THE MOTION OF A PARTICLE AROUND THE AGRANGIAN POINT L4 CONSIDERING RADIATION PRESSURE

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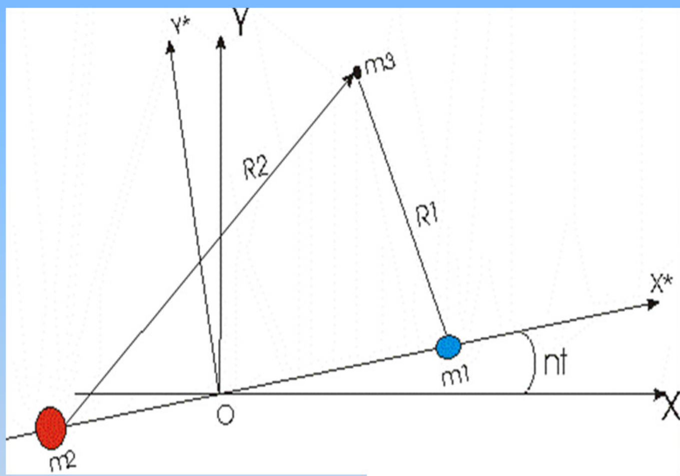
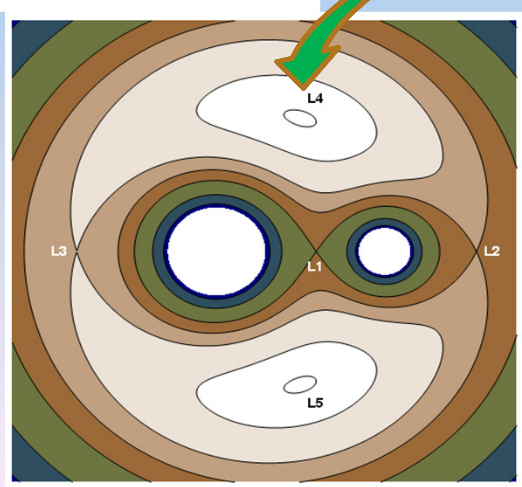
SUMMARY

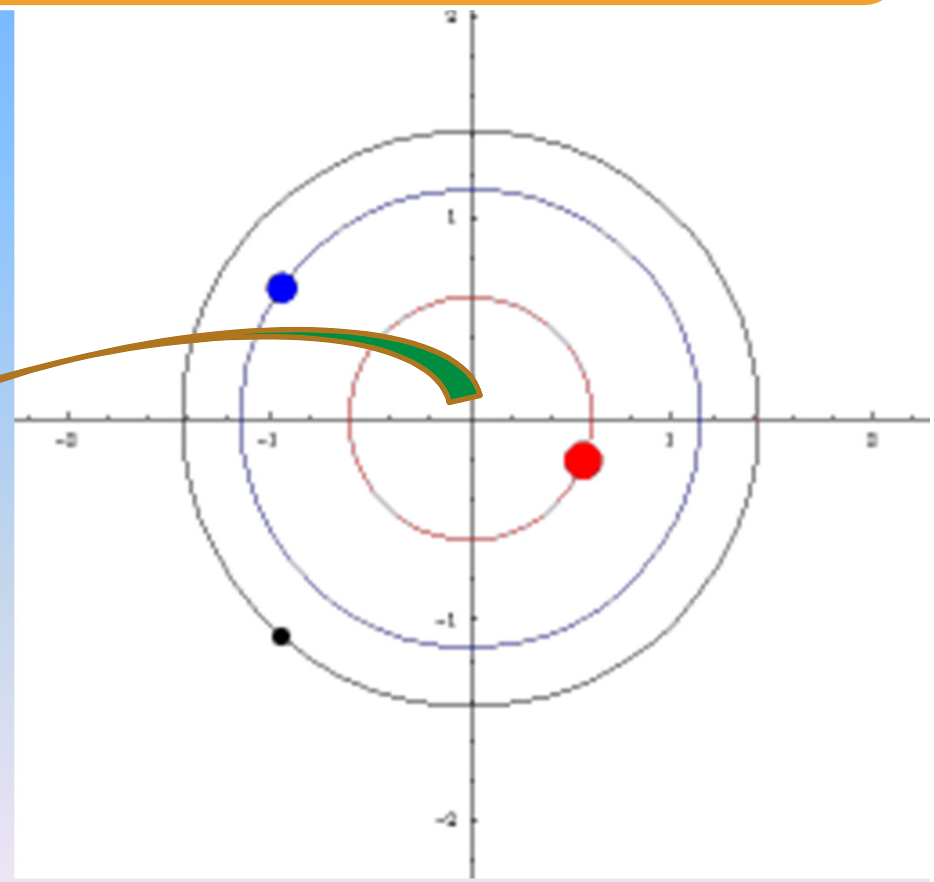
- INTRODUCTION
- WHAT IS THE PCRTBP+RP
- THEOREM OF KOVALEV AND SAVCHENKO (ARNOLD)
- NORMAL FORM
- PROGRAM TO ANALIZE THE STABILITY
- RESULTS

PCRTBP+RP

Restrict Planar Three Body Prolem With Solar Radiation Pressure







Lagrangian Points

The Hamiltonian

(Simmon´s et al.,1985):

$$U(x,y) = -\frac{1}{2}(x^2 + y^2) - \frac{k_1(1-\mu)}{r_1} - \frac{k_2\mu}{r_2}$$

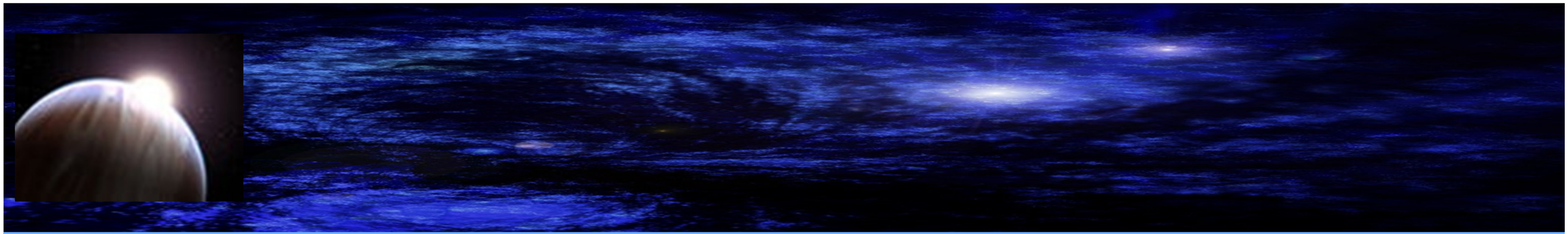
$$\mu_2 = \mu \quad e \quad \mu_1 = 1 - \mu \qquad \mu = \frac{m_2}{m_1 + m_2}$$

$$r_1 = \sqrt{(x-\mu)^2 + y^2} \qquad r_2 = \sqrt{(x+1-\mu)^2 + y^2}$$

PCRTBP + RP

$$k_i = 1 - \frac{F_r}{F_g} = 1 - \frac{3L}{16\pi c \mathrm{GM}\rho s} \quad , i = 1{,}2$$

- $k_i < 0$ ⟹ $F_r < F_g$
- $k_i > 0$ ⟹ $F_r > F_g$
- $k_i = 1$ ⟹ $F_r = 0$



Using canonical variables (x, y, p_x, p_y), (Szebehely, 1967; Kumar and Choudry, 1987):

$$\frac{dx}{dt} = \frac{\partial H}{\partial p_x}, \frac{dy}{dt} = \frac{\partial H}{\partial p_y} \quad \text{and} \quad \frac{dp_x}{dt} = -\frac{\partial H}{\partial x}, \frac{dp_y}{dt} = -\frac{\partial H}{\partial y}$$

where

$$H = \frac{1}{2}({p^2}_x + {p^2}_y) + p_x y - p_y x - \frac{{\kappa^3}_1(1-\mu)}{r_1} - \frac{{\kappa^2}_2\mu}{r_2}$$

and

$$k_1 = \kappa_1^3 \ \text{ and } \ k_2 = \kappa_2^3$$

Non-linear stability

Theorem of Kovalev & Savchenko

Let us consider the normalized Hamiltonian up to the order 4

$$Z = \frac{1}{2}\sum_{k=1}^{n} \omega_k (q_k^2 + p_k^2) + \sum_{i,j=1}^{2} \delta_{ij}(q_k, p_k) + O(5)$$

An equilibrium solution is LYAPUNOV stable if:

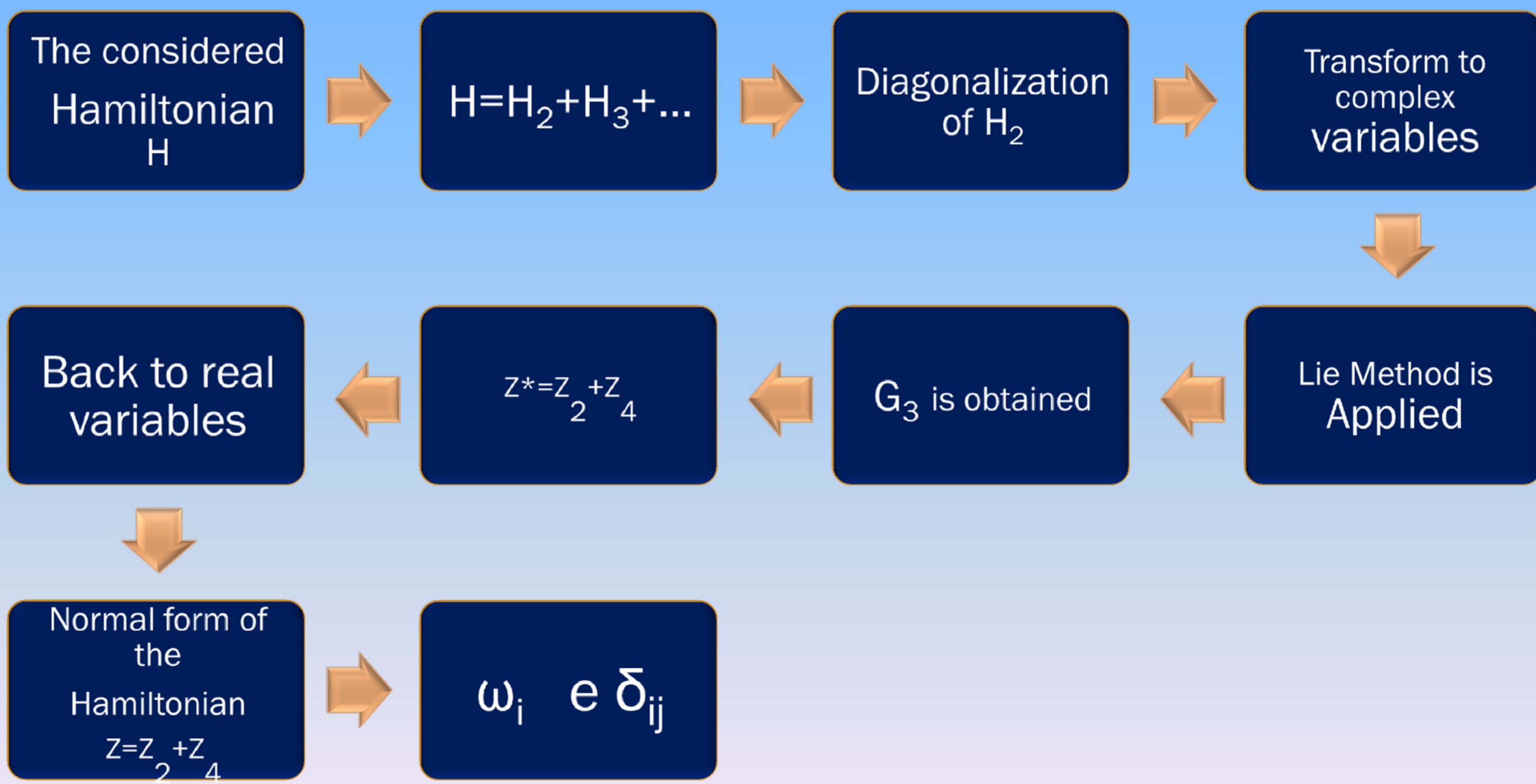
i) The eigenvalues associated to the quadratic part of the Z, are pure imaginary

ii) $k_1\omega_1 + k_2\omega_2 \neq 0, \quad 0 \leq |k_1| + |k_2| \leq 4$

iii) $D = -(\delta_{11}\omega_2^2 - 2\delta_{12}\omega_1\,\omega_2 + \delta_{22}\omega_1^2) \neq 0$

Constructing the model

ALGORITHM



Algorithm

Inputs for normal form and stability criterium

1-Input: equilibrium points

2-Input eigenvalues of Z_2: i=1,2,3 ω_i

3-Input: $h_{3,\alpha,\beta}$ and $h_{4,\alpha,\beta}$ (H_3 and H_4)

Output

$$k_1\omega_1 + k_2\omega_2 \neq 0,$$
$$0 \leq |k_1| + |k_2| \leq 4$$

$$\delta_{ij}$$

Determinant
D
(instability points)

What to do???

Stability criterium

Input: ω_i e δ_{ij}

1st condition is fulfilled: $\alpha_i = i\omega$???

2nd condition? $k_1\omega_1 + k_2\omega_2 \neq 0, \quad 0 \leq |k_1| + |k_2| \leq 4$

Test the 3rd condition: $D \neq 0$

Normal Form

Expanded Hamiltonian

$$H = H_2 + H_3 + H_4 + \ldots., \quad \textbf{(1)}$$

Quadratic part

$$H_2 = \frac{1}{2}\sum_{k=1}^{n}\omega_k(q_k^2 + p_k^2) \quad \textbf{(2)}$$

For j>2,

$$H_j = \sum_{k=1}^{n} h_{j,\alpha,\beta}\, q_k^{\alpha} p_k^{\beta} \quad \textbf{(3)}$$

Where, for H_3 , $\alpha+\beta=3$ and $\alpha+\beta=4$ for H_4

Normal Form

$$(q_k, p_k) \rightarrow (x_k^*, y_k^*)$$

$$q_k = \frac{1}{\sqrt{2}}(x_k^* - iy_k^*), \; p_k = \frac{i}{\sqrt{2}}(x_k^* + iy_k^*), \; k = 1,2,\ldots \quad (5)$$

New Hamiltonian

$$H^* = \sum_{k=1}^{4} \lambda_k x_k^* y_k^* + \sum_{k>2}^{4} h_{k\alpha\beta} x_k^{*\alpha} y_k^{*\beta} \quad \textbf{(6)}$$

Normal Form: Lie method

We will get a normal form using Lie series

$$Z = H^* + \{H^*, G\} + \frac{1}{2!}\{\{H^*, G\}, G\} + \frac{1}{3!}\{\{\{H^*, G\}, G\}, G\} + \frac{1}{4!}\{\{\{\{H^*, G\}, G\}, G\}, G\} + \ldots.,$$

Through the method Lie the Hamiltoniana is rearranged as:

Degree 2: $Z_2 = H_2^*$

Degree 3: $Z_3 = H_3^* + \{H_2^*, G_3\}$ **(7)**

Degree 4: $Z_4 = H_4^* + \{H_3^*, G_3\} + \frac{1}{2!}\{\{H_2^*, G_3\}, G_3\} + \{H_2^*, G_4\}$

where

$$G_3 = \sum (-h_{\alpha,\beta} x^\alpha y^\beta) / < \beta - \alpha, W > \quad \textbf{(8)}$$

Normal Form : Lie Method

Through the homologic equation, we get:

$$F_4 = H_4^* + \frac{1}{2!}\{H_3^*, G_3\}$$

$$Z_4 = F_4 - \frac{1}{2!}\{H_2{}^*, G_3\}$$

Normal Form

Two degrees of freedom

$$Z_2 = \frac{1}{2}\omega_1(q_1^2 + p_1^2) + \frac{1}{2}\omega_2(q_2^2 + p_2^2)$$

$$Z_4 = \delta_{11}(x_1\, y_1\,)^2 + \delta_{12}(x_1\, y_1\, x_2\, y_2\,) + \delta_{12}\,(x_2\, y_2\,)^2$$

δ_{ij} are real coeficients , combination of $h_{k,l,m,n}$ with the frequencies ω_i (i=1,2).

$$Z(\overline{q}_1, \overline{p}_1, \overline{q}_2, \overline{p}_2) = Z_2(\overline{q}_1, \overline{p}_1, \overline{q}_2, \overline{p}_2) + Z_4(\overline{q}_1, \overline{p}_1, \overline{q}_2, \overline{p}_2)$$

Normal Form

Three degrees of freedom

Real coordinates

$$Z_2 = \frac{1}{2}\omega_1(q_1^2 + p_1^2) + \frac{1}{2}\omega_2(q_2^2 + p_2^2) + \frac{1}{2}\omega_3(q_3^2 + p_3^2)$$

$$\begin{aligned} Z_4 = {} & \delta_{11}(q_1^4 + p_1^4 + 2q_1^2 p_1^2) + \delta_{22}(q_2^4 + p_2^4 + 2q_2^2 p_2^2) + \delta_{33}(q_3^4 + p_3^4 + 2q_3^2 p_3^2) \\ & + \delta_{12}(q_1^2 q_2^2 + q_1^2 p_2^2 + p_1^2 q_2^2 + p_1^2 p_2^2) + \delta_{13}(q_1^2 q_3^2 + q_1^2 p_3^2 + p_1^2 q_3^2 + p_1^2 p_3^2) + \\ & + \delta_{23}(q_2^2 q_3^2 + q_2^2 p_3^2 + p_2^2 q_3^2 + p_2^2 p_3^2) \end{aligned}$$

$$Z(q_i, p_i) = Z_2(q_i, p_i) + Z_4(q_i, p_i), \quad \mathrm{i} = 1,2,3$$

Example :CPRTBP with RP

$$H = H_2 + H_3 + H_4$$

$$H_2 = \frac{1}{2}(p_1^2 + p_2^2) + p_1 q_2 - p_2 q_1 + \alpha q_1^2 + \beta q_1 q_2 + \gamma q_2^2$$

$$H_3 = \mathrm{h}_{3,3,0,0,0}\ q_1^3 + \ \mathrm{h}_{3,2,0,1,0}\ q_1^2 q_2 + \mathrm{h}_{3,1,0,2,0}\ q_2^2 q_1 + \mathrm{h}_{3,0,0,3,0} q_2^3$$

$$H_4 = \mathrm{h}_{4,4,0,0,0} q_1^4 + \mathrm{h}_{4,3,0,1,0} q_1^3 q_2 + \mathrm{h}_{4,2,0,2,0}\ q_1^2 q_2^2 + \mathrm{h}_{4,1,0,3,0} q_1\, q_2^3 + \mathrm{h}_{4,0,0,4,0} q_2^4$$

Example

H_2:

$$\alpha = \left[\frac{1}{2} - (1-\mu)\frac{3(\kappa_1^2 + 1 - \kappa_2^2)^2}{8\kappa_1^2} - \mu \frac{3(\kappa_1^2 + 1 - \kappa_2^2)^2}{8\kappa_2^2} \right]$$

$$\beta = -\frac{\sqrt{b}}{\kappa_1 \kappa_2} \left[\frac{-3\kappa_1^2 \mu (\kappa_2^2 + 1 - \kappa_1^2) + 3\kappa_2^2 (1-\mu)(\kappa_1^2 + 1 - \kappa_2^2)}{2} \right]$$

$$\gamma = -\frac{3b\kappa_2^2 (1-\mu) - \mu\kappa_1^2 + 3b}{2}$$

Example

H_3:

$$h_{3,3,0,0,0} = \frac{1}{14}[\frac{\mu}{\kappa_2^4}(\kappa_1^2 - 1 - \kappa_2^2)(5(\kappa_1^2 - 1 - \kappa_2^2)^2 - 12\kappa_2^2) + \frac{(1-\mu)}{\kappa_1^4}(\kappa_1^2 + 1 - \kappa_2^2)$$
$$(5(\kappa_1^2 + 1 - \kappa_2^2)^2 - 12\kappa_1^2)]$$

$$h_{3,2,0,1,0} = \frac{4}{9}\sqrt{b}[\mu\kappa_1\kappa_2^{-3}(5(\kappa_1^2 - 1 - \kappa_2^2)^2 - 4\kappa_2^2) + (1-\mu)\kappa_2\kappa_1^{-3}(5(\kappa_1^2 + 1 - \kappa_2^2)^2 - 4\kappa_1^2)]$$

$$h_{3,1,0,2,0} = \frac{3}{4}[\frac{\mu}{\kappa_2^2}(\kappa_1^2 - 1 - \kappa_2^2)(3b\kappa_1^2 - 1) + \frac{(1-\mu)}{\kappa_1^2}(\kappa_1^2 - 1 - \kappa_2^2)(3b\kappa_2^2 - 1)]$$

$$h_{3,0,0,3,0} = \frac{1}{9}\sqrt{b}[\mu\kappa_1\kappa_2^{-1}(7b\kappa_2^2 - 3) + (1-\mu)\kappa_2\kappa_1^{-1}(5b\kappa_2^2 - 3)]$$

Example

H_4:

$$h_{4,4,0,0,0} = -\frac{1}{9}[(1-\mu)\left(\kappa_1^{-6}(3\kappa_1^4 - \frac{15}{2}(\kappa_1^2+1-\kappa_2^2)^2\kappa_1^2 + \frac{33}{17}(\kappa_1^2+1-\kappa_2^2)^4\right) + \mu\left(\kappa_2^{-6}(3\kappa_2^4 - \frac{15}{2}(\kappa_1^2-1-\kappa_2^2)^2\kappa_2^2 + \frac{35}{16}(\kappa_1^2-1-\kappa_2^2)^4\right)]$$

$$h_{4,3,0,1,0} = -\frac{7}{5}\sqrt{b}[\kappa_2\kappa_1^{-3}(1-\mu)(\kappa_1^2+1-\kappa_2^2)(3-\frac{7}{5}(\kappa_1^2+1-\kappa_2^2)^2\kappa_1^{-1}) + \mu\,\kappa_1\kappa_2^{-3}(\kappa_1^2-1-\kappa_2^2)(3-\frac{7}{4}(\kappa_1^2-1-\kappa_2^2)^2\kappa_2^{-2})]$$

$$h_{4,2,0,2,0} = \frac{4}{5}[(1-\mu)\kappa_1^{-2}(5\kappa_2 b + \frac{5}{4}b\kappa_2^2 + \frac{5}{4}(\kappa_1^2+1-\kappa_2^2)^2\kappa_1^{-2} - \frac{37}{5}(\kappa_1^2+1-\kappa_2^2)^2 b\kappa_2^{-2}\kappa_1^{-2} - 1) + \mu\kappa_2^{-2}(5\kappa_1 b + \frac{5}{4}b\kappa_1^2 + \frac{5}{4}(\kappa_1^2-1-\kappa_2^2)^2\kappa_2^{-2}$$
$$-\frac{35}{4}(\kappa_1^2-1-\kappa_2^2)^2 b\kappa_1^{-2}\kappa_2^{-2} - 1)]$$

$$h_{4,1,0,3,0} = -\frac{5}{4}\sqrt{b}[\kappa_2\kappa_1^{-3}(1-\mu)(\kappa_1^2+1-\kappa_2^2)(3-7\kappa_2^2 b) + \mu\,\kappa_1\kappa_2^{-3}(\kappa_1^2-1-\kappa_2^2)(3-7\kappa_1^2 b)]$$

$$h_{4,0,0,4,0} = \frac{-3}{9}[[(1-\mu)\kappa_1^{-2}(3-30\kappa_2^2 b + 35\kappa_2^4 b^2) + \mu\kappa_2^{-2}(3-30\kappa_1^2 b + 35\kappa_1^4 b^2)]$$

Example

Coeficients

$$\begin{aligned}
\delta_{11} = (3(-96b(&\frac{\kappa 2(-4\kappa 1^2+5(1+\kappa 1^2-\kappa 2^2)^2)(1-\mu)}{\kappa 1^3}+\frac{\kappa 1(-4\kappa 2^2+5(1-\kappa 1^2+\kappa 2^2)^2)\mu}{\kappa 2^3})^2\omega_1^3 \\
&-20(\frac{(1+\kappa 1^2-\kappa 2^2)(-12\kappa 1^2+5(1+\kappa 1^2-\kappa 2^2)^2)(1-\mu)}{\kappa 1^4} \\
&+\frac{(-1+\kappa 1^2-\kappa 2^2)(-12\kappa 2^2+5(1-\kappa 1^2+\kappa 2^2)^2)\mu}{\kappa 2^4})^2\omega_1^2\omega_2 \\
&+256(-\frac{(3\kappa 1^4+\frac{35}{16}(1+\kappa 1^2-\kappa 2^2)^4-\frac{15}{2}(\kappa 1+\kappa 1^3-\kappa 1\kappa 2^2)^2)(1-\mu)}{\kappa 1^6} \\
&-\frac{(3\kappa 2^4+\frac{35}{16}(1-\kappa 1^2+\kappa 2^2)^4-\frac{15}{2}(\kappa 2-\kappa 1^2\kappa 2+\kappa 2^3)^2)\mu}{\kappa 2^6})\omega_1^3\omega_2 \\
&+36b(\frac{\kappa 2(-4\kappa 1^2+5(1+\kappa 1^2-\kappa 2^2)^2)(1-\mu)}{\kappa 1^3}+\frac{\kappa 1(-4\kappa 2^2+5(1-\kappa 1^2+\kappa 2^2)^2)\mu}{\kappa 2^3})^2\omega_1\omega_2^2 \\
&+5(\frac{(1+\kappa 1^2-\kappa 2^2)(-12\kappa 1^2+5(1+\kappa 1^2-\kappa 2^2)^2)(1-\mu)}{\kappa 1^4} \\
&+\frac{(-1+\kappa 1^2-\kappa 2^2)(-12\kappa 2^2+5(1-\kappa 1^2+\kappa 2^2)^2)\mu}{\kappa 2^4})^2\omega_2^3 \\
&-64(-\frac{(3\kappa 1^4+\frac{35}{16}(1+\kappa 1^2-\kappa 2^2)^4-\frac{15}{2}(\kappa 1+\kappa 1^3-\kappa 1\kappa 2^2)^2)(1-\mu)}{\kappa 1^6} \\
&-\frac{(3\kappa 2^4+\frac{35}{16}(1-\kappa 1^2+\kappa 2^2)^4-\frac{15}{2}(\kappa 2-\kappa 1^2\kappa 2+\kappa 2^3)^2)\mu}{\kappa 2^6})\omega_1\omega_2^3))/(4096(4\omega_1^3\omega_2-\omega_1\omega_2^3))
\end{aligned}$$

Example

$$
\begin{aligned}
\delta_{12} = {} & (3(-48b(-\frac{\kappa2(-3+5b\kappa2^2)(-1+\mu)}{\kappa1} + \frac{\kappa1(-3+5b\kappa1^2)\mu}{\kappa2})(\frac{\kappa2(-4\kappa1^2+5(1+\kappa1^2-\kappa2^2)^2)(1-\mu)}{\kappa1^3} \\
& + \frac{\kappa1(-4\kappa2^2+5(1-\kappa1^2+\kappa2^2)^2)\mu}{\kappa2^3})\omega_1^5 - 12(-\frac{(1+\kappa1^2-\kappa2^2)(-1+5b\kappa2^2)(-1+\mu)}{\kappa1^2} \\
& + \frac{(-1+5b\kappa1^2)(-1+\kappa1^2-\kappa2^2)\mu}{\kappa2^2})(\frac{(1+\kappa1^2-\kappa2^2)(-12\kappa1^2+5(1+\kappa1^2-\kappa2^2)^2)(1-\mu)}{\kappa1^4} \\
& + \frac{(-1+\kappa1^2-\kappa2^2)(-12\kappa2^2+5(1-\kappa1^2+\kappa2^2)^2)\mu}{\kappa2^4})\omega_1^4\omega_2 \\
& - 6b(\frac{\kappa2(-4\kappa1^2+5(1+\kappa1^2-\kappa2^2)^2)(1-\mu)}{\kappa1^3} + \frac{\kappa1(-4\kappa2^2+5(1-\kappa1^2+\kappa2^2)^2)\mu}{\kappa2^3})^2\omega_1^4\omega_2 \\
& + 64(\frac{(1-5b\kappa2^2+\frac{5(1+\kappa1^2-\kappa2^2)^2}{4\kappa1^2}-\frac{35b(\kappa2+\kappa1^2\kappa2-\kappa2^3)^2}{4\kappa1^2})(1-\mu)}{\kappa1^2} \\
& + \frac{(1+5b\kappa1^2+\frac{5(1-\kappa1^2+\kappa2^2)^2}{4\kappa2^2}-\frac{35b(\kappa1-\kappa1^3+\kappa1\kappa2^2)^2}{4\kappa2^2})\mu}{\kappa2^2})\omega_1^5\omega_2 \\
& + 96(\frac{(1+\kappa1^2-\kappa2^2)(-1+5b\kappa2^2)(-1+\mu)}{\kappa1^2} - \frac{(-1+5b\kappa1^2)(-1+\kappa1^2-\kappa2^2)\mu}{\kappa2^2})^2\omega_1^3\omega_2^2 \\
& + 204b(-\frac{\kappa2(-3+5b\kappa2^2)(-1+\mu)}{\kappa1} + \frac{\kappa1(-3+5b\kappa1^2)\mu}{\kappa2})(\frac{\kappa2(-4\kappa1^2+5(1+\kappa1^2-\kappa2^2)^2)(1-\mu)}{\kappa1^3} \\
& + \frac{\kappa1(-4\kappa2^2+5(1-\kappa1^2+\kappa2^2)^2)\mu}{\kappa2^3})\omega_1^3\omega_2^2 + 51(-\frac{(1+\kappa1^2-\kappa2^2)(-1+5b\kappa2^2)(-1+\mu)}{\kappa1^2} \\
& + \frac{(-1+5b\kappa1^2)(-1+\kappa1^2-\kappa2^2)\mu}{\kappa2^2})(\frac{(1+\kappa1^2-\kappa2^2)(-12\kappa1^2+5(1+\kappa1^2-\kappa2^2)^2)(1-\mu)}{\kappa1^4} \\
& + \frac{(-1+\kappa1^2-\kappa2^2)(-12\kappa2^2+5(1-\kappa1^2+\kappa2^2)^2)\mu}{\kappa2^4})\omega_1^2\omega_2^3))/(256(4\omega_1^5\omega_2 - 17\omega_1^3\omega_2^3 + 4\omega_1\omega_2^5))
\end{aligned}
$$

Example

$$
\begin{aligned}
\delta_{22} = {} & \frac{1}{256(-\omega_1^3\omega_2 + 4\omega_1\omega_2^3)} 3(20b(\frac{\kappa2(-3+5b\kappa2^2)(-1+\mu)}{\kappa1} + \frac{\kappa1(3-5b\kappa1^2)\mu}{\kappa2})^2\omega_1^3 \\
& + 9(\frac{(1+\kappa1^2-\kappa2^2)(-1+5b\kappa2^2)(-1+\mu)}{\kappa1^2} \\
& - \frac{(-1+5b\kappa1^2)(-1+\kappa1^2-\kappa2^2)\mu}{\kappa2^2})^2\omega_1^2\omega_2 \\
& - 4(\frac{(3+5b\kappa2^2(-6+7b\kappa2^2))(-1+\mu)}{\kappa1^2} + \frac{(-3+5b\kappa1^2(6-7b\kappa1^2))\mu}{\kappa2^2})\omega_1^3\omega_2 \\
& - 80b(\frac{\kappa2(-3+5b\kappa2^2)(-1+\mu)}{\kappa1} + \frac{\kappa1(3-5b\kappa1^2)\mu}{\kappa2})^2\omega_1\omega_2^2 \\
& - 24(\frac{(1+\kappa1^2-\kappa2^2)(-1+5b\kappa2^2)(-1+\mu)}{\kappa1^2} \\
& - \frac{(-1+5b\kappa1^2)(-1+\kappa1^2-\kappa2^2)\mu}{\kappa2^2})^2\omega_2^3 + 16(\frac{(3+5b\kappa2^2(-6+7b\kappa2^2))(-1+\mu)}{\kappa1^2} \\
& + \frac{(-3+5b\kappa1^2(6-7b\kappa1^2))\mu}{\kappa2^2})\omega_1\omega_2^3)
\end{aligned}
$$

Linear Stability

Eigenvalues (quadratic part)

$$\lambda_{1,2} = \pm\left\{-1-\alpha-\gamma+\sqrt{\alpha^2+\beta^2-2\alpha(\beta-2)+\gamma(\gamma+4)}\right\}^{\frac{1}{2}}$$

$$\lambda_{3,4} = \pm\left\{-1-\alpha-\gamma-\sqrt{\alpha^2+\beta^2-2\alpha(\beta-2)+\gamma(\gamma+4)}\right\}^{\frac{1}{2}}$$

(9)

Linear stability: complex e.v. ($\lambda_{1,2} = \pm i\omega_1$, $\lambda_{3,4} = \pm i\omega_2$)

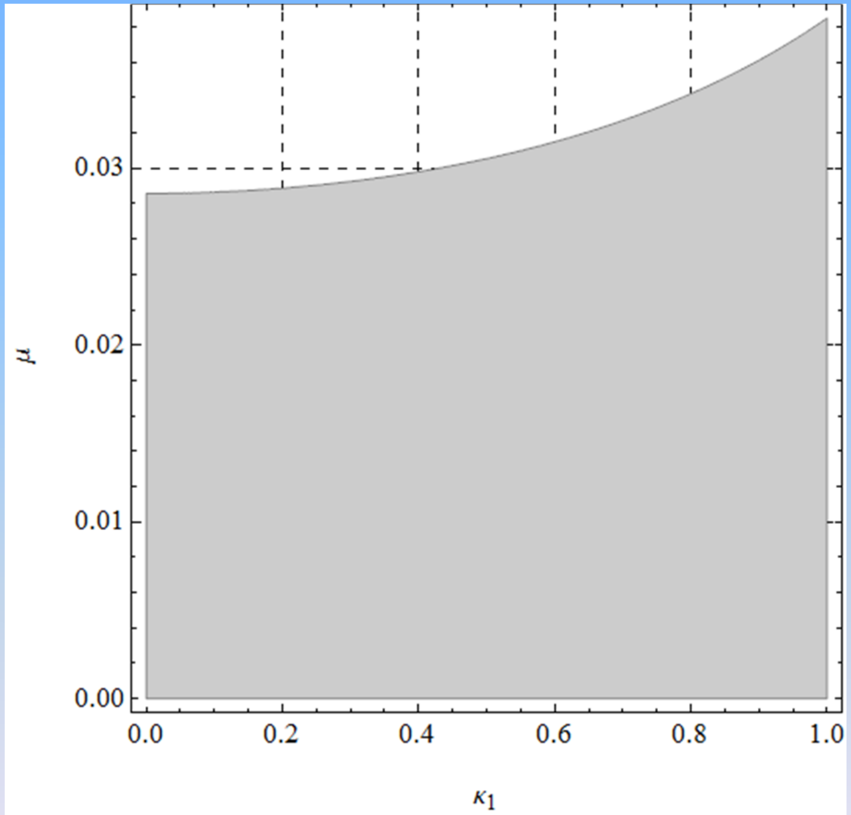
$$\alpha^2+\beta^2-2\alpha(\beta-2)+\gamma(\gamma+4)<0$$

Example

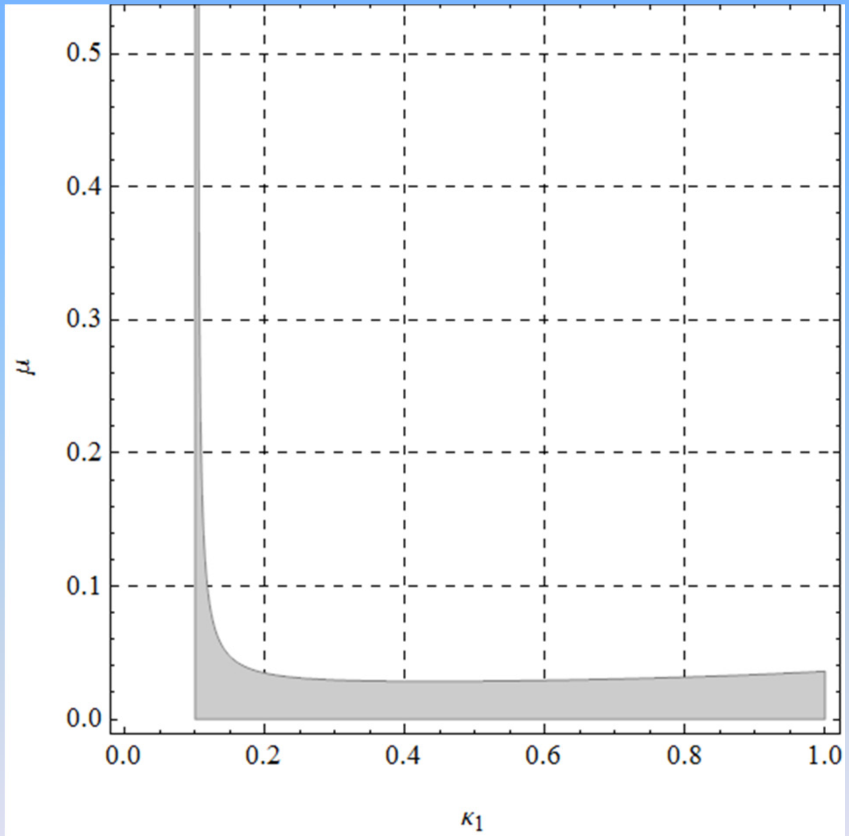
$$
\begin{aligned}
&\frac{3}{32\kappa_1^2\kappa_2^4}(-1+\kappa_1-\kappa_2)(1+\kappa_1-\kappa_2)(-1+\kappa_1+\kappa_2)(1+\kappa_1\\
&\quad+\kappa_2)(3\kappa_1^2(-1+\kappa_1^2)^2-2(-6+5\kappa_1^2+9\kappa_1^4)\kappa_2^2\\
&\quad+3(8+9\kappa_1^2)\kappa_2^4-12\kappa_2^6)\\
&\quad+\frac{3\mu}{32\kappa_1^2\kappa_2^4}(-1+\kappa_1-\kappa_2)(1+\kappa_1-\kappa_2)(-1+\kappa_1+\kappa_2)(1\\
&\quad+\kappa_1+\kappa_2)(3\kappa_1^2(-1+\kappa_1^2)^2-2(-6+5\kappa_1^2+9\kappa_1^4)\kappa_2^2\\
&\quad+3(8+9\kappa_1^2)\kappa_2^4-12\kappa_2^6)\\
&\quad+\frac{9\mu^2}{64\kappa_1^2\kappa_2^4}(-1+\kappa_1-\kappa_2)(1+\kappa_1-\kappa_2)(-1+\kappa_1+\kappa_2)(1\\
&\quad+\kappa_1+\kappa_2)(3\kappa_1^2(-1+\kappa_1^2)^2+2(6+\kappa_1^2-5\kappa_1^4)\kappa_2^2\\
&\quad+(8+11\kappa_1^2)\kappa_2^4-4\kappa_2^6)<0
\end{aligned}
$$

Results

Results

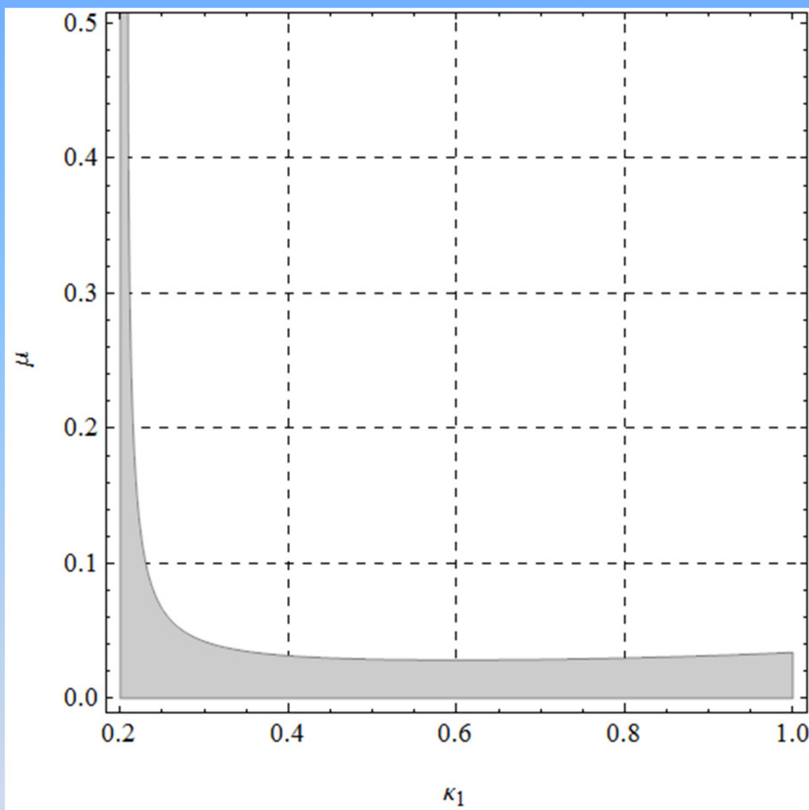


Linear stability for κ_2=1.

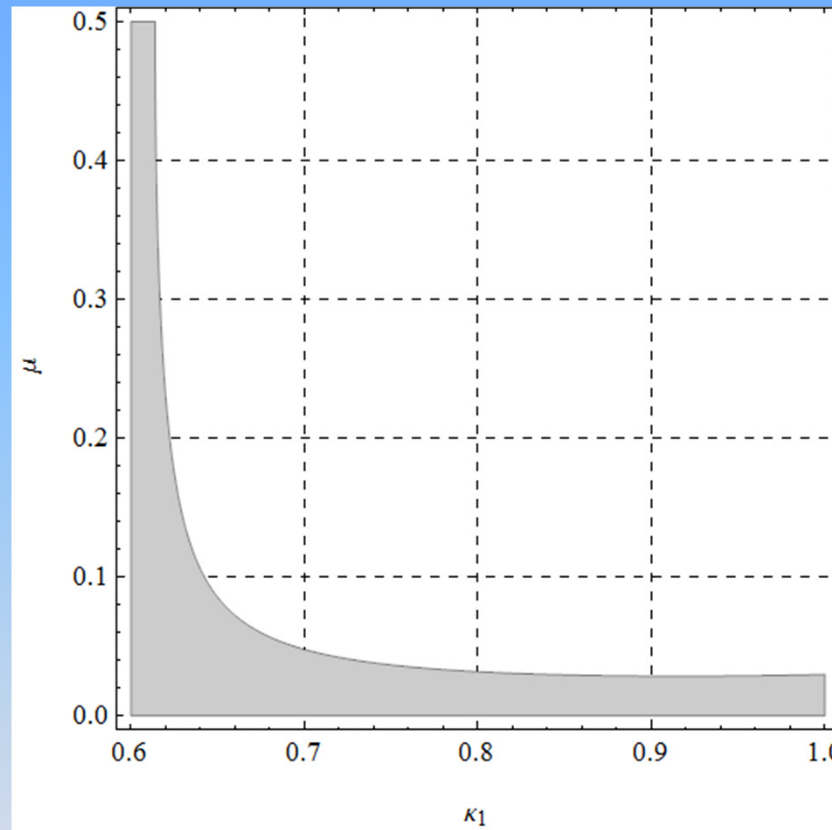


Linear stability for κ_2=0,9.

Results



Linear stability for κ_2=0,8.



Linear stability for κ_2=0,4.

Results

Existence of Resonances

From eq. (9):

Resonance $\omega_1 = \omega_2$

$$\mu = \frac{1}{2} - \sqrt{\frac{1}{4} - \frac{4\kappa_1^2\kappa_2^2}{144\kappa_1^2\kappa_2^2 - 36(-1+\kappa_1^2+\kappa_2^2)^2}}$$

$$\mu = \frac{1}{2} - \sqrt{\frac{1}{4} - \frac{4\kappa_1^2\kappa_2^2}{900\kappa_1^2\kappa_2^2 - 225(-1+\kappa_1^2+\kappa_2^2)^2}}$$

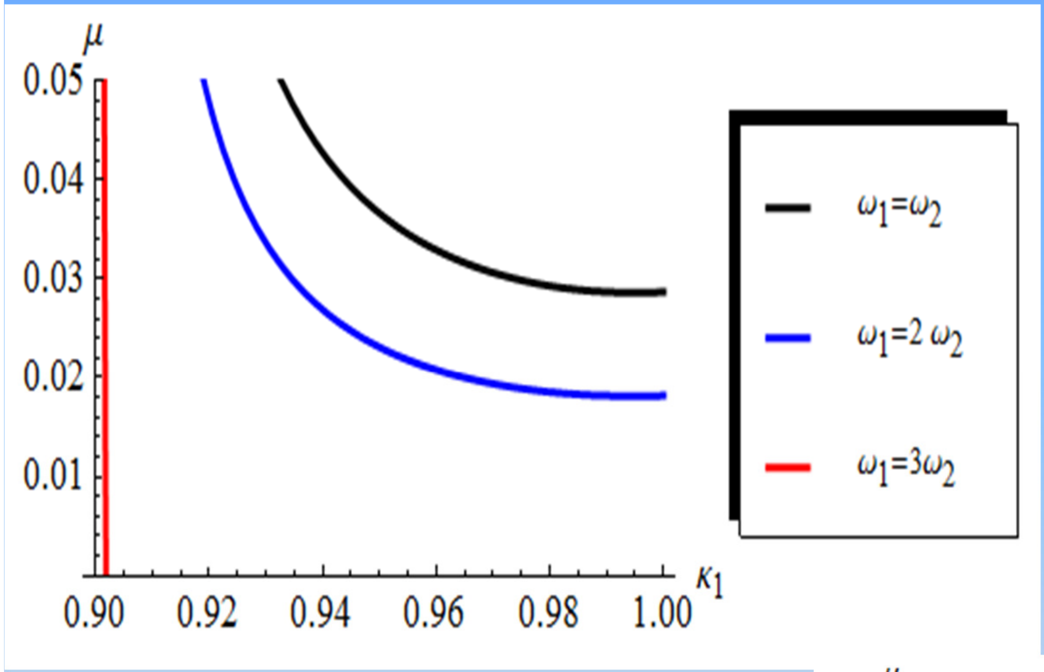
Resonance $\omega_1 = 2\omega_2$

Resonance $\omega_1 = 3\omega_2$

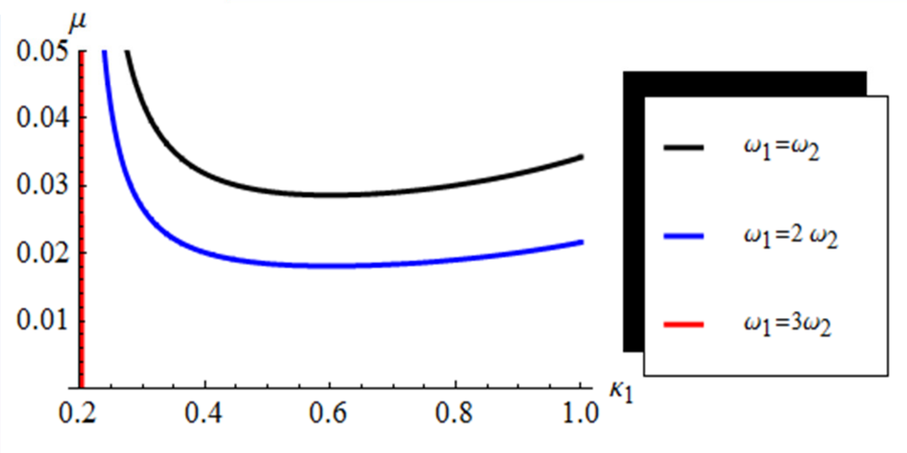
$$\mu = \frac{1}{2} - \sqrt{\frac{1}{2} - \frac{4\kappa_1^2\kappa_2^2}{400\kappa_1^2\kappa_2^2 - 100(-1+\kappa_1^2+\kappa_2^2)^2}}$$

Results

κ_2=0,1



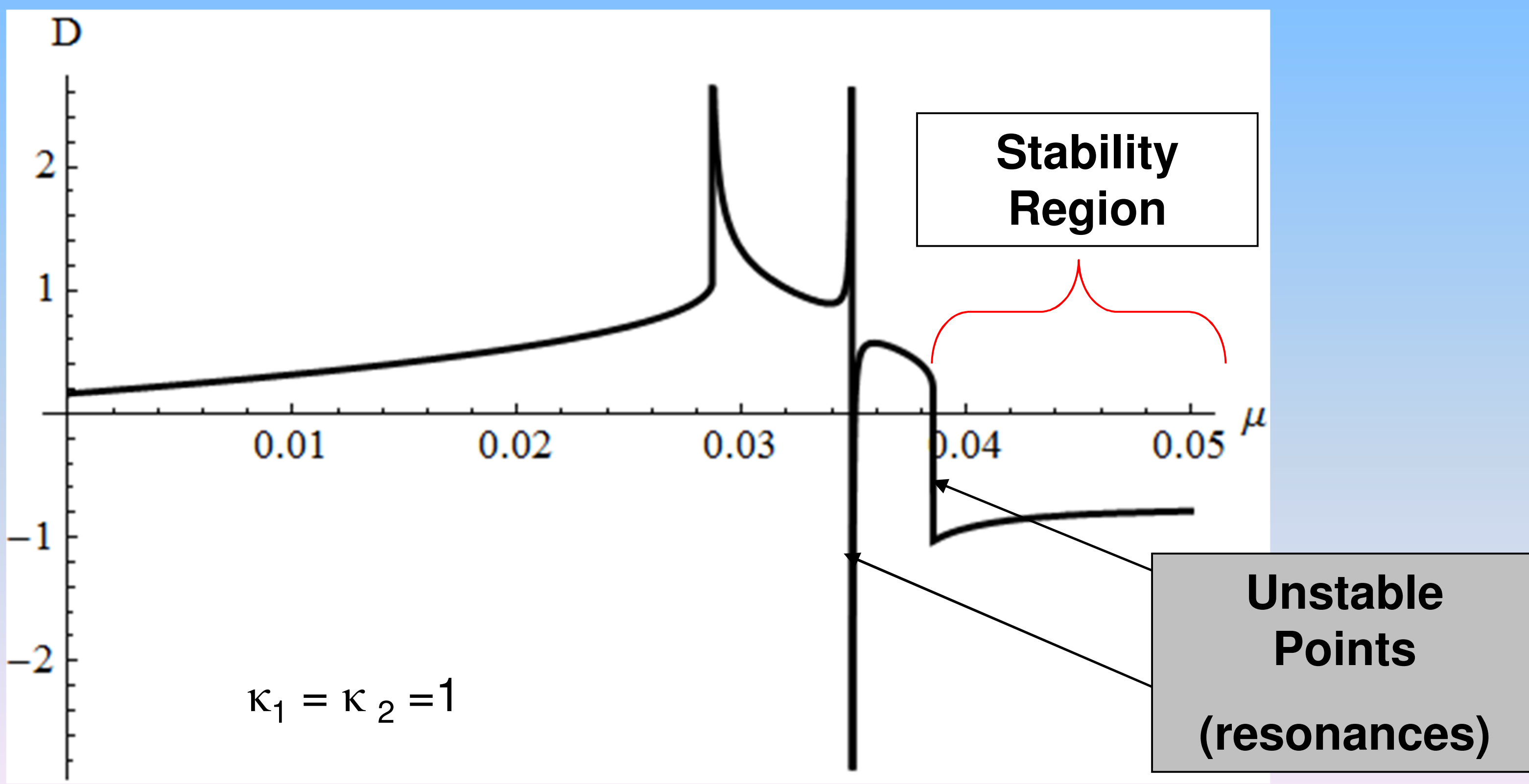
κ_2=0,8



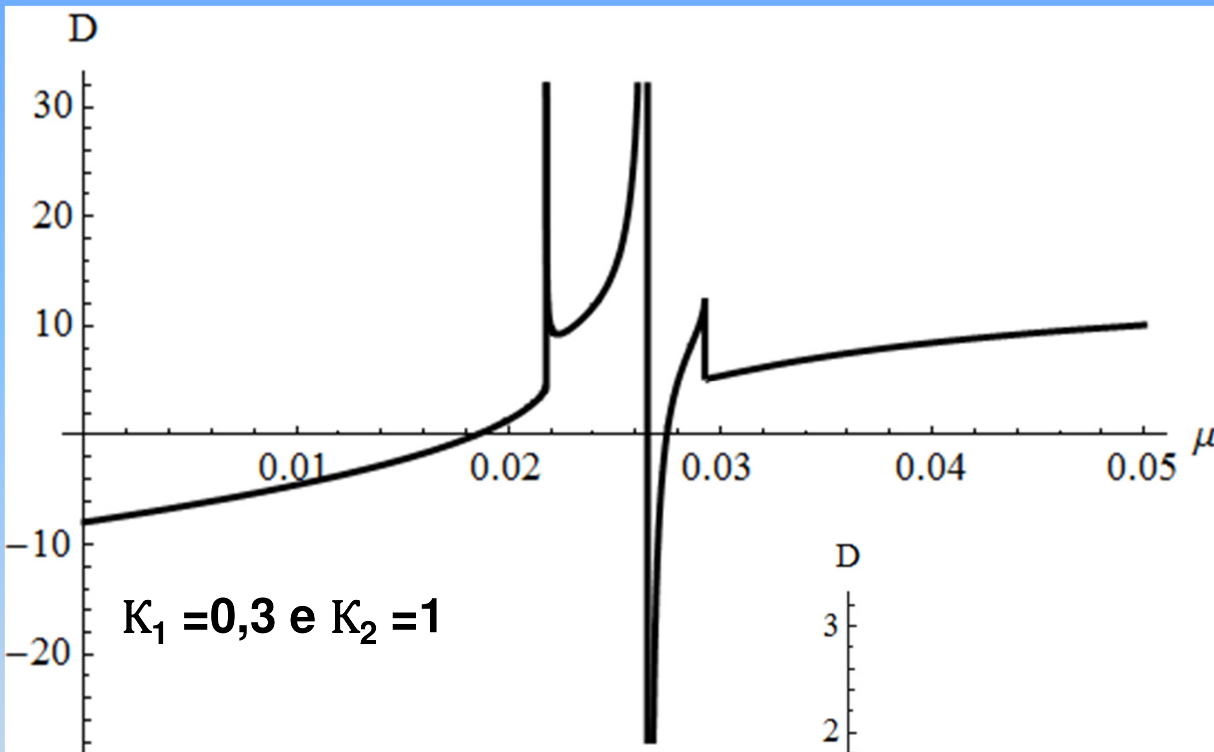
Results

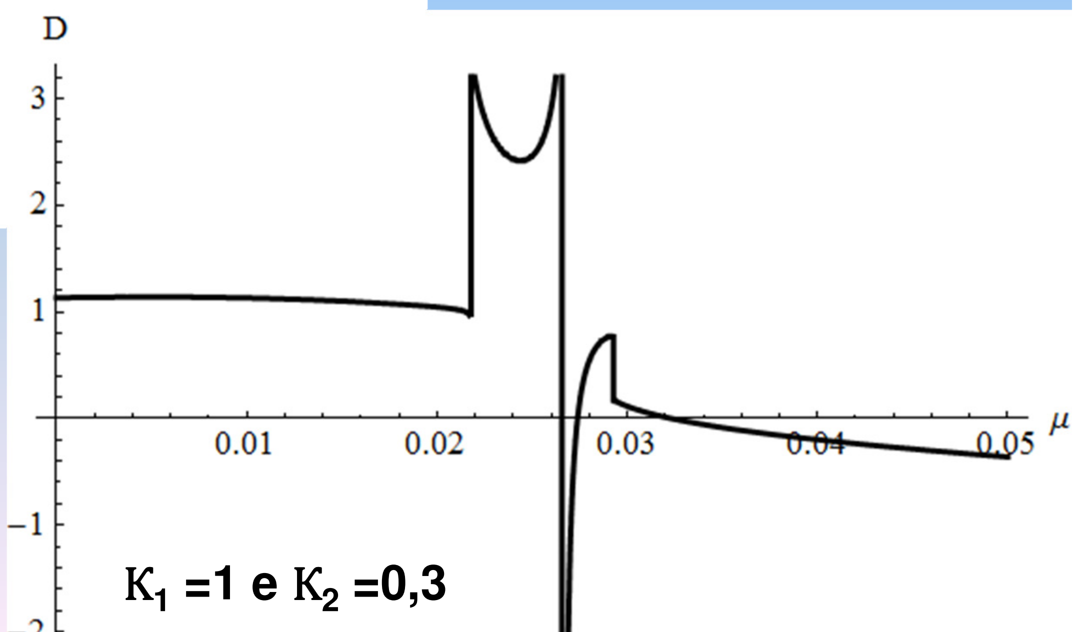
Results

$$D = -(\overline{\delta_{11}}\omega_2^2 - 2\overline{\delta_{12}}\omega_1\,\omega_2 + \overline{\delta_{22}}\omega_1^2) \neq 0$$

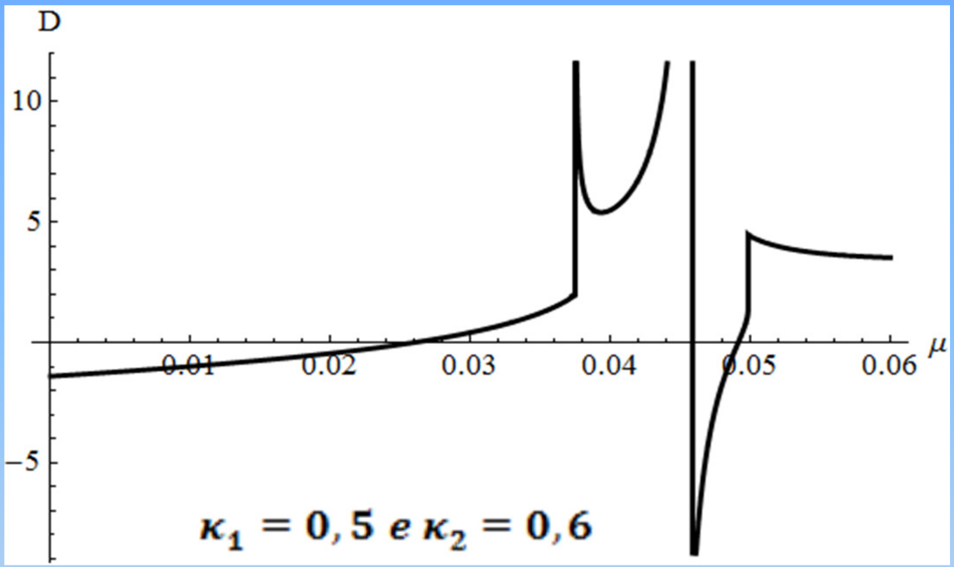


Results

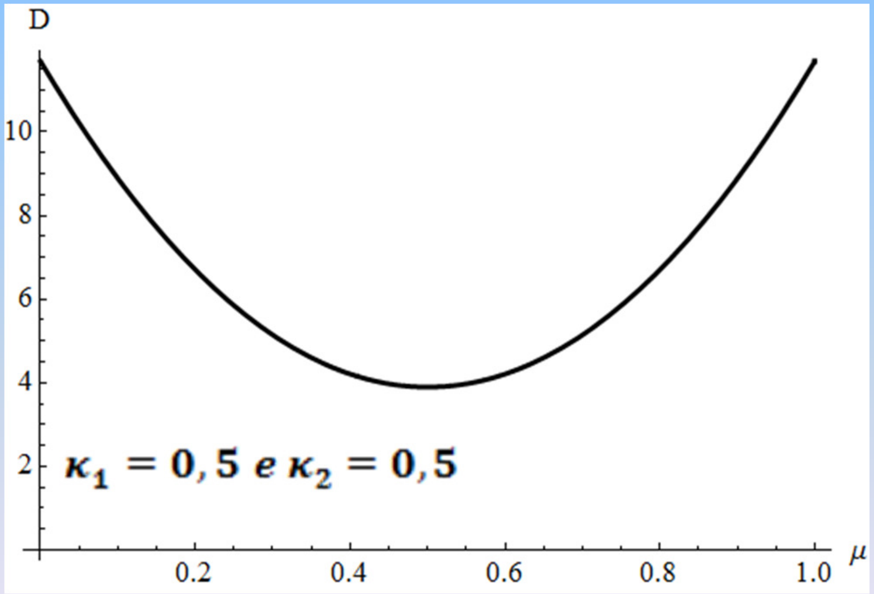

D
30
20
10
−10
−20
0.01
0.02
0.03
0.04
0.05
μ
K1 =0,3 e K2 =1


D
3
2
1
−1
−2
0.01
0.02
0.03
0.04
0.05
μ
K1 =1 e K2 =0,3

Results



D
10
5
−5
0.01
0.02
0.03
0.04
0.05
0.06
μ
$\kappa_1 = 0,5\ e\ \kappa_2 = 0,6$



D
10
8
6
4
2
0.2
0.4
0.6
0.8
1.0
μ
$\kappa_1 = 0,5\ e\ \kappa_2 = 0,5$

Conclusions

Considering the radiation emitted by one or both primaries the following results in the neighborhood of L_4 were obtained:

1. Only the small primary m_1 radiates:

•When, is valid for the small primary, the inequality $0< fr_1/fg_1<0,5$, there exists a mass ratio $\mu = m_1/m_2$ in the interval $0,016<\mu<0,023$ **where L_4 is Liapunov unstable**

•The non stability is due to the 2:1 resonance for all cases studied .

•We don't have instability in L_4 due the 1:1 resonance for $0,1<f_{r1}/f_{g1}<=0,2$.

Conclusions

2. Only the most massive body radiates:

•When 0< fr_2/fg_2 <0,5 there exists a mass ratio μ = m1/m2 in the interval 0,012<μ<0,023 where L_4 is Liapunov unstable.

•The non stability is due to the 2:1 resonance for all cases studied.

•We don't have instability in L_4 due the 1:1 resonance for 0,1<fr2/fg2<=0,5.

Conclusions

Both primaries radiate:

- L_4 is stable for fri/fgi= 0,5 (i=1,2)
- A small variation in the the ratio fri/fgi (i=1,2) about 0,5 can break the stability in L_4.
- When fr_1/fg_1=0,5 and fr_2/fg_2 =0,8 L_4 is Liapunov unstable for 4 value of μ in the interval $0,0138<\mu<0,02876$ (one of this value corresponds to the 2:1 resonance.

THANKS FOR YOUR ATTENTION

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