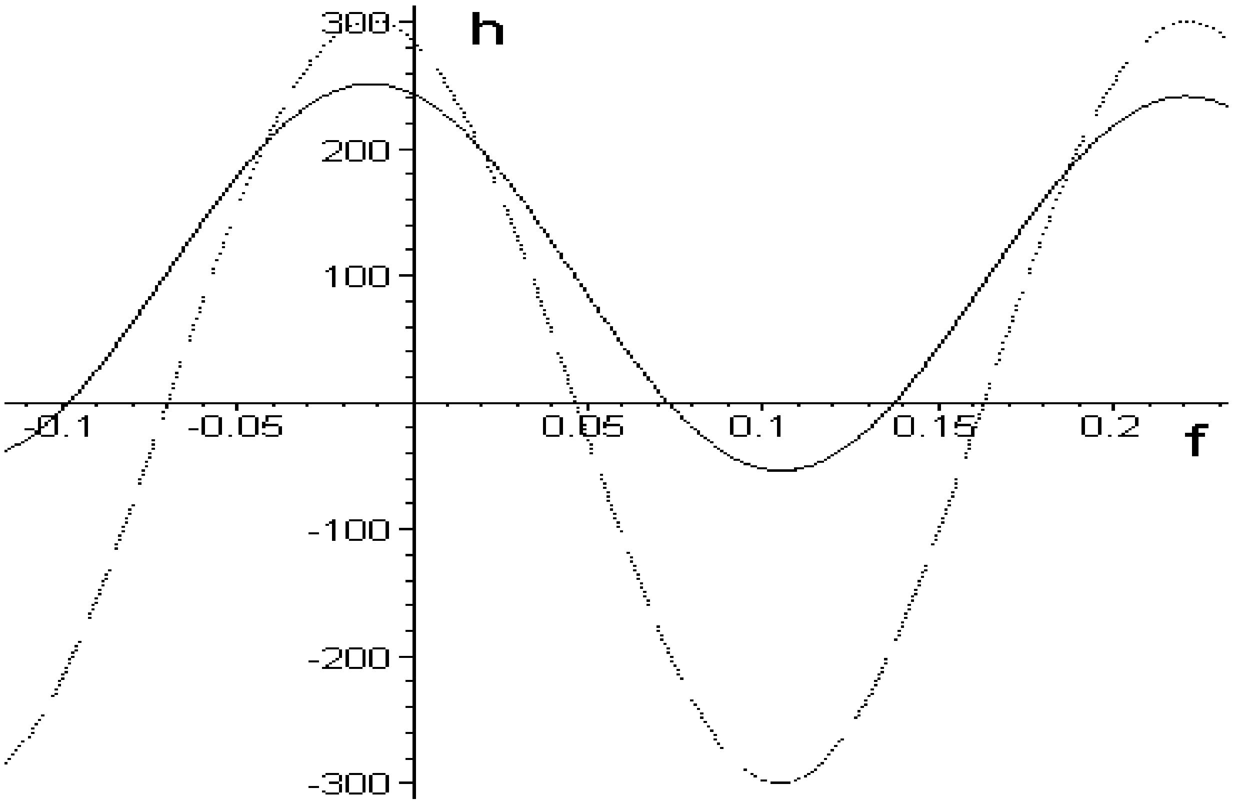


Fig. Test particle relative energy (*h23*) of on phase *f* dependence.
Solid line: *m4/m3*=1, dashed line: *m4/m3*=1000.

Main Results

- The expression for test particle energy in restricted four body problem is derived:

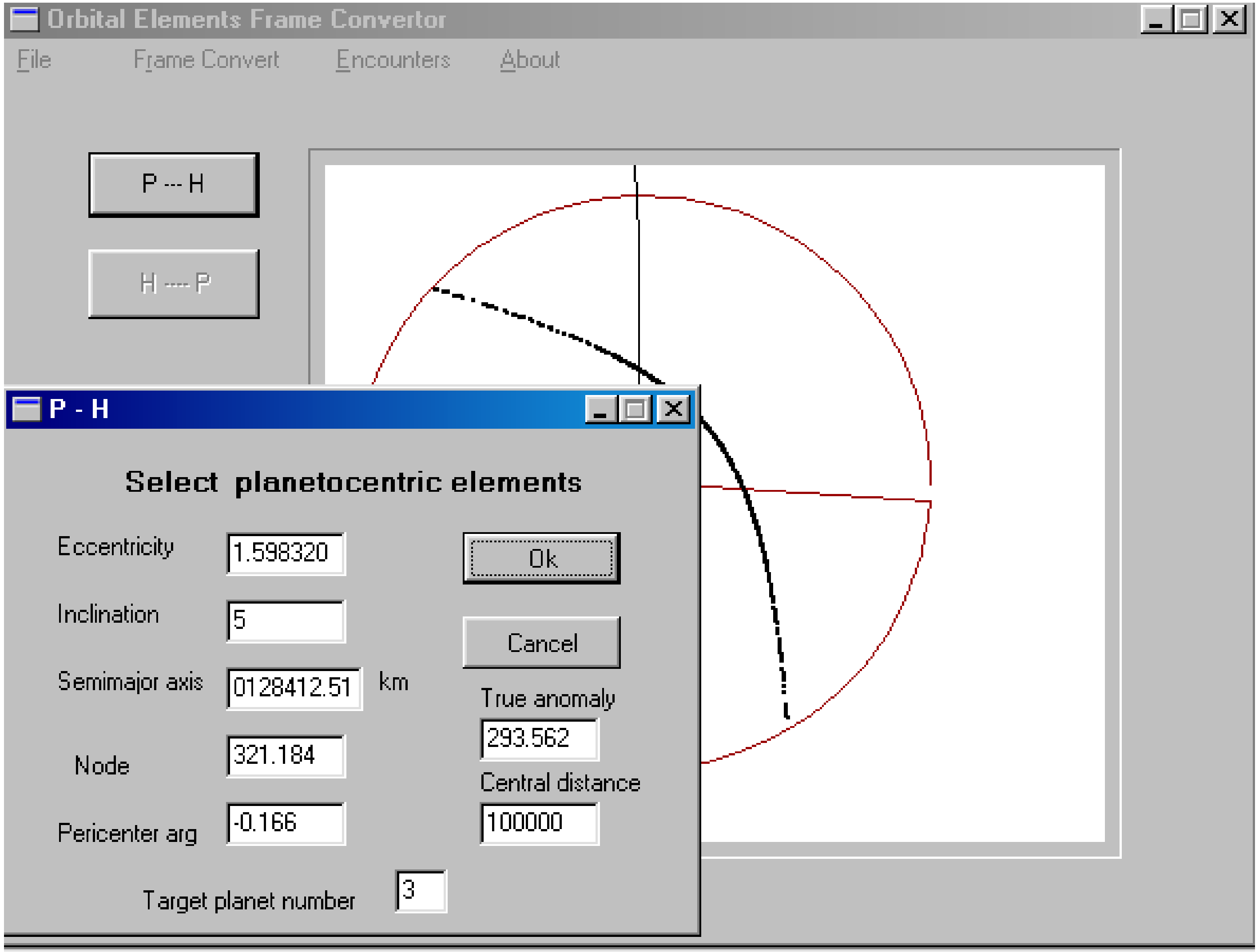
$$h_{23} = H' \pm m_3 V_2 \sqrt{\frac{m_2}{r_{23}}(1+e_{23})}\cos\alpha_b + \frac{m_3}{\sqrt{r_{12}^2 + r_{23}^2 - 2r_{23}r_{12}\cos\alpha_b}} \pm m_4 V_2 \sqrt{\frac{m_2}{r_{24}}}\cos\beta_b +$$

$$+\frac{m_4}{\sqrt{r_{12}^2 + r_{24}^2 - 2r_{24}r_{12}\cos\beta_b}} + \frac{m_4 m_3}{\sqrt{r_{24}^2 + r_{23}^2 - 2r_{23}r_{24}\cos\gamma}}$$

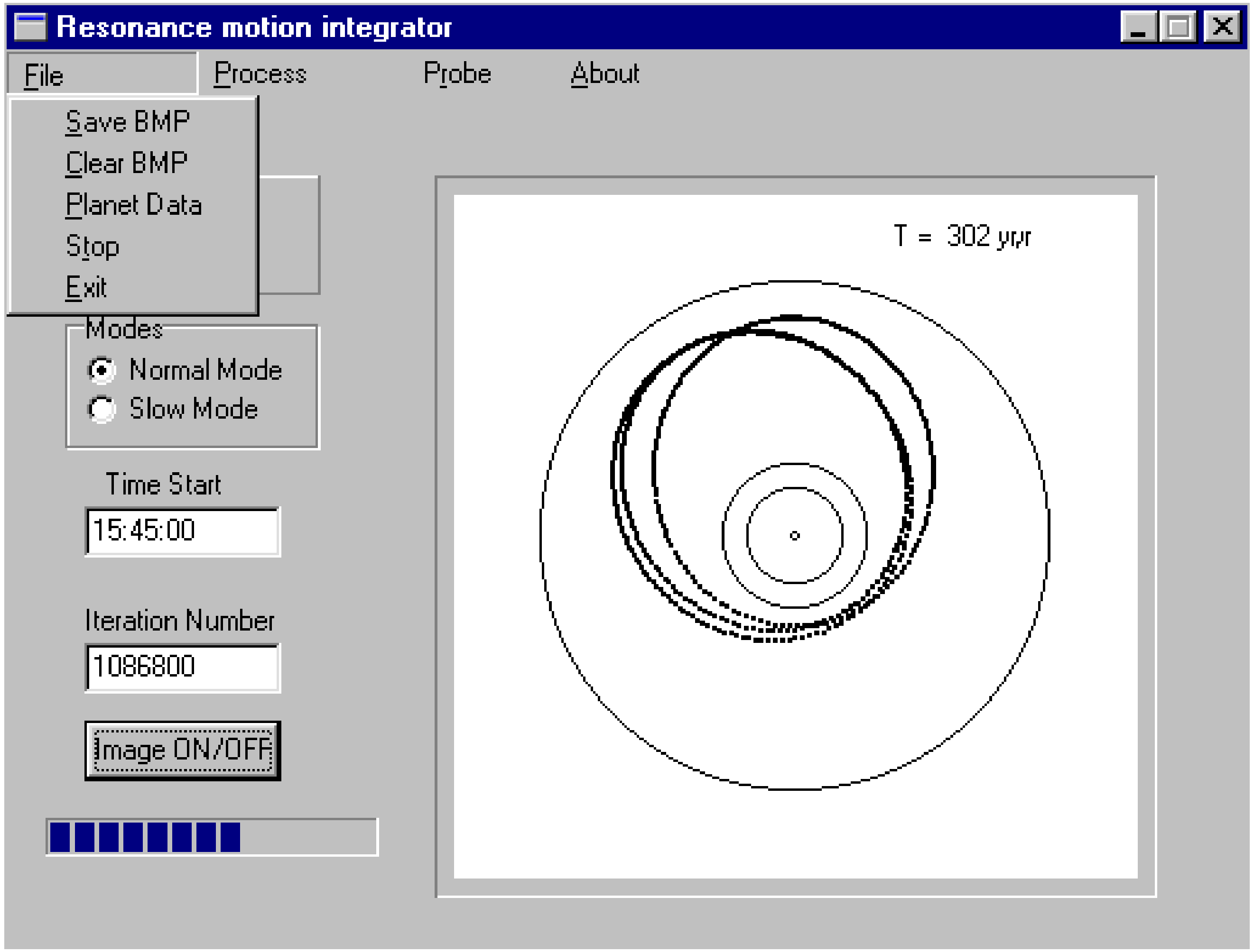
- Possible new ability for energy dissipation relative 3 body problem
- The possibility of capture large object on retrograde orbit is found

NUMERIC TOOLS

View of action sphere program screen

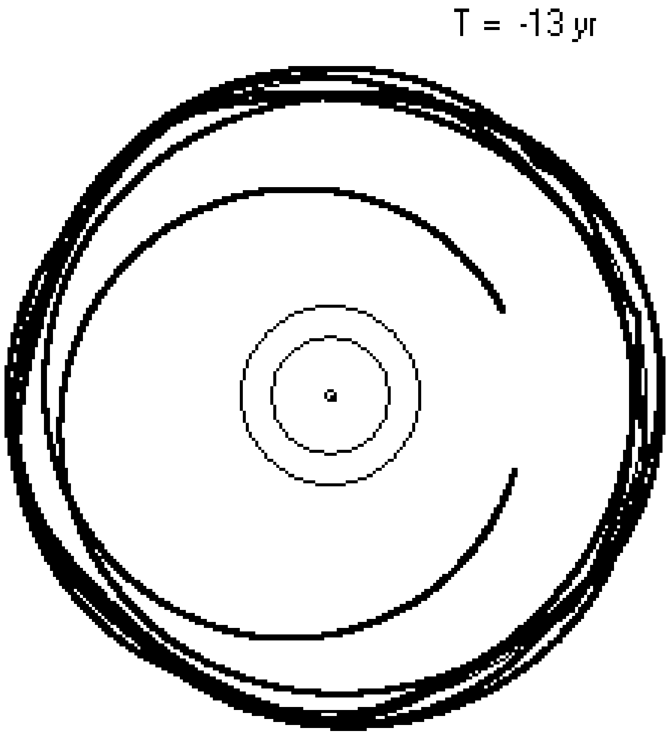


View of numeric integration program screen

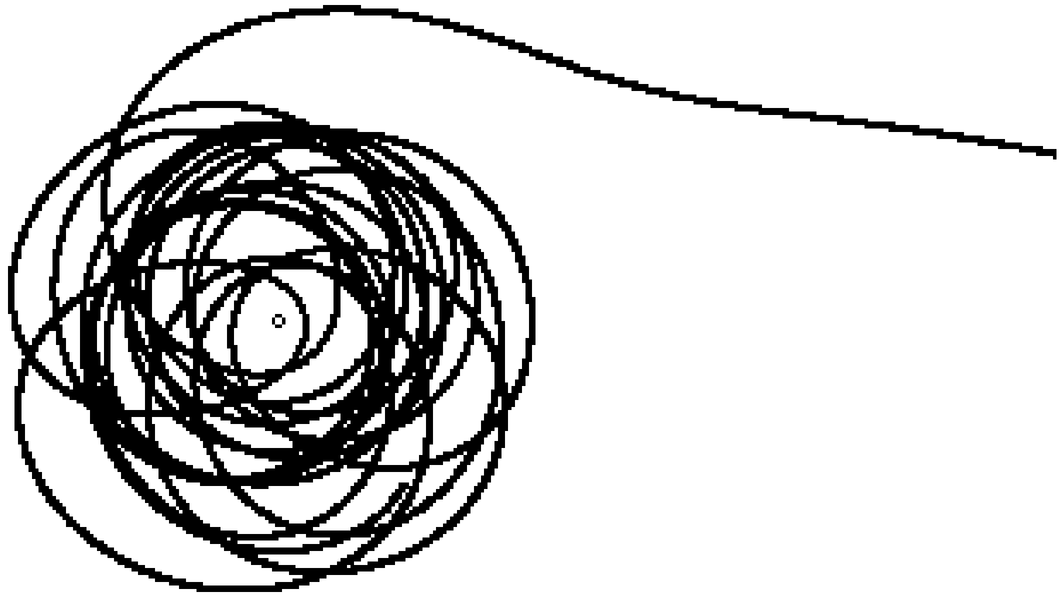


Heliocentric orbit of temporary Jupiter satellite capture

	Elements before capture	Elements after capture
a, a.u	3.8955513	3.8893871
e	0.18052845	0.17267844



Planetocentric orbit of temporary Jupiter satellite capture



Conclusions

The capture process in restricted four body problem (RFBP) is researched. There are two kinds of capture possibility. We consider motion infinitesimal particle at a boundary of action sphere of planet, far from a perturbing satellite.

The expression for test particle energy is derived. The map in a phase space in RFBP model is much more complex, than in a three body problem. Possible it gives new abilities for energy dissipation.

A significant difference between case prograde and retrograde orbits is found, maybe capture in four body problem is more probable on retrograde than on prograde orbit.

Future works

- Eccentricity $e<1$ is not a criteria of capture. Is $h<0$ a true criteria?
- Numeric proof of retrograde capture preference