Equilibrium Figure and tides (Part. II, orbital revolution)

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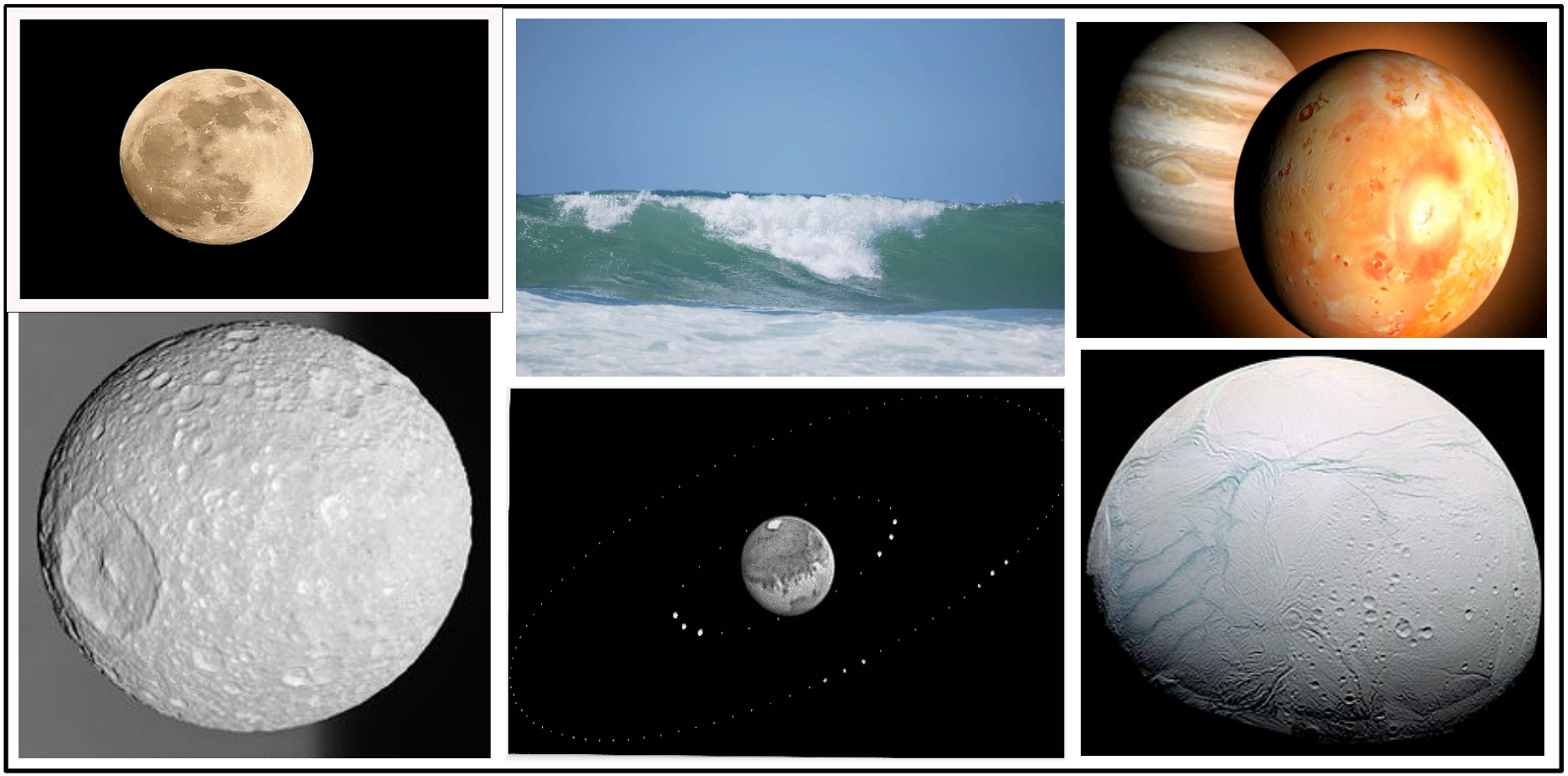

Introduction

Few numbers

1 000 W

Coal power plant
100-1 000 MW

Nuclear power plant
0.9 – 1.3 GW

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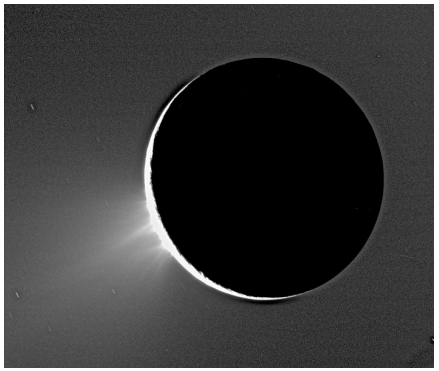

Geyser on
Enceladus
15.8 +/- 3.1 GW

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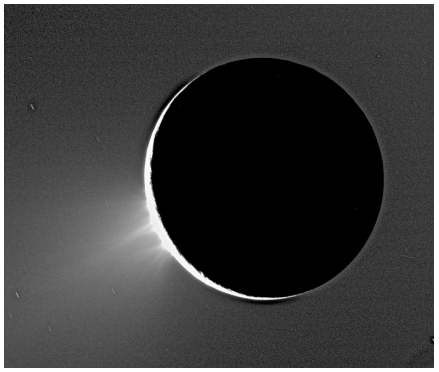

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Earth tidal
dissipation
3.75 TW

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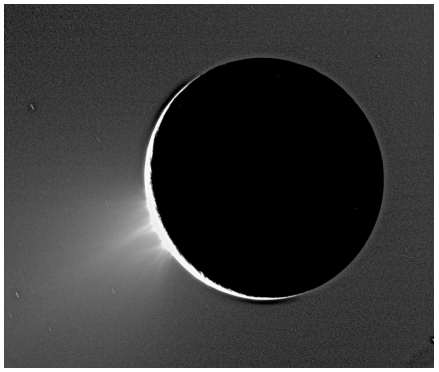

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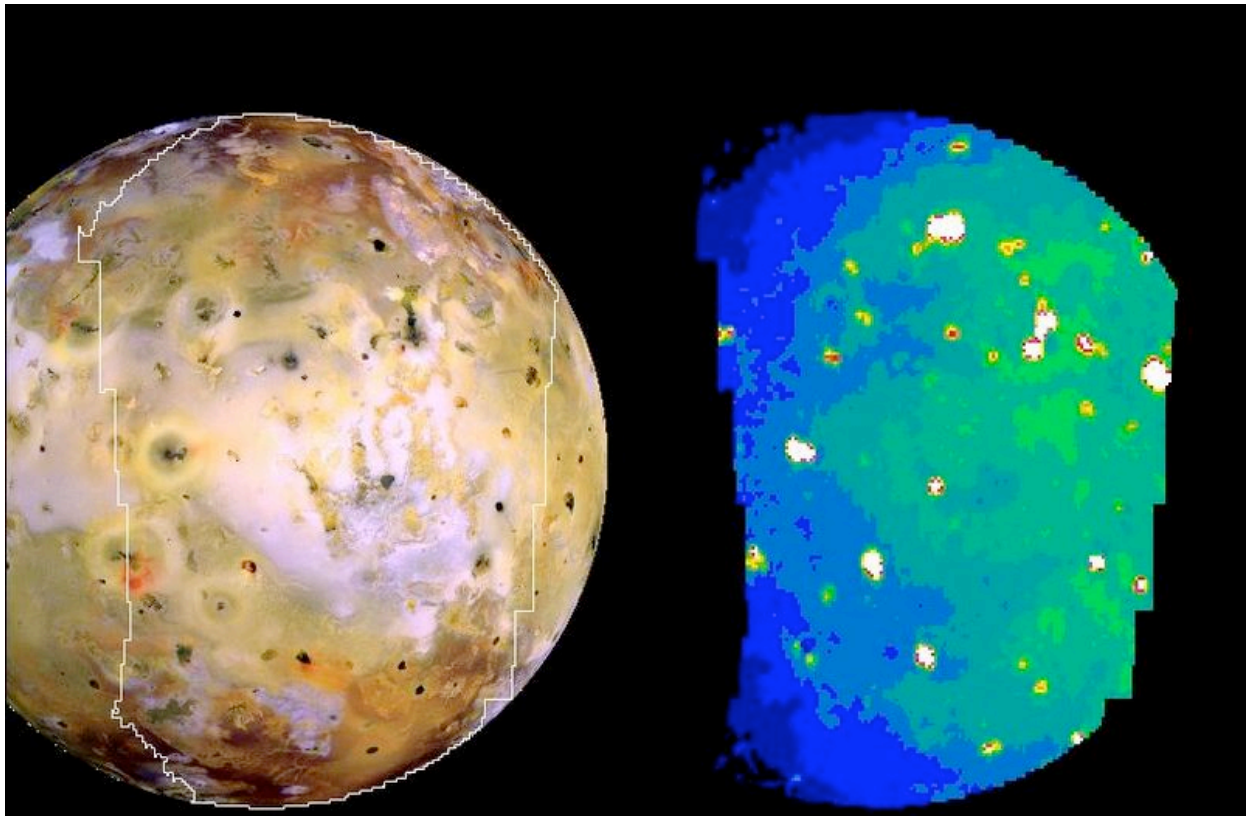
Volcanism on Io
(9.33 +/- 1.87) x 10^1 TW

How can we quantify tidal dissipation?

By direct IR measurements…



Thermal emission map by NIMS/Galileo

How can we quantify tidal dissipation?

Three important issues:

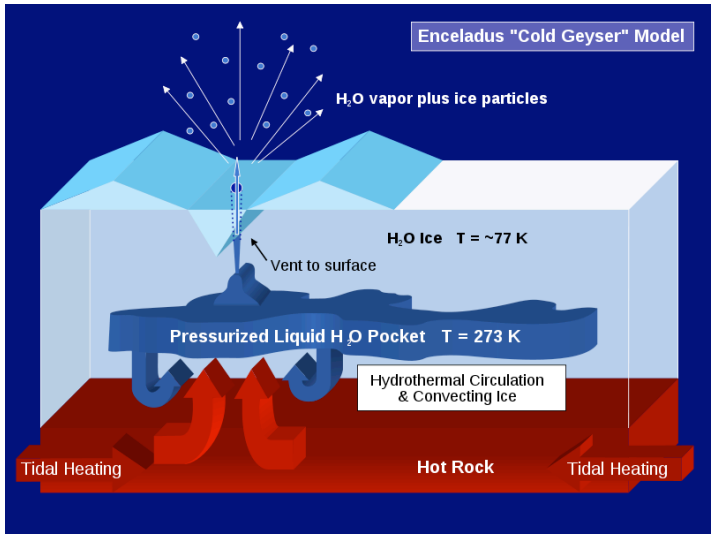
1- Only hot spots are quantified

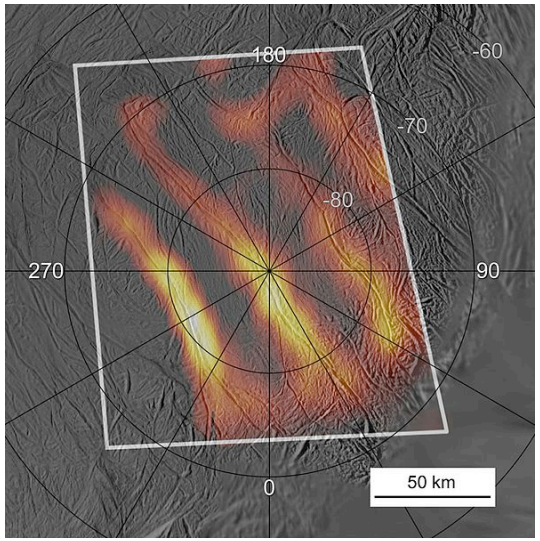
2- Barely feasible in general!

Low signal, other sources of heating (radioactive decay, Solar heat, gravitational contraction…)

3- Thermal equilibrium is not sure

Orbital elements and interior properties may change while heat is transported at the surface





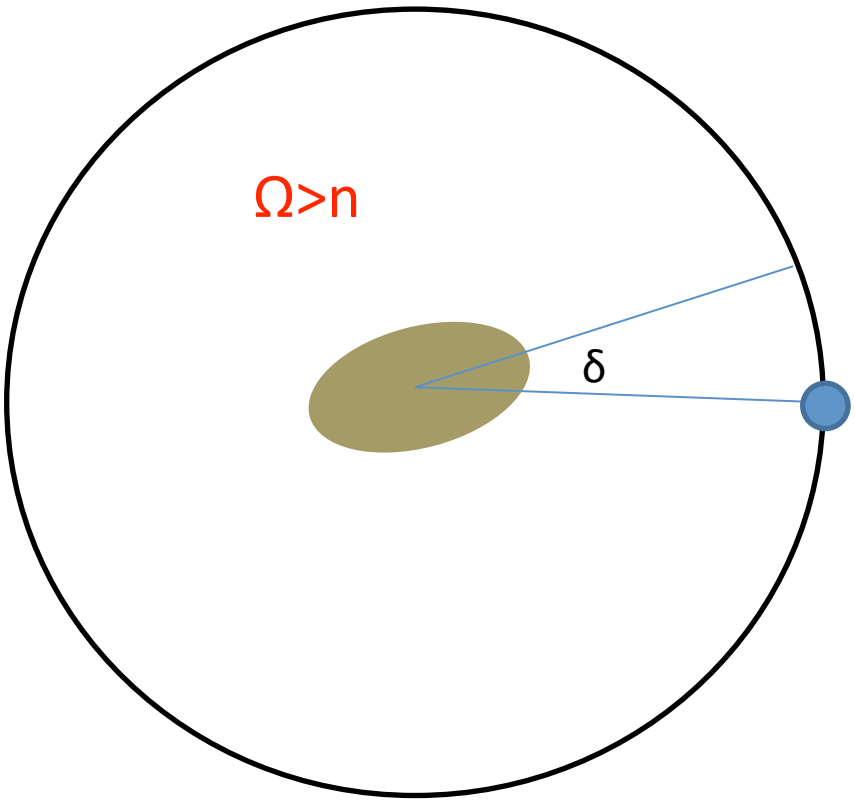
→ **A more dynamical way to derive heat production is needed**

The tidal effects and the estimation of Q from long term evolution

Tidal effects on the primary

Let's consider a satellite raising tides on its primary

Dissipation implies time lag Δt which is connected to the geometrical lag δ by: $2(\Omega-n)\Delta t=\delta$



$L \approx mna^2 + I\Omega$

$dL/dt=0 \Rightarrow \quad I\dot{\Omega} = -\frac{1}{2} mna\dot{a}$

$E \approx -GMm/2a + 1/2\, I\Omega^2$

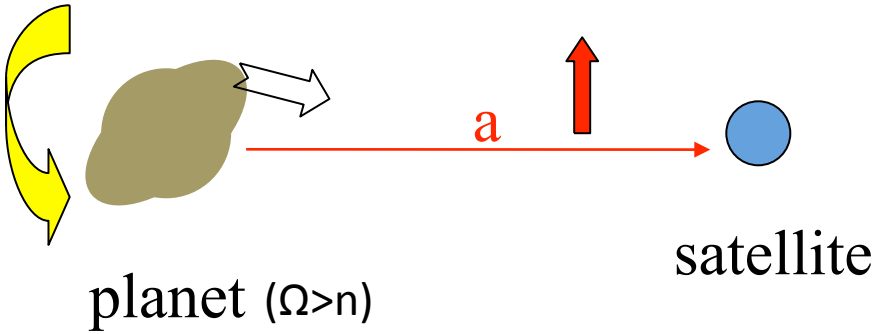
$\dot{E} = I\Omega\dot{\Omega} + \frac{1}{2} mn^2 a\dot{a}$

Introducing k_2 and Q:

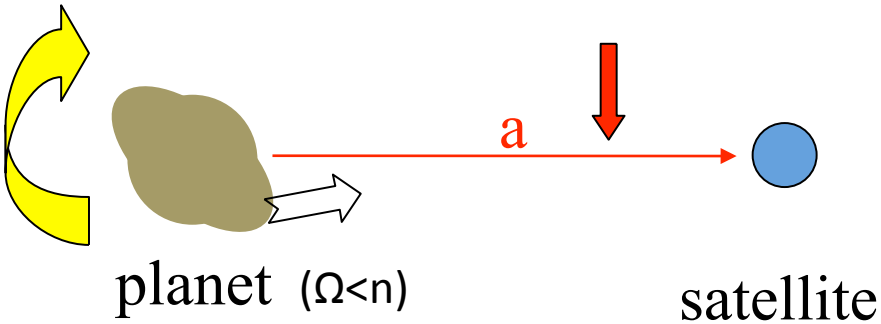
$$\frac{da}{dt} = \frac{3k_2 m}{QM}\left(\frac{Er}{a}\right)^5 na$$

Tidal effects on the primary

The tides raised in the planet → exchange of angular momentum



Secular deceleration on the mean motion



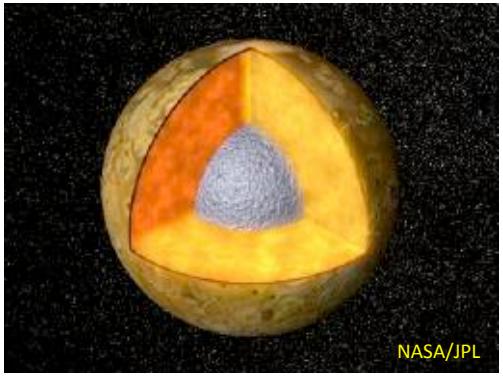
Secular acceleration on the mean motion

Tidal effects on the primary

Q is defined by:

$$Q^{-1}=\Delta E/(2\pi E)$$

Where ΔE is the energy loss during one cycle and E is the maximum energy stored in the tidal distortion



Low Q→high dissipation, high Q→low dissipation

A second parameter of interest is the Love number k_2

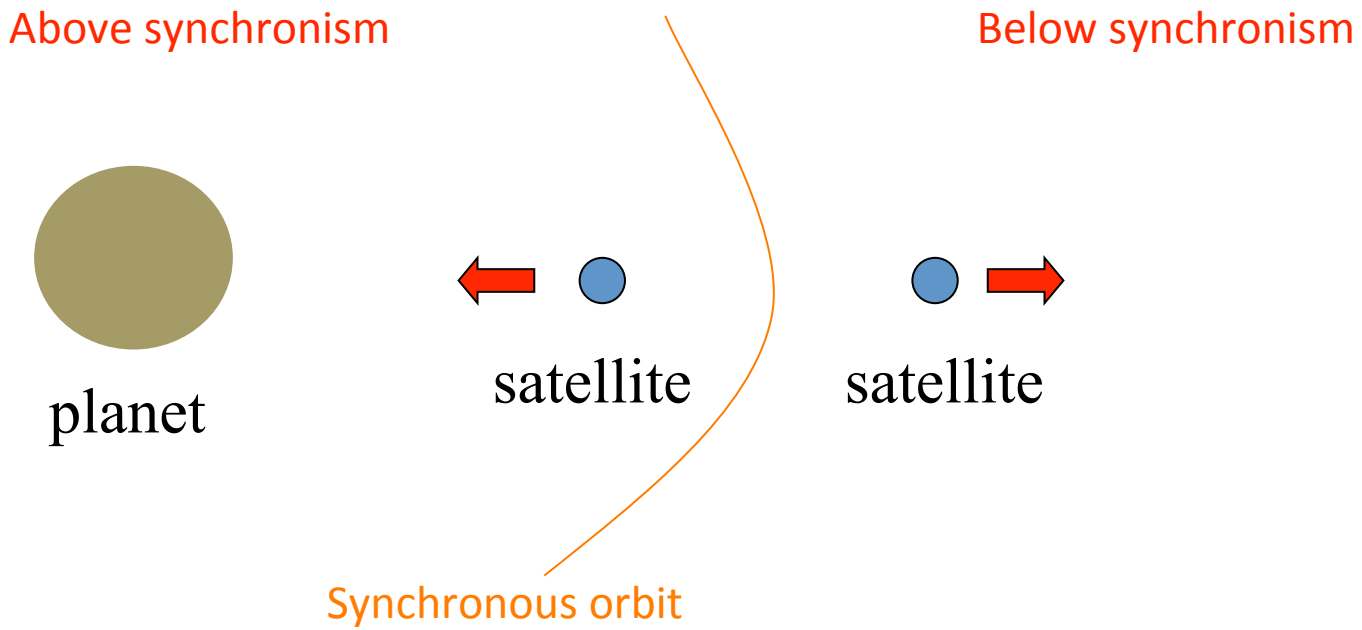
which characterizes the response of the body to the tidal potential

The orbital tidal acceleration/deceleration of a satellite are dependent on the ratio k_2/Q

An example: tidal dissipation in giant planets

A paper of reference that estimates Q for the giant planets is Goldreich and Soter (1966)

Assuming that the main satellites were formed beyond the synchronous orbit, one can give a lower bound for Q



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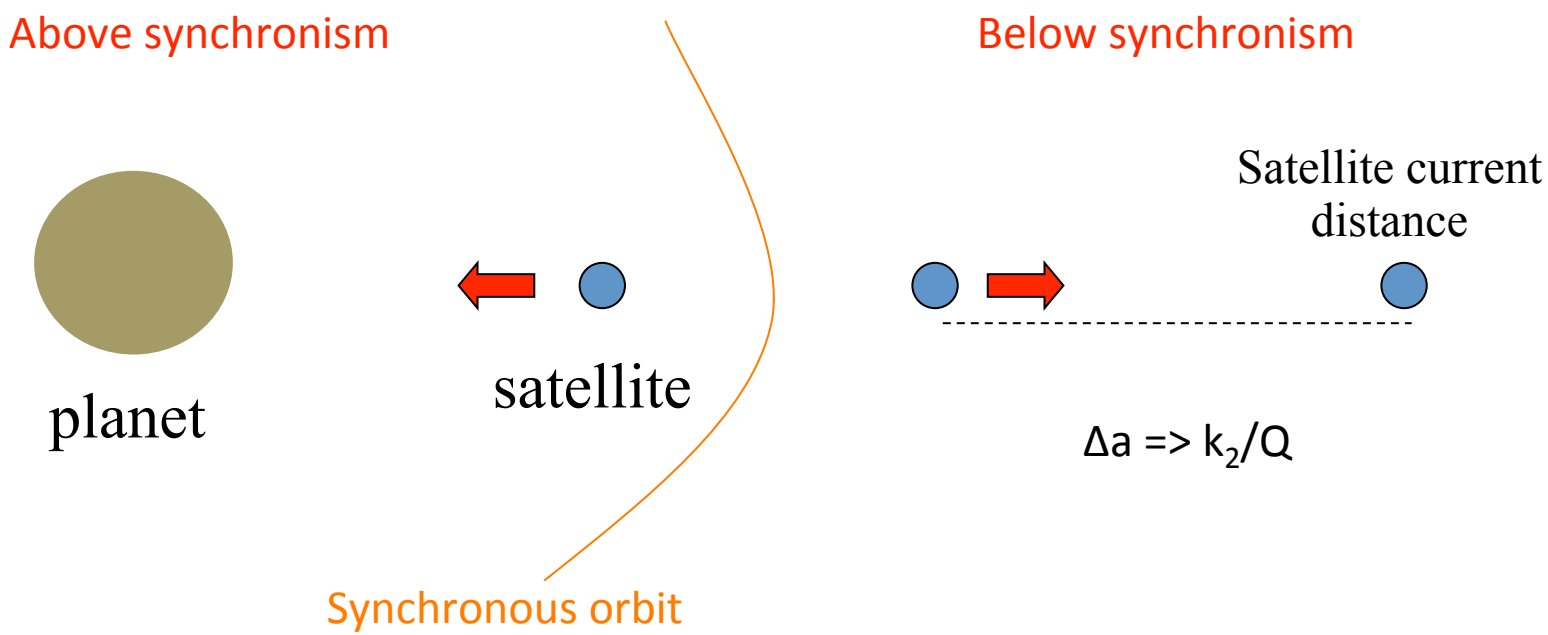


An example: tidal dissipation in giant planets

Estimation of lower bound of Q for the giant planets by Goldreich and Soter (1966)

$Q_{Jupiter} \geq 1.0\ 10^5$ (Io)

$Q_{Saturn} \geq 6.0\ 10^4$ (Mimas)

$Q_{Uranus} \geq 7.2\ 10^4$ (Miranda)

Let's remind that:

$Q_{Mars} = 80$

$Q_{Earth} = 260$

$Q_{Io} = 20$

Values further improved by Gavrilov and Zharkov (1977)

$Q_{Jupiter} \geq 2.5\ 10^4$ (Io)

$Q_{Saturn} \geq 1.4\ 10^4$ (Mimas)

$Q_{Uranus} \geq 5.0\ 10^3$ (Miranda)

These values are still good references!

An example: tidal dissipation in giant planets

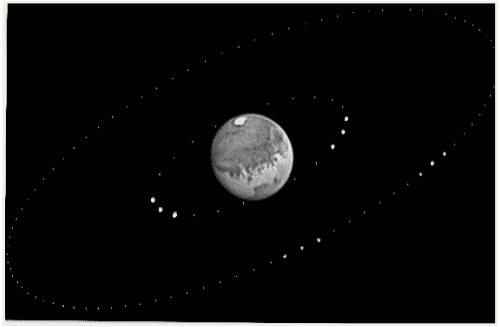
Caveats:

1- Strong assumption on the past evolution of the system (formation, resonances, dissipation in satellites…)

2- Only averaged value of Q over 4.5 Byr is obtained. In particular, current tidal dissipation may be somewhat different (Q≠cste)

A second example: the terrible fate of Phobos

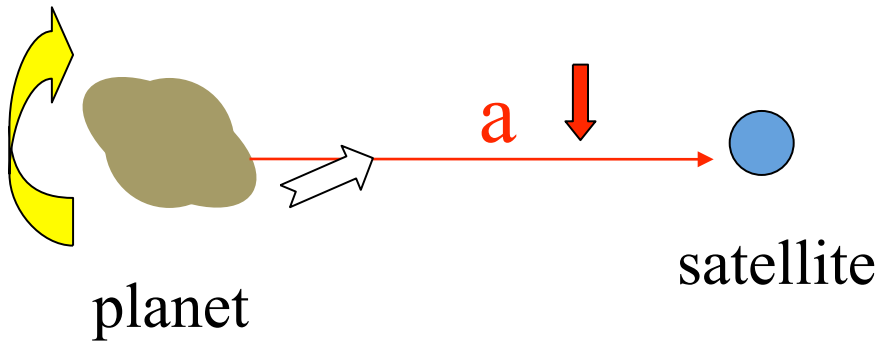
Phobos and Deimos have been discovered by A.Hall in 1877.

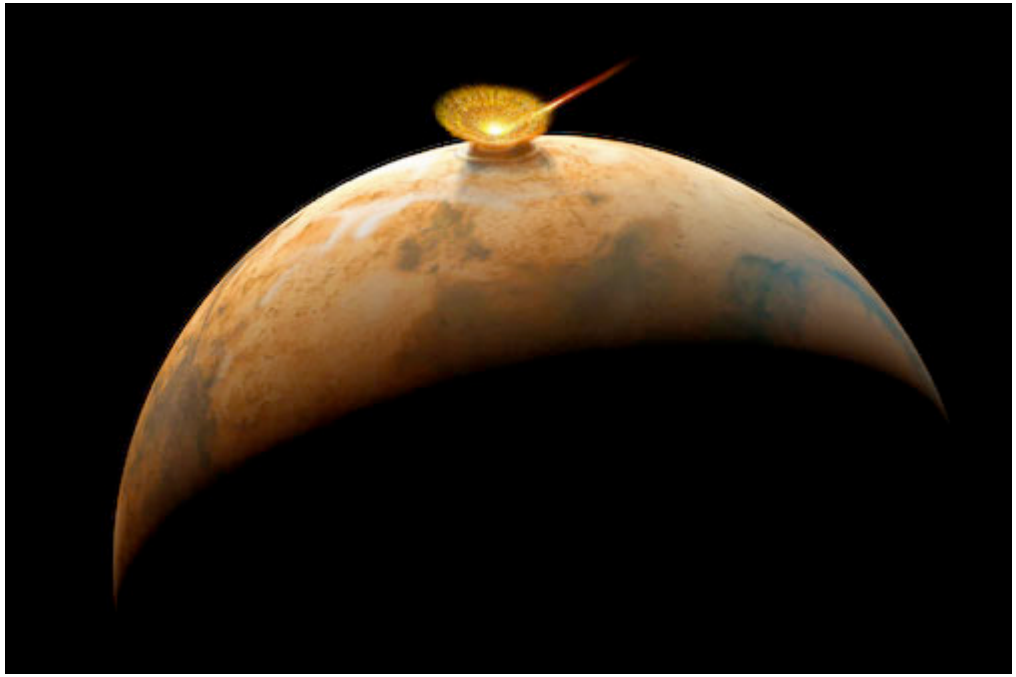


Sharless (1945) discovered an unexpected acceleration on Phobos…

A second example: the terrible fate of Phobos

Why Phobos is doomed…





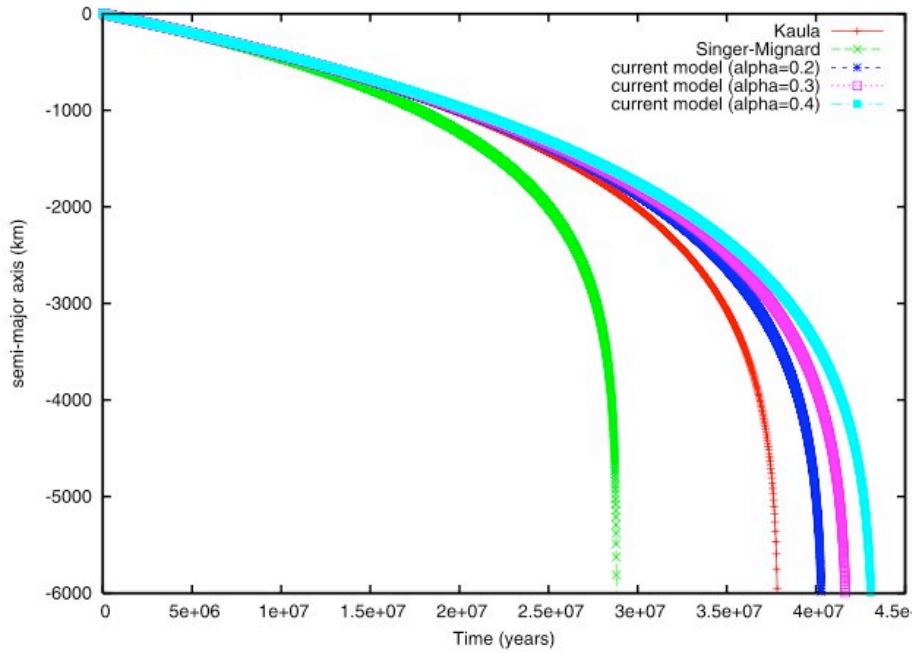
Question: how much time will that take place?

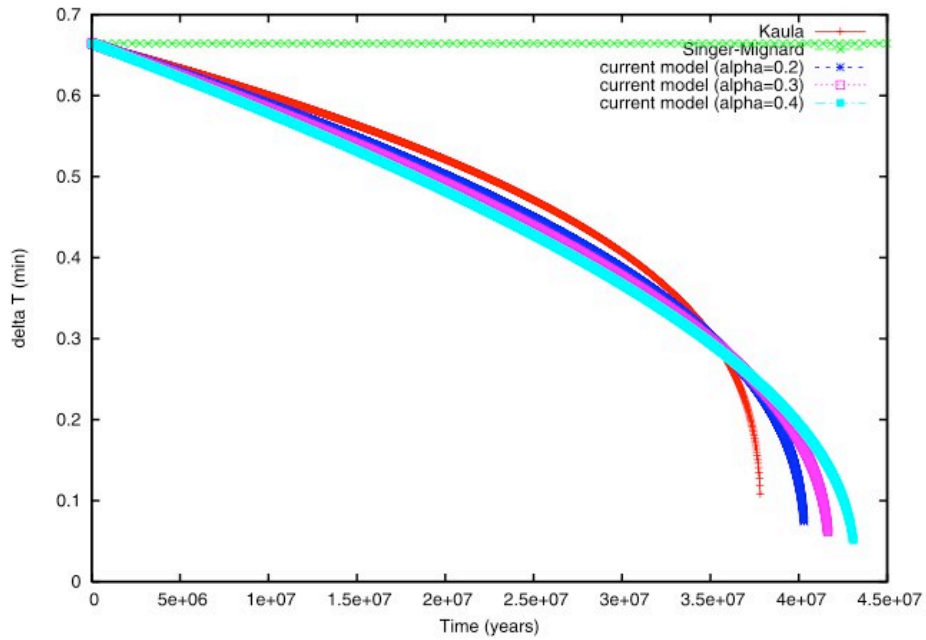
$$\frac{da}{dt} = \frac{3k_2 m}{QM}\left(\frac{Er}{a}\right)^5 na$$

A second example: the terrible fate of Phobos

So far, only two frequency dependence of Q have been used: Q=cst (most of the time)
Q α 1/f (Singer/Mignard model)

BUT: recent experiments strongly suggest: Q α f^{α} (Karato 2009)





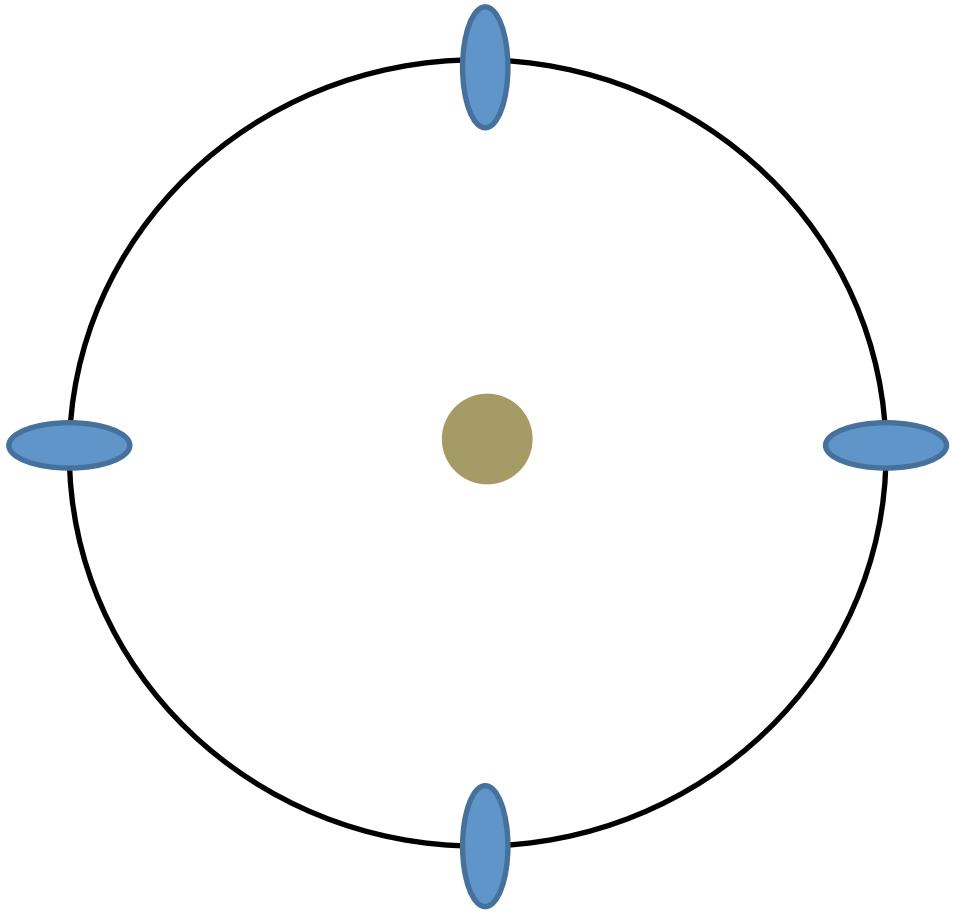
Efroimsky, Lainey (2007)

→ Depending of the rheology model, Phobos' lifetime changes by 50%!

The tidal effects and the estimation of Q from short term evolution

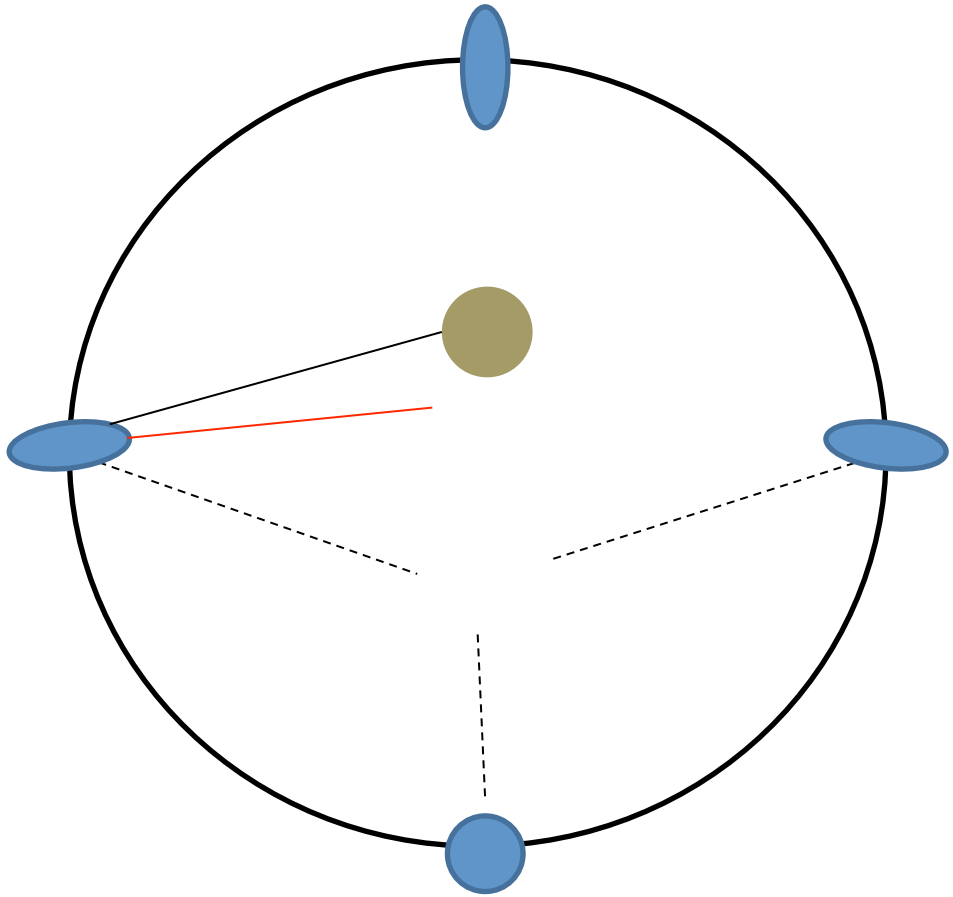
Satellite in spin-orbit resonance

If e=0 → no internal friction: the tidal bulges are facing the COM of the primary



Satellite in spin-orbit resonance

If e≠0 → internal friction: the tidal bulges are facing the COM of the primary



Orbital energy is dissipated as consequence of:

a- distance variation (radial dissipation)

b- Kepler 's 2nd law (tangential dissipation)

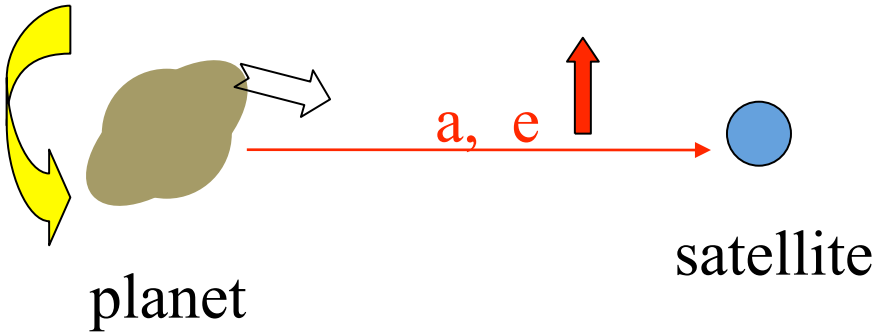
$E \approx -GMm/2a$

$\dot{E} \approx \frac{GMm}{2a^2}\dot{a}$ => a increases

$$L \approx mna^2\sqrt{1-e^2} + I_s\Omega_s \approx mna^2\sqrt{1-e^2}$$

$dL/dt=0$ => e decreases

Competition between tidal dissipation effects



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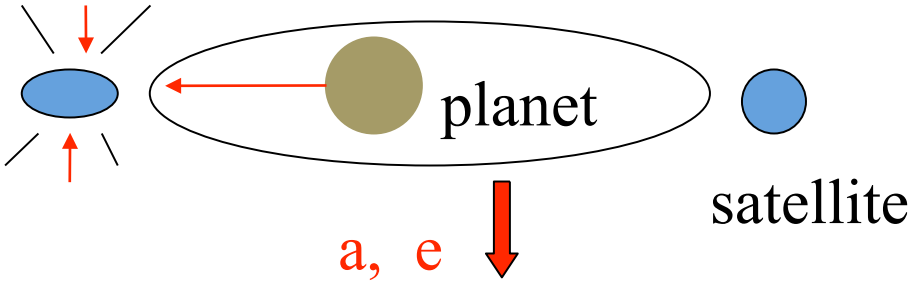

Secular acceleration on the mean motion

Orbit determination of natural satellites

→ *Orbit determination of natural satellites and spacecraft follow exactly the same methodology…*

Method in three steps:

1- modeling of the dynamical system

2- gathering the observations

3- fitting the model to the observations

Today, this kind of work is done completly numerically

S/C: GINS, DPODP, GEODYN, …

SAT: NOE, SATELORB…

Orbit determination of natural satellites

Step 1: Modeling of the dynamical system

Equations of motion

$$\ddot{\vec{r}}_i = -G(m_0 + m_i)\left(\frac{\vec{r}_i}{r_i^3} - \nabla_i U_{\bar{i}\hat{0}} + \nabla_0 U_{\bar{0}\hat{i}}\right) + \sum_{j=1, j\neq i}^{N} G m_j \left(\frac{\vec{r}_j - \vec{r}_i}{r_{ij}^3} - \frac{\vec{r}_j}{r_j^3} - \nabla_j U_{\bar{j}\hat{i}} + \nabla_i U_{\bar{i}\hat{j}} + \nabla_j U_{\bar{j}\hat{0}} - \nabla_0 U_{\bar{0}\hat{j}}\right)$$

$$+ \frac{(m_0 + m_i)}{m_i m_0}\left(\vec{F}_{\bar{i}\hat{0}}{}^T - \vec{F}_{\bar{0}\hat{i}}{}^T\right) - \frac{1}{m_0}\sum_{j=1, j\neq i}^{N}\left(\vec{F}_{\bar{j}\hat{0}}{}^T - \vec{F}_{\bar{0}\hat{j}}{}^T\right)$$

Orbit determination of natural satellites

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$$+\frac{(m_0+m_i)}{m_i m_0}\left(\vec{F}_{\bar{i}\hat{0}}{}^T - \vec{F}_{\bar{0}\hat{i}}{}^T\right) - \frac{1}{m_0}\sum_{j=1, j\neq i}^{N}\left(\vec{F}_{\bar{j}\hat{0}}{}^T - \vec{F}_{\bar{0}\hat{j}}{}^T\right)$$

N-body problem

Orbit determination of natural satellites

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N-body problem

Extended gravity fields

Orbit determination of natural satellites

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Orbit determination of natural satellites

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N-body problem
Extended gravity fields
Tidal effects
Relativistic terms

Orbit determination of natural satellites

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N-body problem
Extended gravity fields
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Relativistic terms

Initial conditions are required to solve for equations of motion…

Problem: How will we find the « real » initial conditions of the system we consider?

Orbit determination of natural satellites

Problem: How can we find the « real » initial conditions of the system we consider?

Clearly we need to know at every observation time *t* the quantity $\left.\frac{\partial \vec{r}_i}{\partial c_l}\right|_t$

Variational equations

$$\frac{d^2 \vec{r}_i}{dt^2} = \frac{1}{m_i} \vec{F}_i$$

Orbit determination of natural satellites

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$$\frac{\partial}{\partial c_l}\left(\frac{d^2\vec{r}_i}{dt^2}\right) = \frac{1}{m_i}\left[\sum_j\left(\frac{\partial \vec{F}_i}{\partial \vec{r}_j}\frac{\partial \vec{r}_j}{\partial c_l} + \frac{\partial \vec{F}_i}{\partial \dot{\vec{r}}_j}\frac{\partial \dot{\vec{r}}_j}{\partial c_l}\right) + \frac{\partial \vec{F}_i}{\partial c_l}\right]$$

Orbit determination of natural satellites

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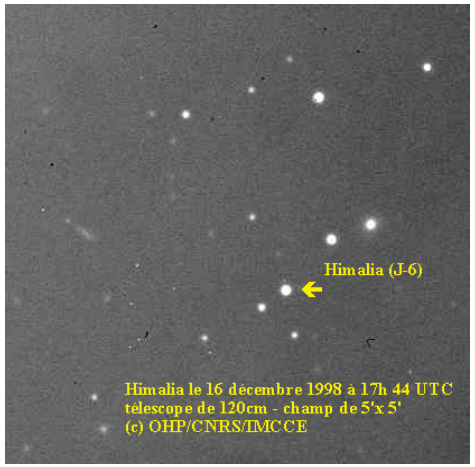
Computation of variational equations can be time consuming and requires a lot of development time!

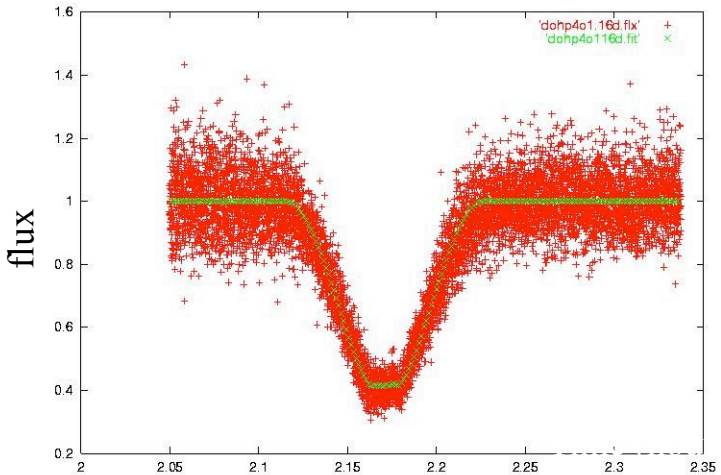
Orbit determination of natural satellites

Step 2: gathering the observations

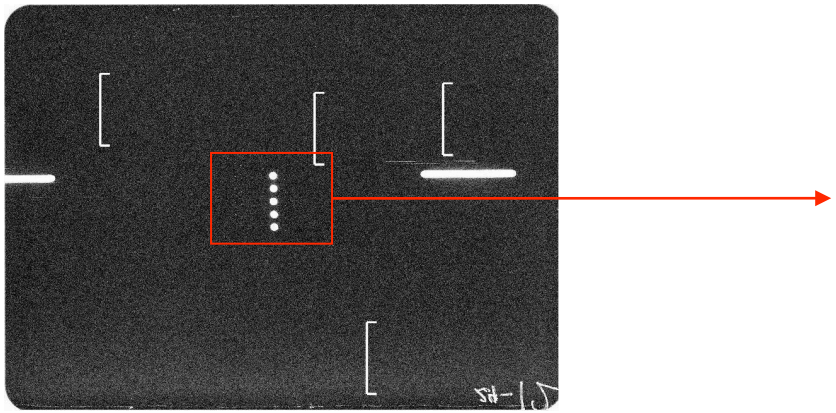
Direct astrometric measurement

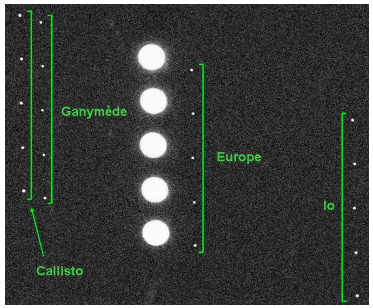


Undirect astrometric measurement (photometry)



Astrometric remeasurement (benefit from modern scanning machine)





Orbit determination of natural satellites

Step 2: gathering the observations

Accuracy of the observations?

Direct astrometric measurement

rule of thumb: 100 mas

≈30 km (Mars), 300 km (Jupiter), 600 km (Saturne), 1200 km (Uranus)

Undirect astrometric measurement (photometry)

≈few tens of km (not dependent on the distance, but require larger telescopes for large distance)

Astrometric remeasurement (benefit from modern scanning machine)

35 mas (intersatellite) and 80 mas (RA, DEC) for good photographic plates (Robert et al. 2011). But may greatly depend on the quality of the plates…

Orbit determination of natural satellites

Step 3: Fitting the model to the observations

Step 1 (Integration of the equations of motion + variational equations)

Step 2 (observations)

Step 3 (fitting the model)
Approximation by a linear system
=> least squares method

$$\left(O - C \right) = \sum_{l=1}^{N} A_i(c_l) \cdot \Delta c_l + \cancel{O((\Delta c_l)^2)}$$

unknown

Example of the Mars system

Former works:

Sinclair, 1972

Shor, 1975

Sinclair, 1989

Jacobson et al. 1989

Chapront-Touzé, 1990

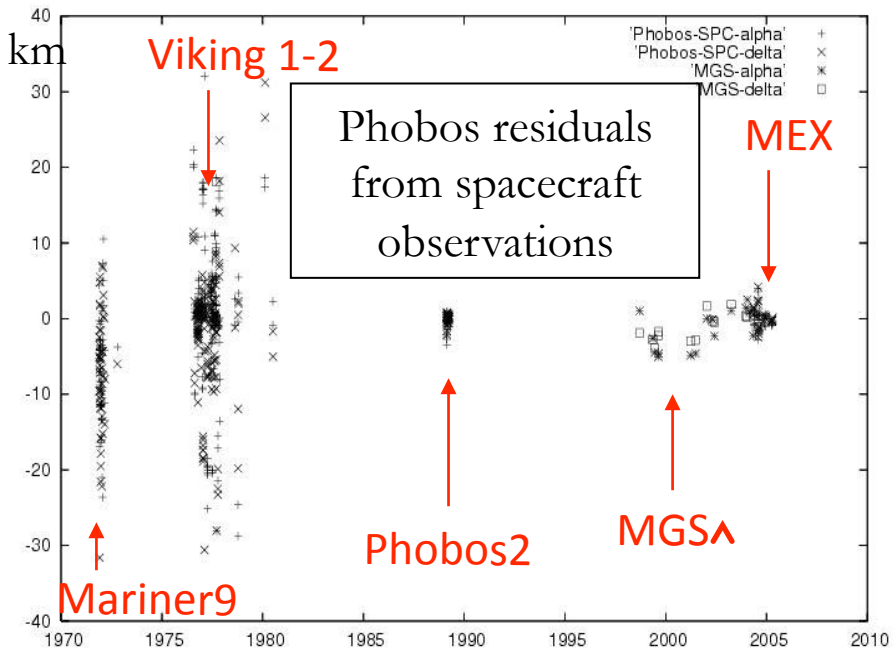
Morley, 1990

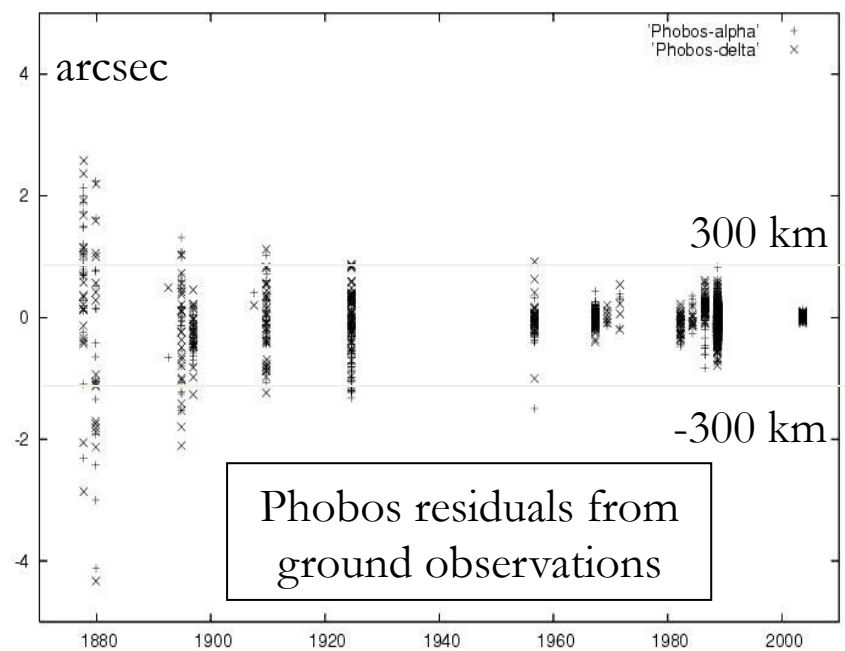
Emelianov et al. 1993

→All these models were analytic

Example of the Mars system

Astrometric post-fit residuals for Phobos after fit of initial state vectors, Mars **dissipation factor Q** and Phobos' oblate parameters c_{20}, c_{22}.





Lainey, Dehant and Pätzold (2007)

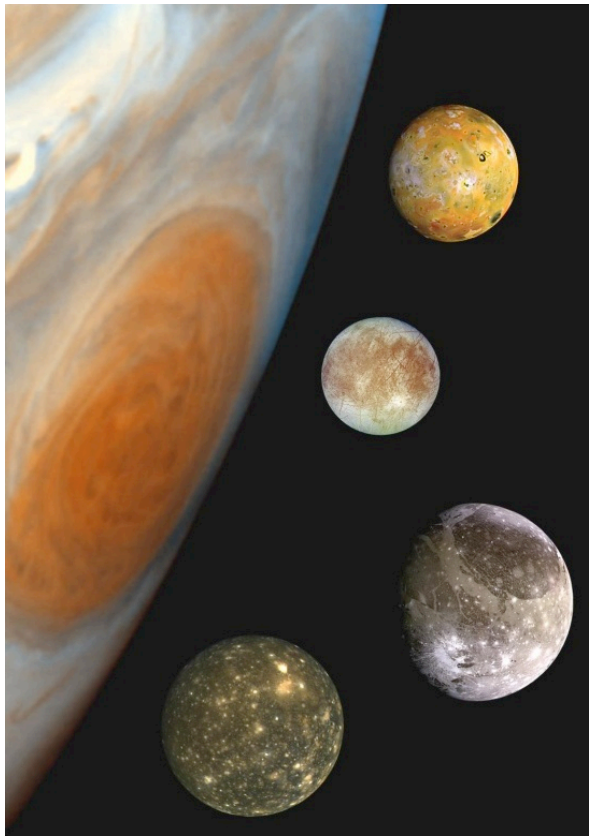
Example of the Mars system

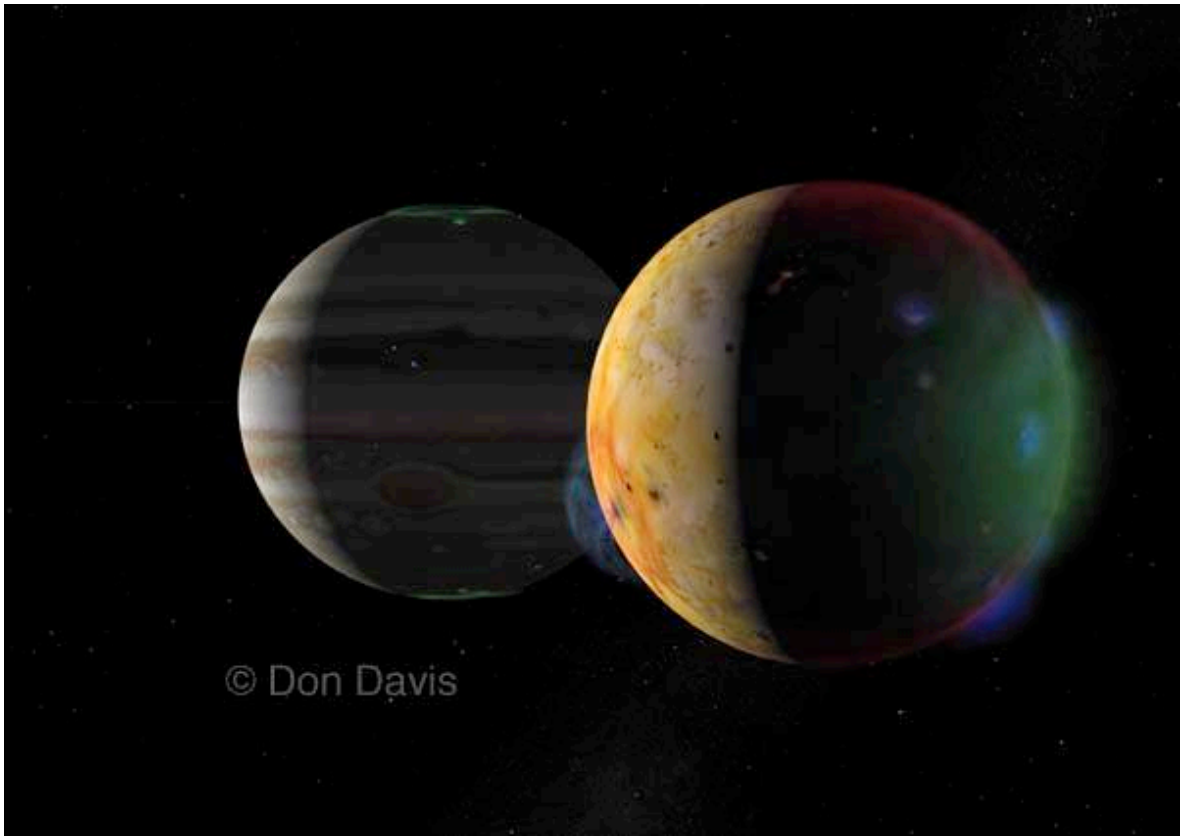
Estimation of Phobos tidal acceleration over time (Jacobson 2010):

Reference	$s \times 10^{-3}$ (deg yr^{-2})	κ_2	Q	γ (deg)
Sharpless (1945)	1.882 ± 0.171			
Shor (1975)	1.427 ± 0.147			
Sinclair (1978)	1.326 ± 0.118			
Jacobson et al. (1989)	1.249 ± 0.018			
Chapront-Touzé (1990)	1.270 ± 0.008			
Emelyanov et al. (1993)	1.290 ± 0.010			
Bills et al. (2005)	1.367 ± 0.006	0.163	85.6 ± 0.4	$0^\circ.3346 \pm 0^\circ.0014$
Rainey & Aharonson (2006)	1.334 ± 0.006	0.153	78.6 ± 0.8	$0^\circ.3645 \pm 0^\circ.0039$
Lainey et al. (2007)	1.270 ± 0.015	0.152	79.9 ± 0.7	$0^\circ.3585 \pm 0^\circ.0031$
Current	1.270 ± 0.003	0.152	82.8 ± 0.2	$0^\circ.3458 \pm 0^\circ.0009$

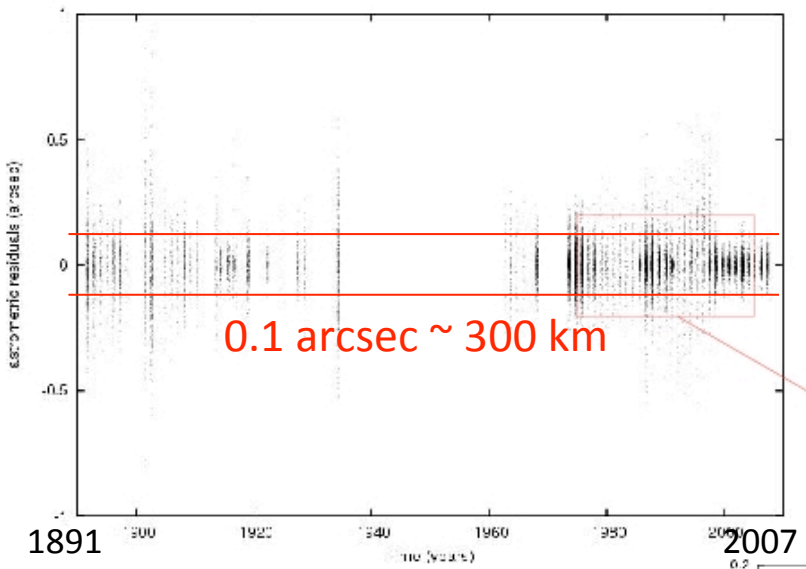
Pretty good agreement since decades!

Example of the Jovian system

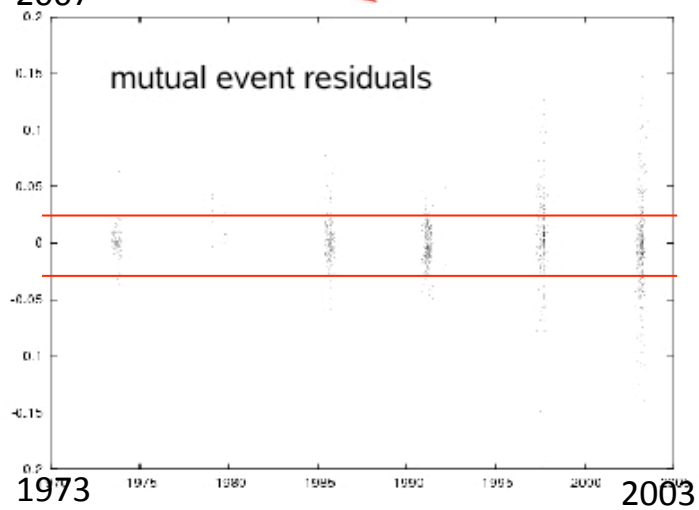




Example of the Jovian system



→Residuals after fitting the initial state vectors of all the Galilean moons and the ratios k_2/Q inside Io and Jupiter



0.025 arcsec ~ 75 km

Lainey, Arlot, Karatekin, Van Hoolst (Nature, 2009)

Example of the Jovian system

Table of residuals

Remind: 0.1 arcsec ~ 300 km

$<\mu_{\alpha cos\delta}>$	$\sigma_{\alpha cos\delta}$	$<\mu_{\delta}>$	σ_{δ}	N
-0.0023	0.0655	0.0009	0.0661	2405
0.0003	0.0638	0.0036	0.0635	2423
0.0065	0.0817	0.0070	0.0884	2625
0.0005	0.0971	-0.0052	0.1106	2824

Photographic and CCD observations

$<\mu_s>$	σ_s	$<\mu_p>$	σ_p	N_s	N_p
-0.0375	0.1570	0.0042	0.1817	274	287
-0.0531	0.1515	0.0184	0.2311	186	194
-0.0402	0.1403	0.0375	0.3094	106	106
-	-	-	-	-	-

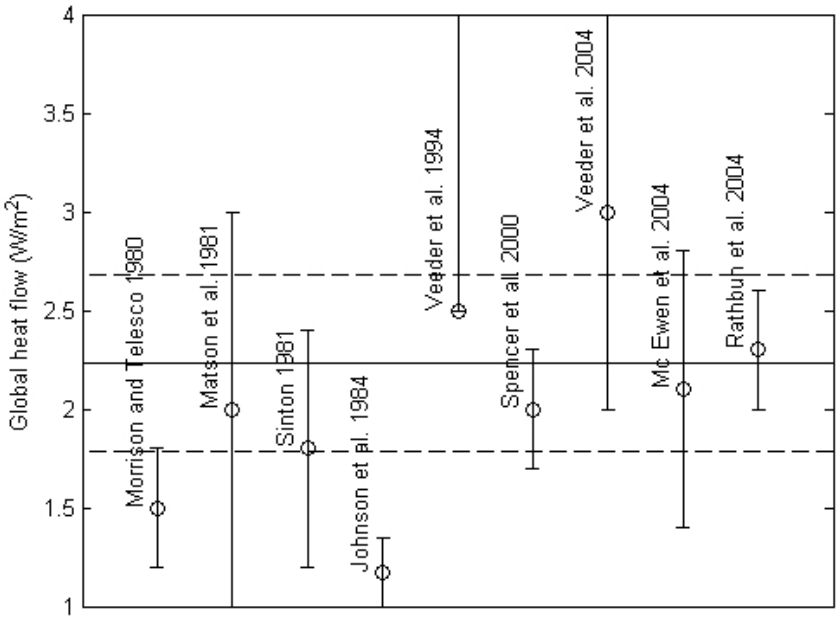
Heliometer observations

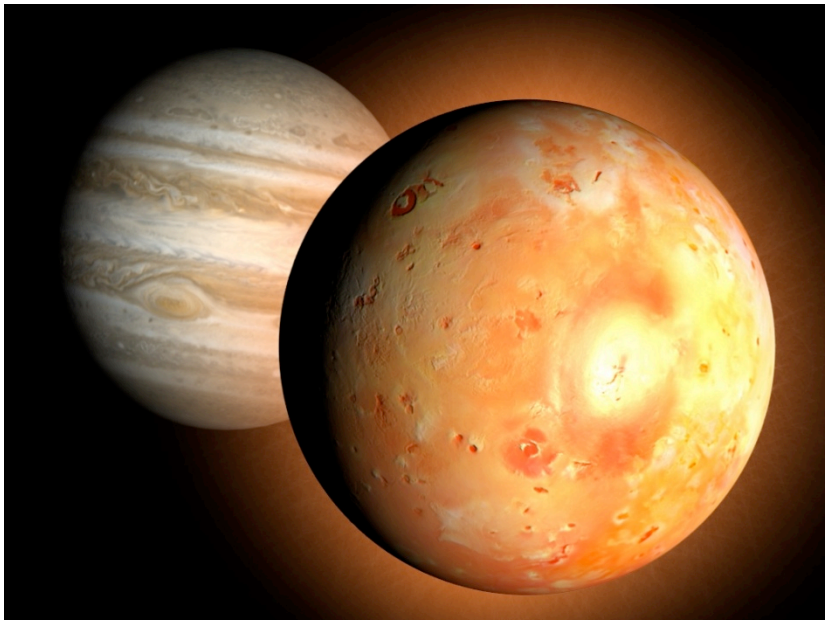
(before 1902)

Example of the Jovian system

Our fit of Io's dissipation provides $k_2/Q = 0.015 \pm 0.003$

One can compare our value with the ones derived from IR emission





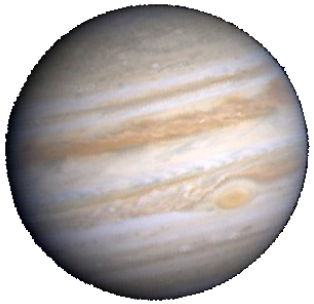
We obtained a very good agreement and confirm the values derived from heat flux observations!

Example of the Jovian system

Our value of the Jovian dissipation is Q=35600 ± 6600 assuming k_2=0.379

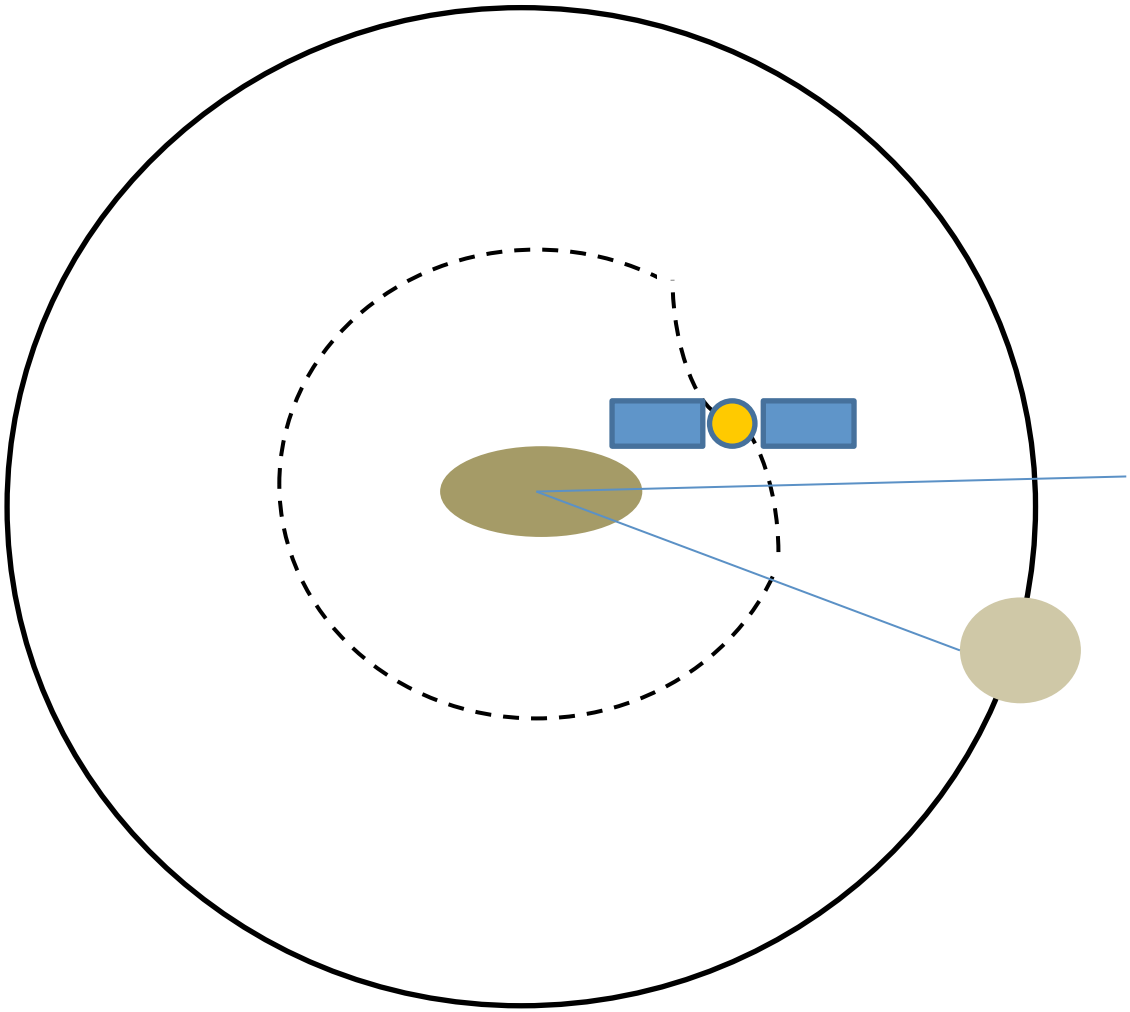
The estimation from (Goldreich and Soter (1966), Gavrilov and Zharkov (1977))

$$2.5\ 10^4 < Q_{jupiter} < 2.5\ 10^5$$



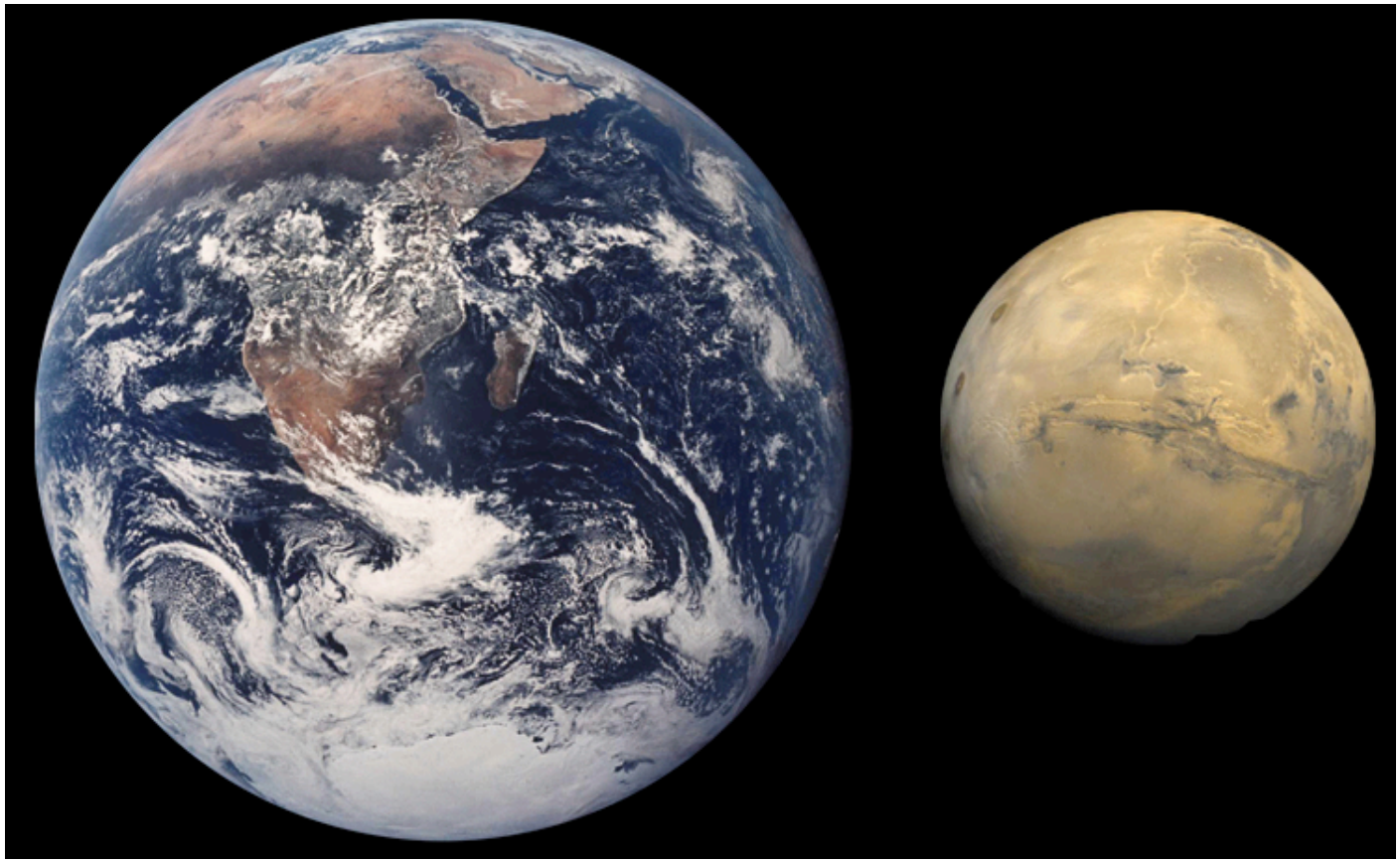
Our estimation gets a much smaller error bar AND it is derived from observations

Another way to determine tidal dissipation on short time scale



Tidal bulges induce short periodic effect on artifical satellite motions!

Another way to determine tidal dissipation on short time scale



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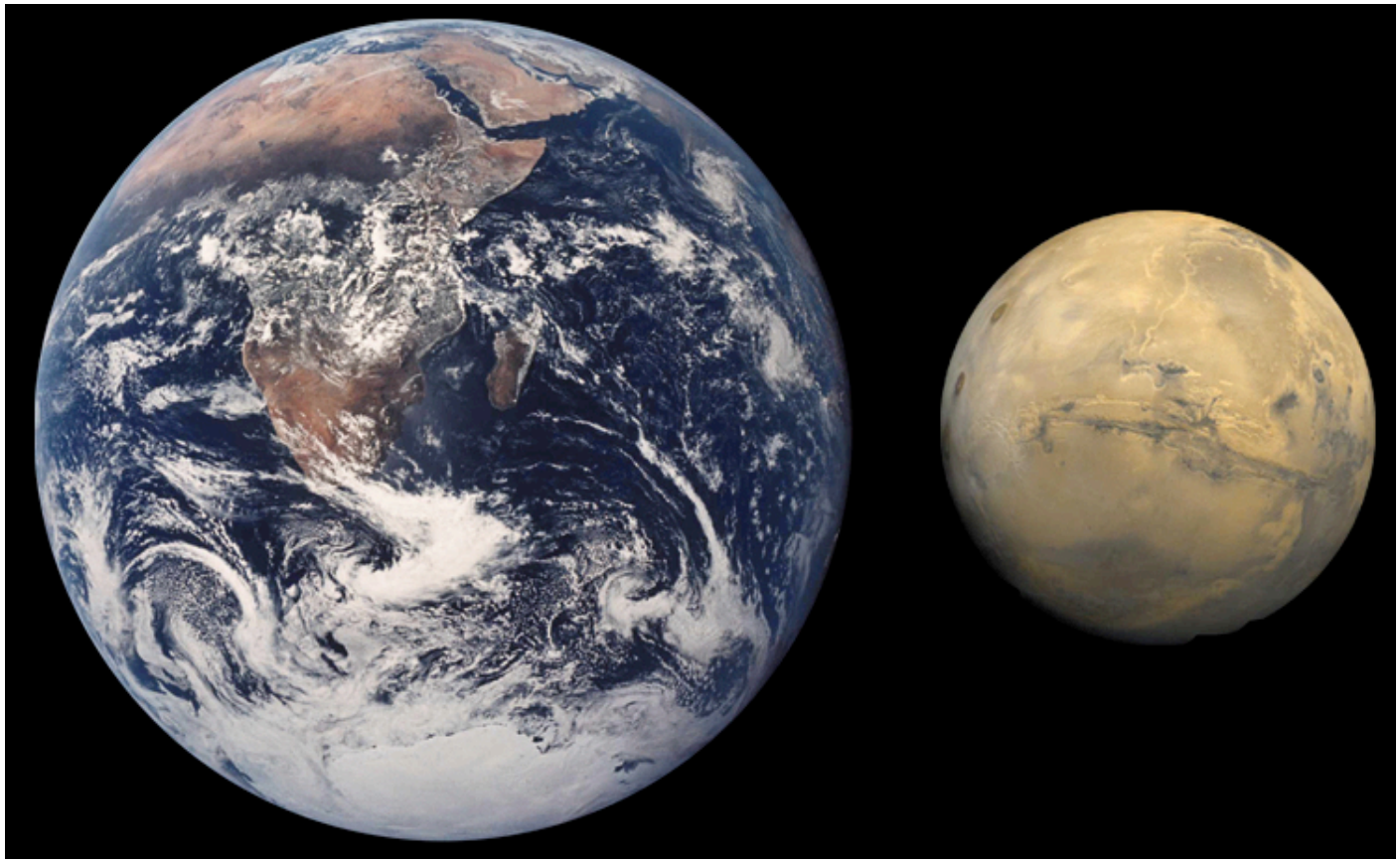
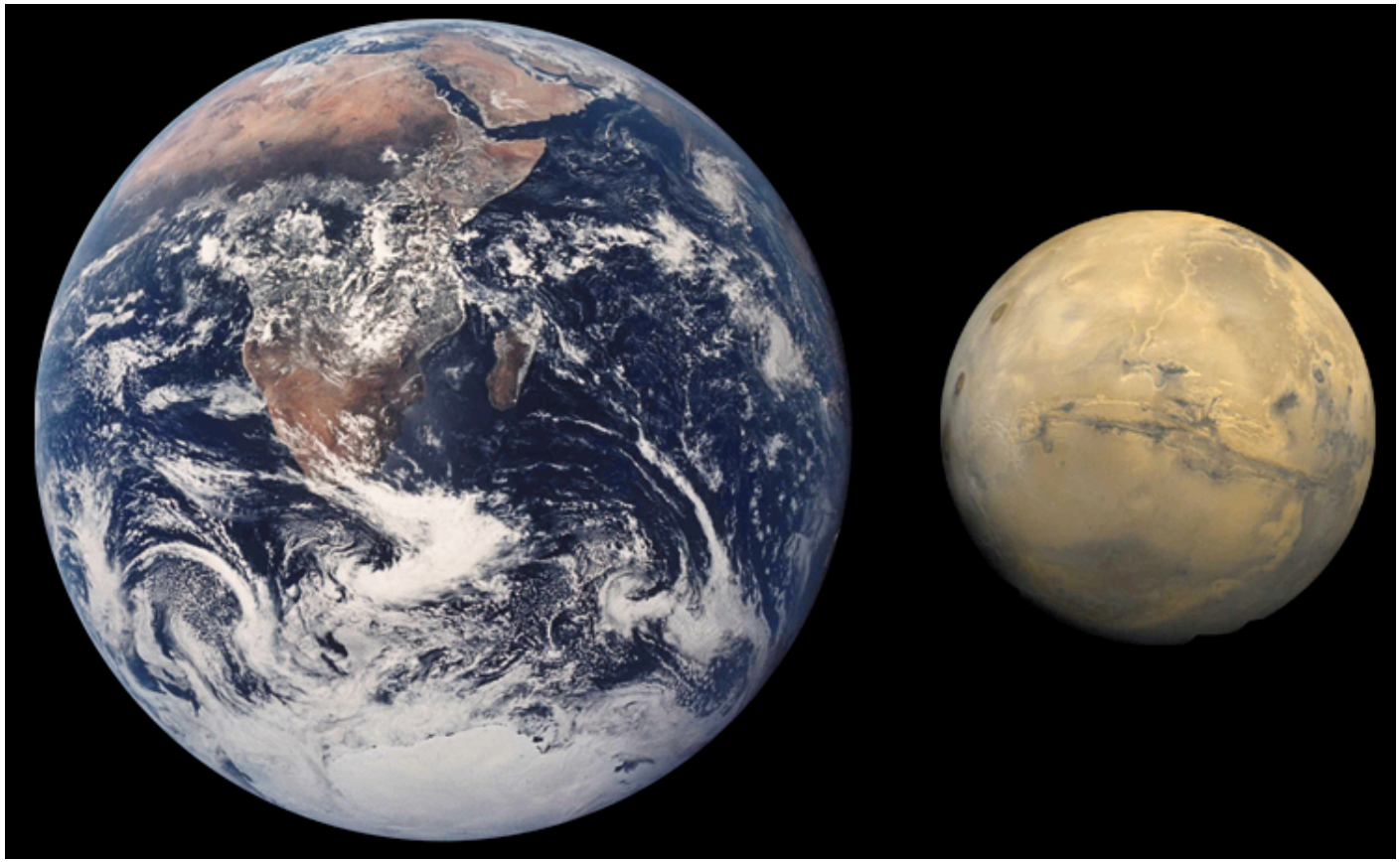


Ray et al. (2001) provided the first estimation of Q (terrestrial tides) Q=260 +/- 80

Another way to determine tidal dissipation on short time scale



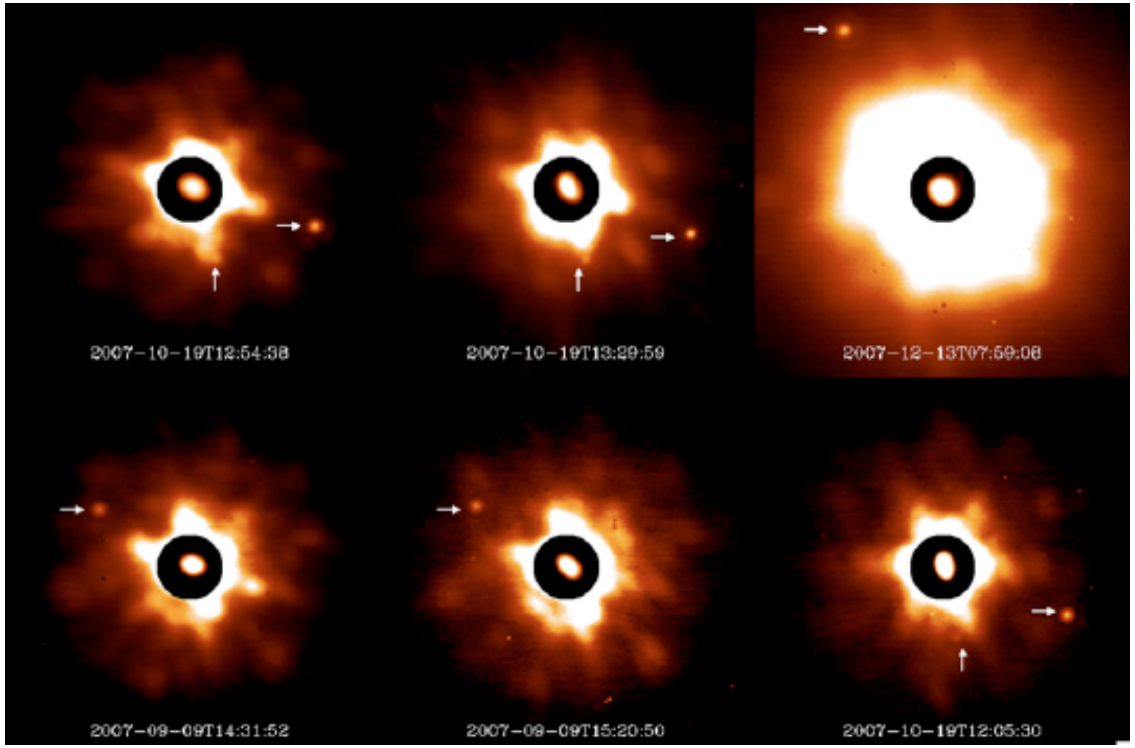
Ray et al. (2001) provided the first estimation of Q (terrestrial tides) Q=260 +/- 80

Konopliv et al. (2011) still cannot derive Q from S/C

Coming to granular material…

Coming to granular media (short term dynamics)

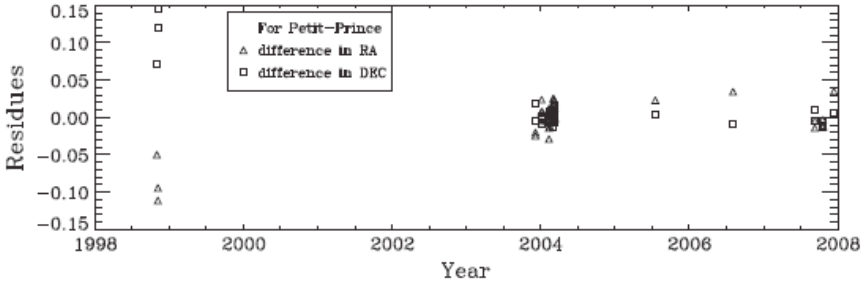
First application of full dynamical model to MBA in 2010: (Marchis et al. 2010)



So far, the models consisted in a precessing orbit... New observations required a more sophisticated model

Coming to granular media (short term dynamics)

We have introduced a full dynamical model, including Eugania's J_2, Solar and planetary perturbations



	Petit–Prince	Princesse
Semi-major axis		
$\langle a \rangle$ in km	1164.6	611.1
min, max (a)	1164.4, 1164.8	610.8, 611.6
1-σ (a)	0.1	0.2
Mean motion		
$\langle n \rangle$ in rad/day	1.3322	3.5047
min, max (n)	1.3318, 1.3326	3.5008, 3.5077
1-σ (n)	0.0002	0.0016
Eccentricity		
$\langle e \rangle$	0.0051	0.0708
min, max (e)	0.0040, 0.0062	0.0682, 0.0738
1-σ (e)	0.0006	0.0018
Inclination		
$\langle i \rangle$ in deg	9.22	18.10
min, max (i)	8.97, 9.35	17.98, 18.19
1-σ (i)	0.10	0.05
Mean rate for the node		
$\langle d\Omega/dt \rangle$ in deg/day	−0.006	−0.550
min, max ($d\Omega/dt$)	−0.068, −0.050	−0.563, −0.537
1-σ ($d\Omega/dt$)	0.004	0.006
Mean rate for periapsis		
$\langle d\omega/dt \rangle$ in deg/day	−0.003	1.0
min, max ($d\omega/dt$)	−2.16, −2.65	0.4, 1.6
1-σ ($d\omega/dt$)	1.42	0.4

We could initial state vectors, Eugenia's J_2 and its polar coordinates…

Precision vs. Accuracy:

Our polar coordinates are close to the expected one from photometrie

But, our value of J_2 is much smaller than expected assuming constant density).

Eugenia is far from a simple oblate body, our estimation is biased by other harmonics (c_{22}, …)

This work is going on adding more observations and studying different systems (thesis of L.Beauvalet)

Coming to granular media (long term dynamics)

What about Phobos' history if it is a rubble pile? (ρ=1.876 g/cm^3)

Significant tidal dissipation may have occured in the past (possible high eccentricity phase) → capture scenario still possible?