Rotation and Reshaping of Self-Gravitating Aggregates

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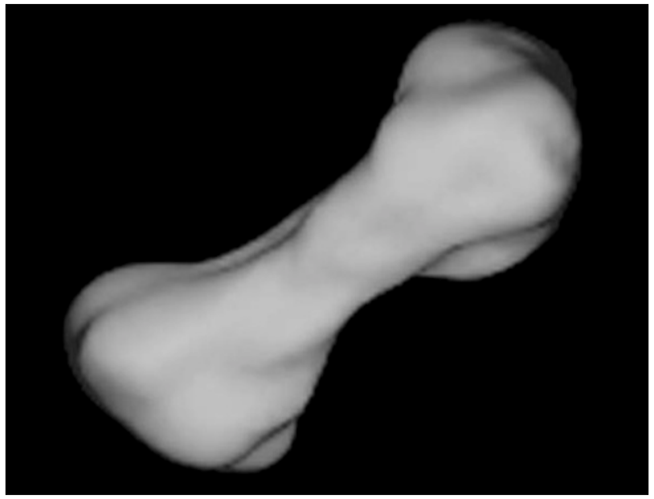
University of Colorado **Boulder**

Asteroid Shapes and Rotation

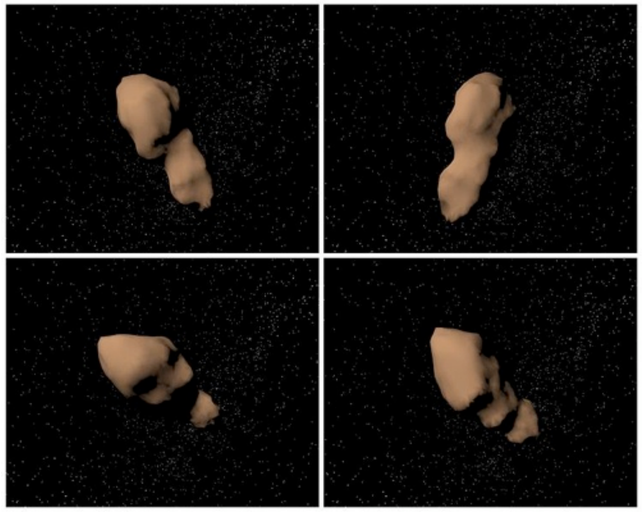
Itokawa



Kleopatra

Ida

Eros



Toutatis

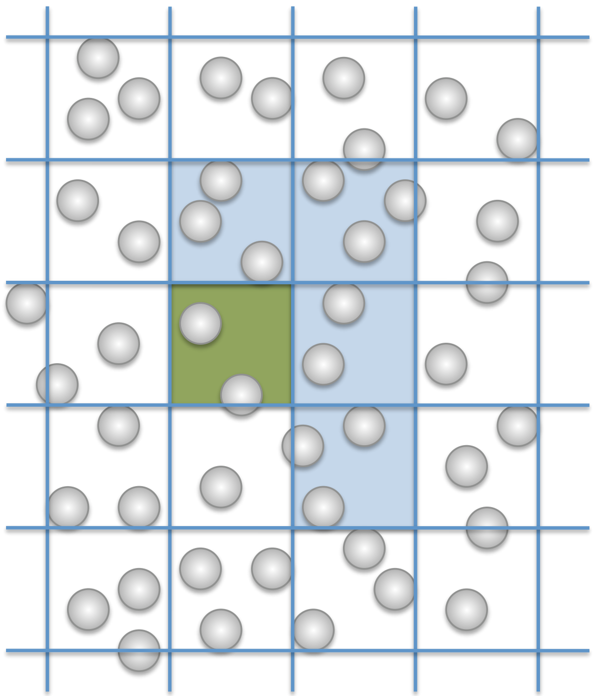
KW_4

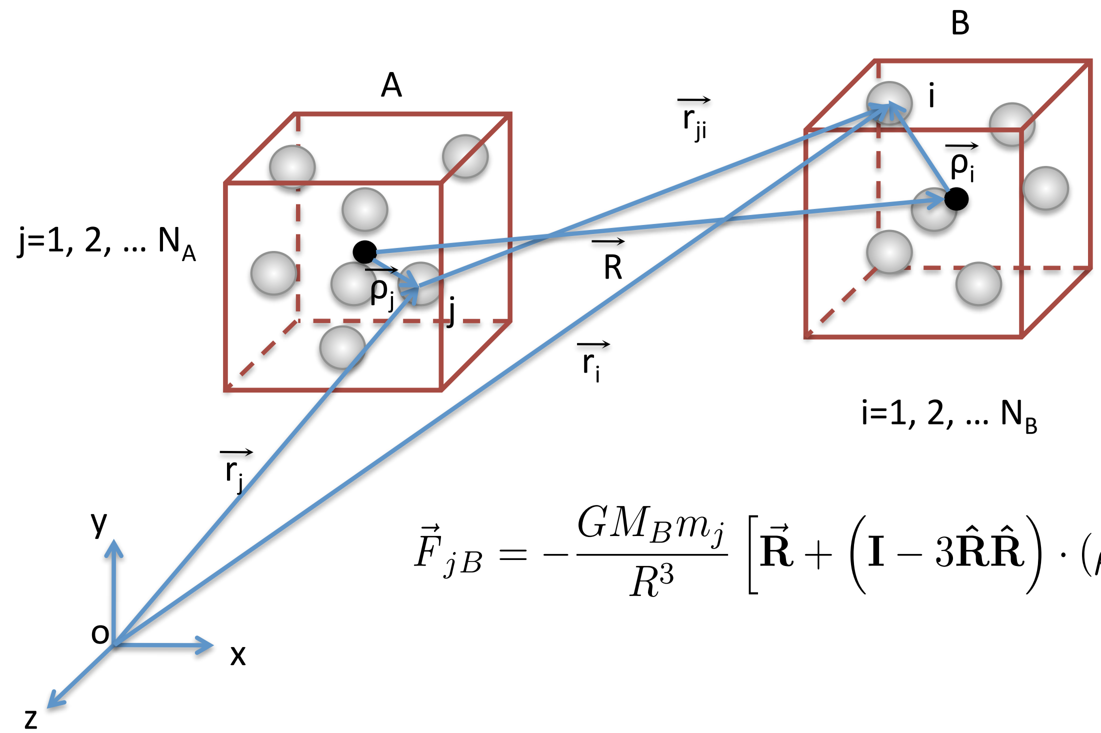
Simulation Method

- Soft-Sphere DEM:
  - Discretizes time, making use of a time-step (δt) for the integration of the equations of motion.
  - Given a set of initial conditions, the code integrates the equations of motion to find positions and velocities at the time $t+\delta t$.
  - This new configuration allows for the calculation of forces/accelerations which in turn are used for the integration.
  - A collision occurs when **particles overlap**.
  - Collisions are handled through a repulsive potential which cut-off distance is the radius of the particle (linear and non-linear springs and dashpots are commonly used).
- Gravitational Forces:
  - The simulation uses a structure similar to that of a tree-code, but with a static grid.
  - Gravity between close neighbours is calculated exactly.
- Simulations:
  - Periodic boundary conditions.
  - The simulations used 2000, 3000, 4000 and 8000 spherical particles (8-9m).
  - The particle density changed from 3200 up until 5200 kg/m^3.
  - Angular velocity is increased in big-fixed steps.

Gravity Algorithm

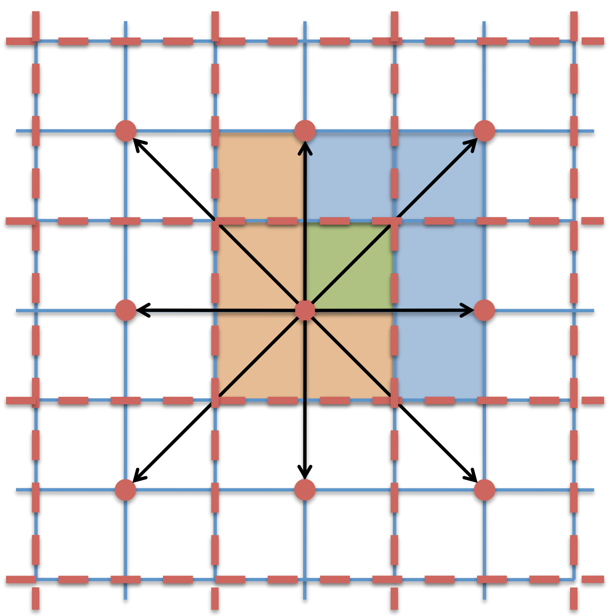
Link List





$$\vec{F}_{jB} = -\frac{GM_B m_j}{R^3}\left[\vec{\mathbf{R}} + \left(\mathbf{I} - 3\hat{\mathbf{R}}\hat{\mathbf{R}}\right)\cdot(\vec{\rho}_B - \vec{\rho}_j)\right]$$

2D Gravity (example)



- Same idea as for a tree code, but the space is divided using a static-regular grid.
- This allows the code to store distances and positions in memory avoiding their calculation.
- The linking-list is used to calculate gravity among close neighbours.
- Each cell in the gravitational grid is twice as big as a linking-list cell.
- Gravitational forces are calculated between groups of particles in the bigger cells.
- Contact forces, approximated gravitational forces and exact gravitational forces are calculated at different intervals.

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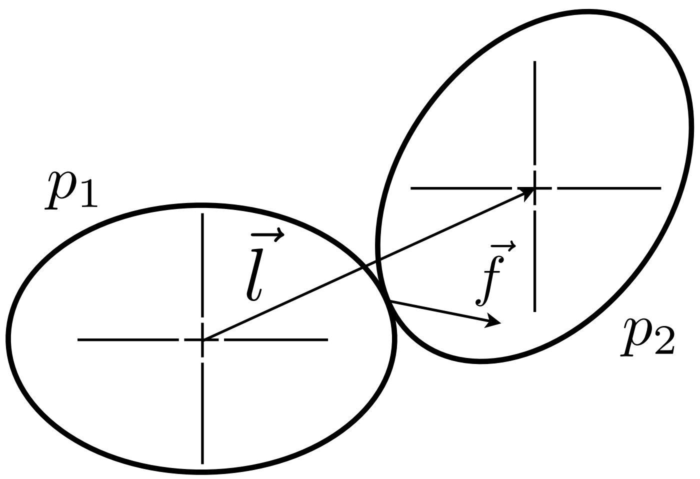
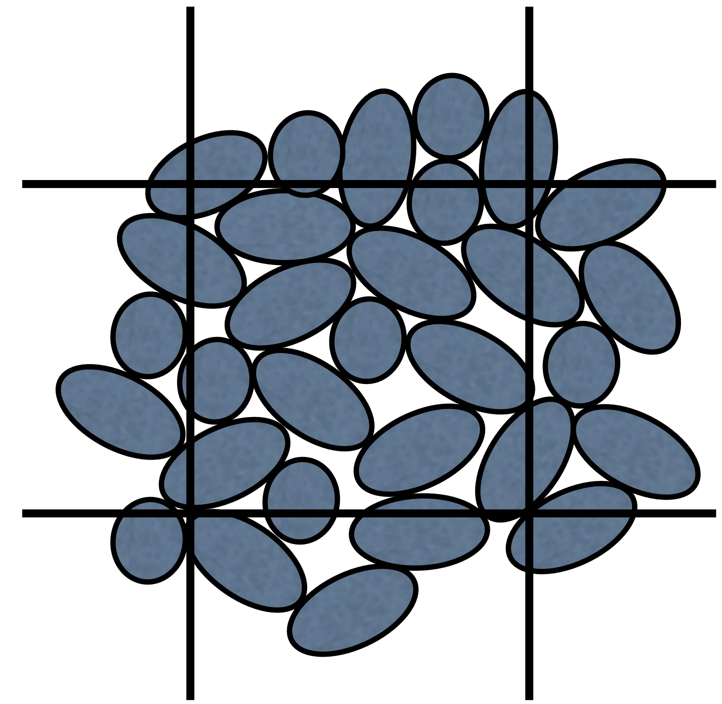
Average-Cauchy Stress Tensor





$$\bar{\sigma} = \frac{w}{V} \sum_{c \in K_V} \vec{f} \otimes \vec{l}$$

$w = 1$ within the volume

$w = \frac{1}{2}$ in the edge the volume

Principal stresses: $\sigma_1,\ \sigma_2$ and σ_3

Drucker-Prager Yield Criterion

$$\sqrt{\frac{1}{6}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right]}-Y-3sp\le 0$$

Y is related to the cohesive forces (zero for this research)

$$p=\frac{\sigma_1+\sigma_2+\sigma_3}{3}$$ is the hydrostatic pressure

$$s=\frac{2\sin\theta}{\sqrt{3}(3-\sin\theta)}$$

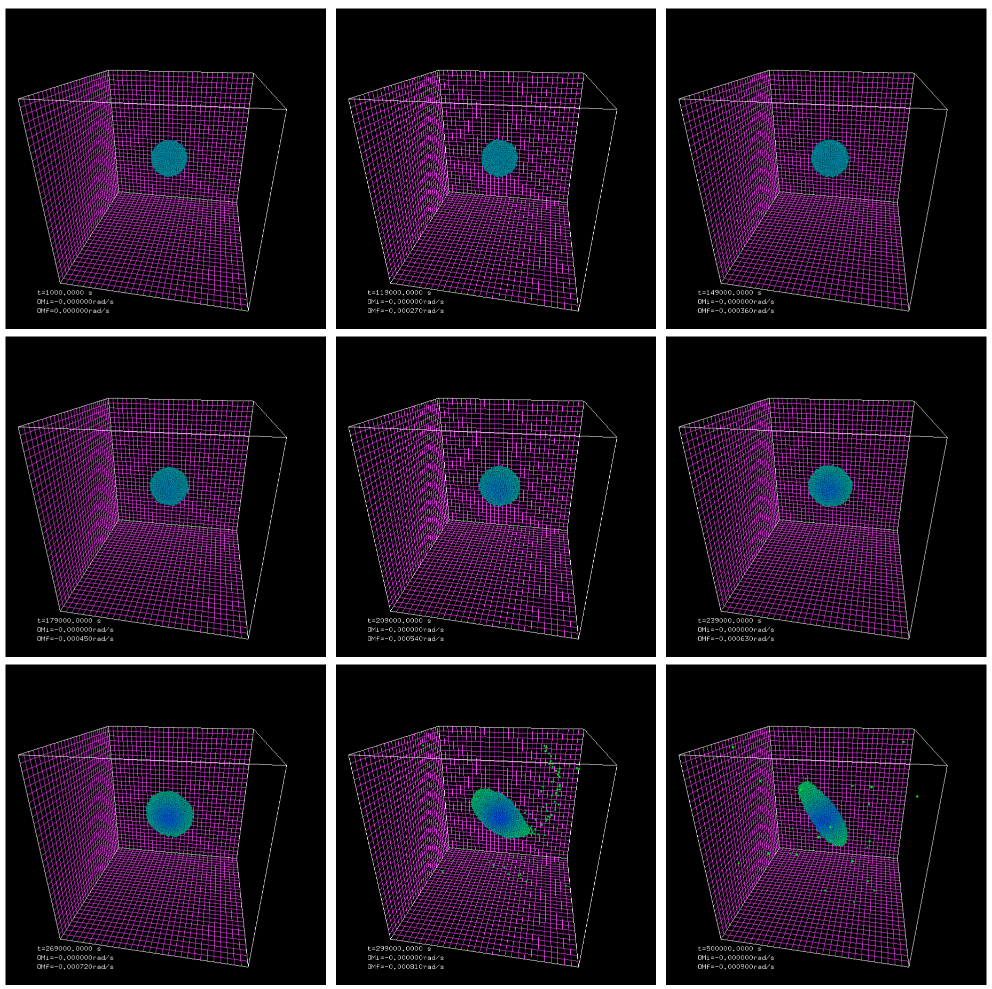
$$s=\frac{2\sin\theta}{\sqrt{3}(3+\sin\theta)}$$

There is no unique relation between s and the angle of friction, so here we provide the two formulas that furnish two limit possibilities.

Spherical Aggregates

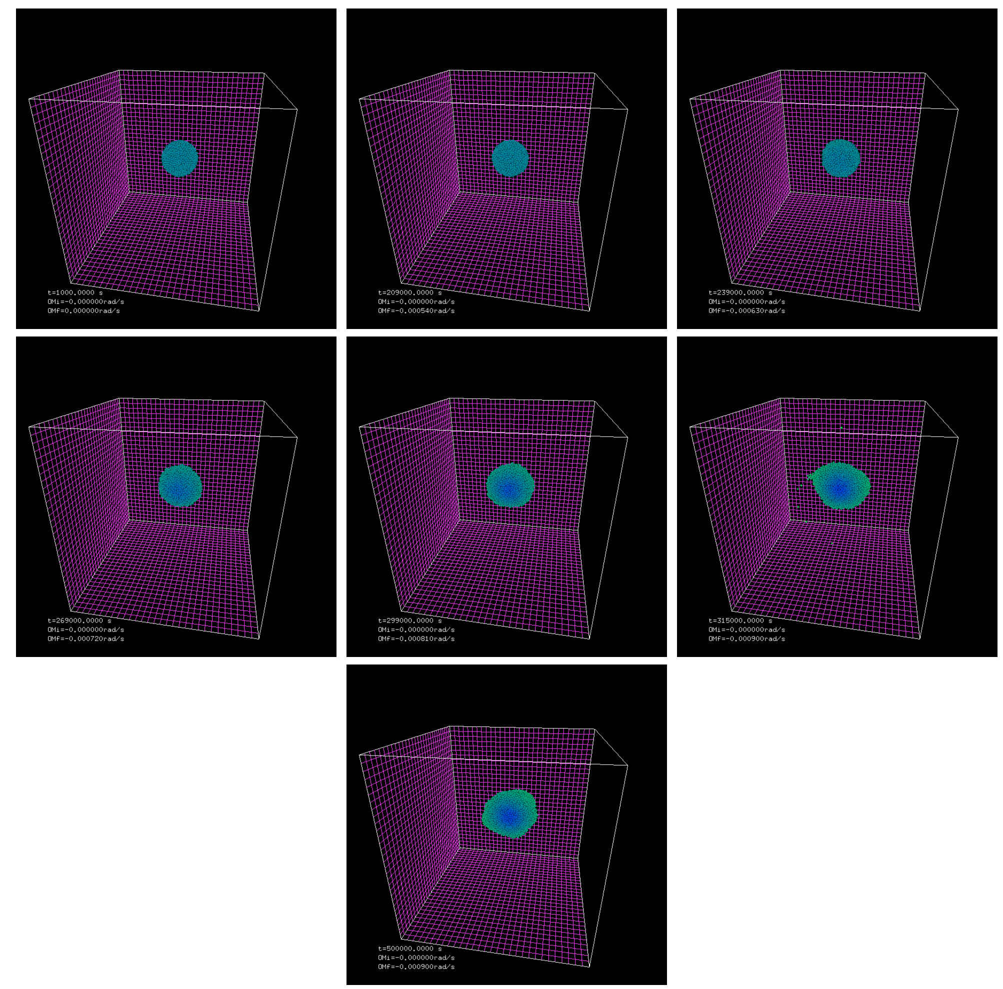
N=3000 - no friction

Initially the body becomes an oblate spheroid, then it sheds material, becomes prolate and keeps on shedding material until it becomes highly prolate.



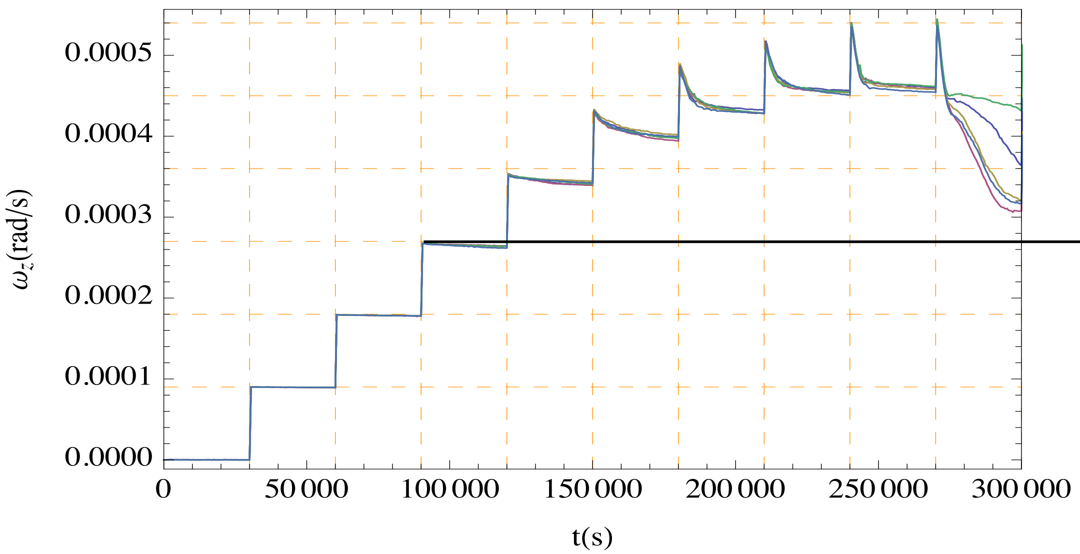
N=3000 - friction

Initially the body becomes an oblate spheroid, then it sheds material, but it remains highly oblate even at the maximum spin rate applied.



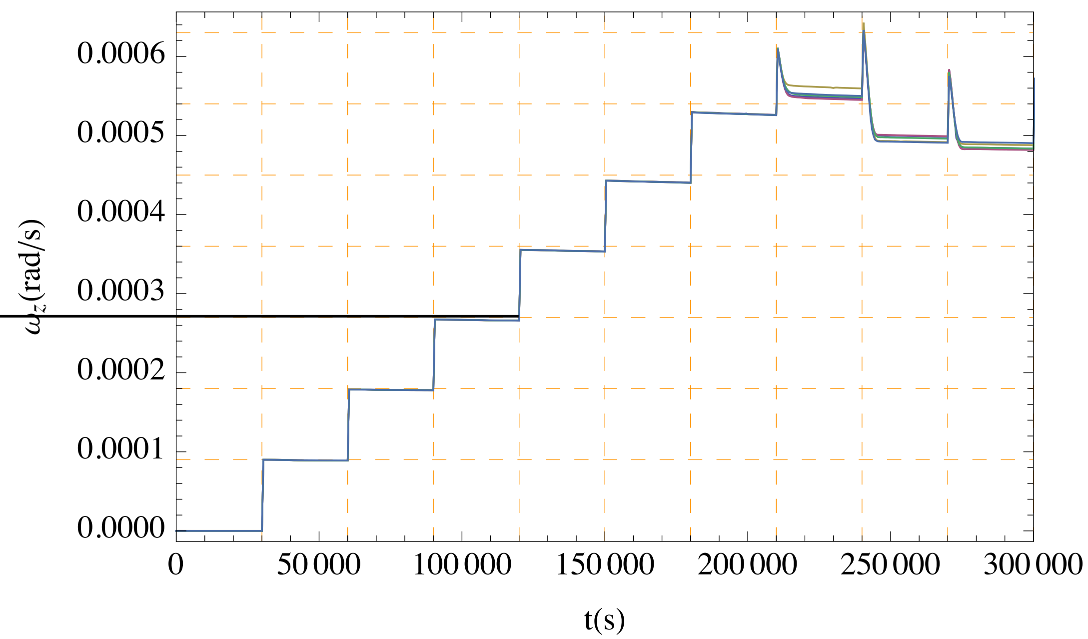
Spherical Aggregates

N=3000 - no friction



Angle of friction: 14°-16° (Drucker-Prager).
First reshaping event: 3.6x10^{-4} rad/s

N=3000 - friction

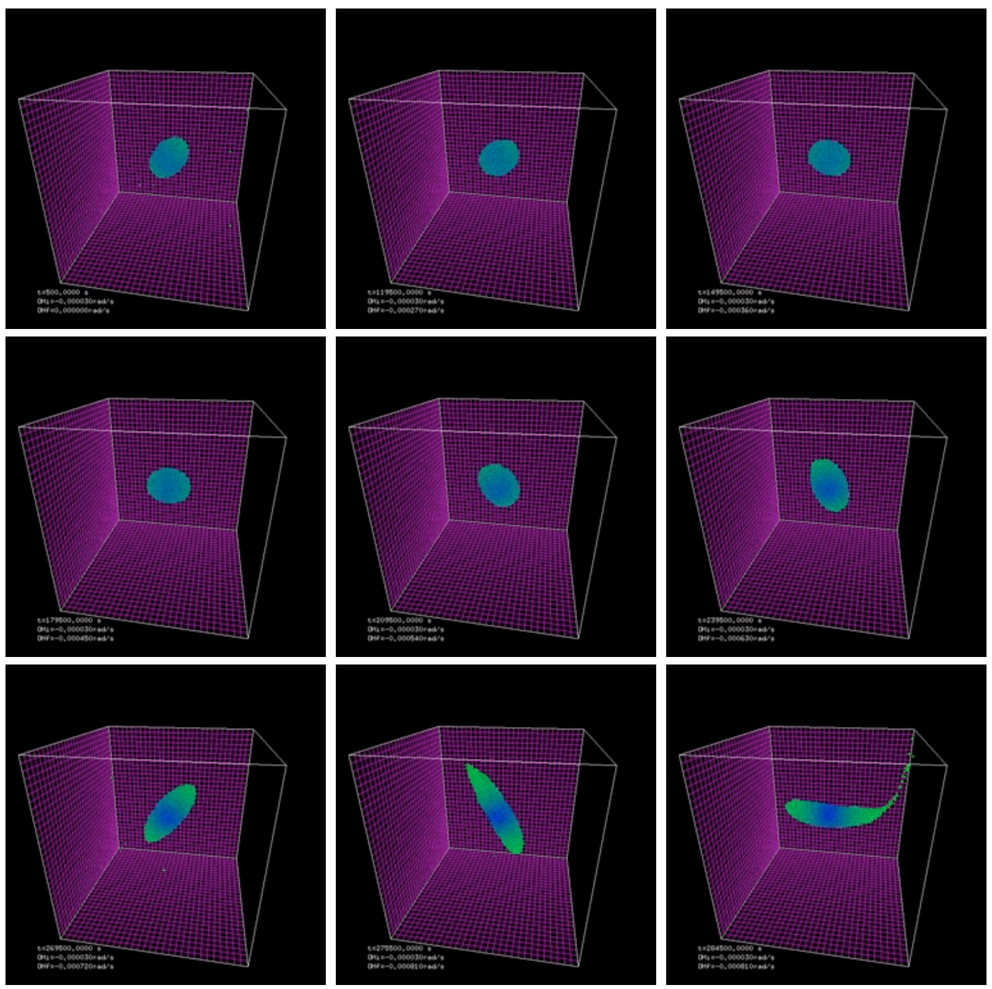


Angle of friction: 30°-45° (Drucker-Pager).
First reshaping event: >5.3x10^{-4} rad/s

Ellipsoidal Aggregates

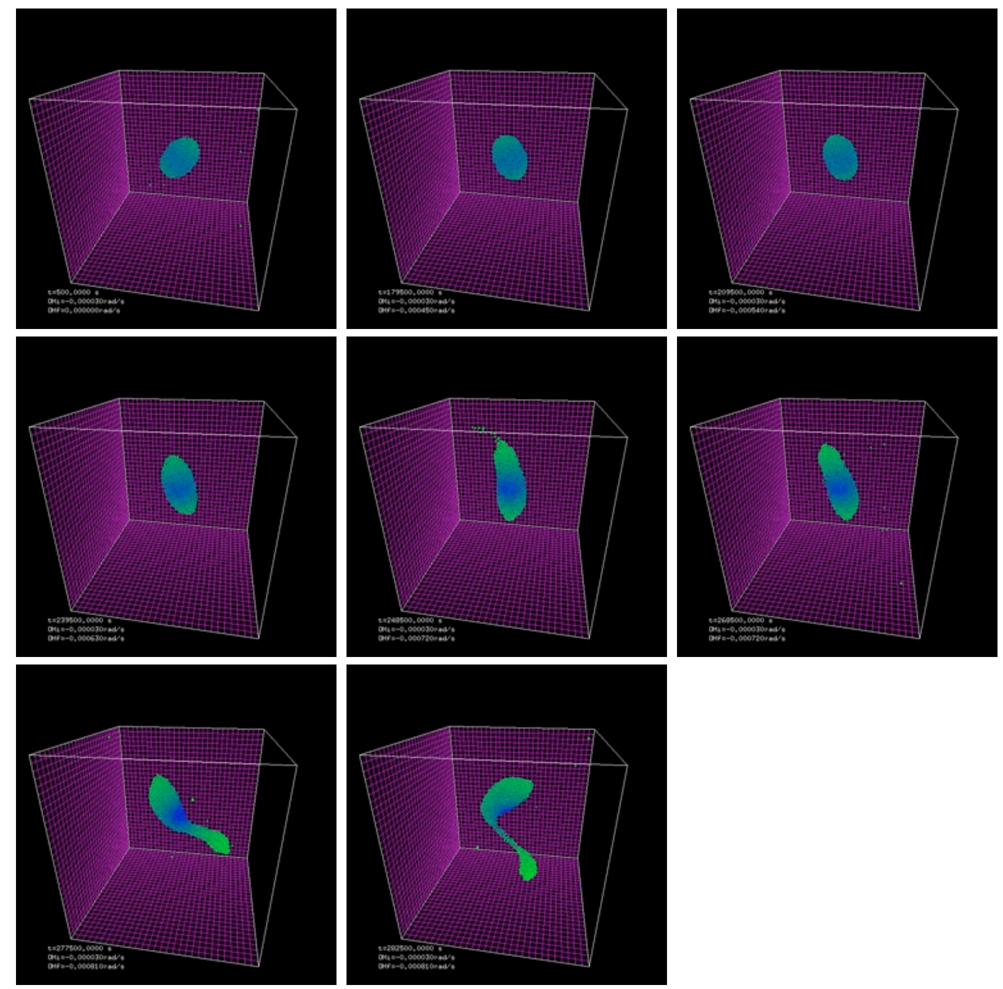
N=3000 - no friction

At a density-dependent spin rate the aggregate elongates, material shedding starts and is continued until the body is disrupted.



N=3000 - friction

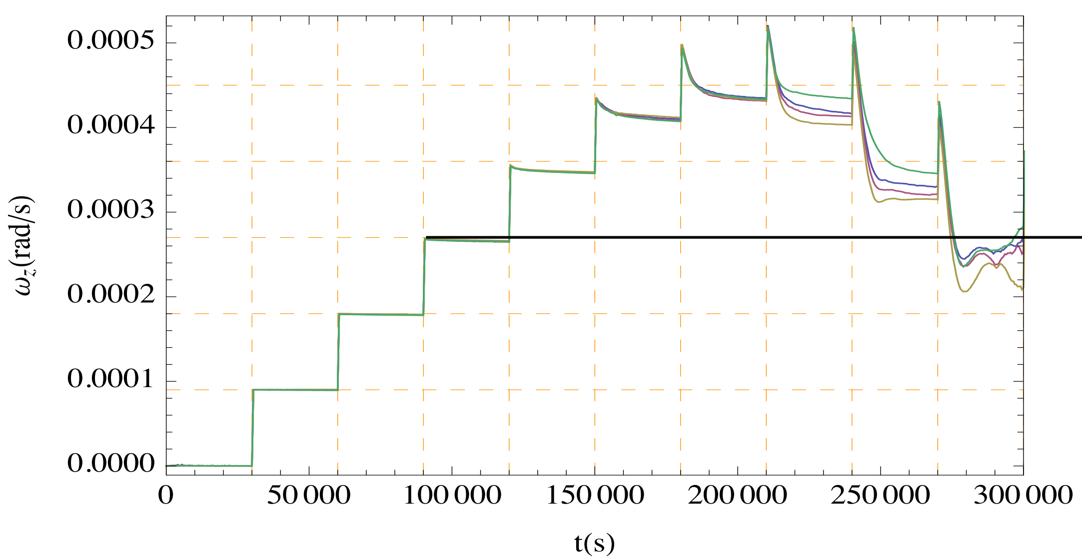
At a higher density-dependent spin rate the aggregate elongates, material shedding takes place, but disruption takes place by fission.



Ellipsoidal Aggregates

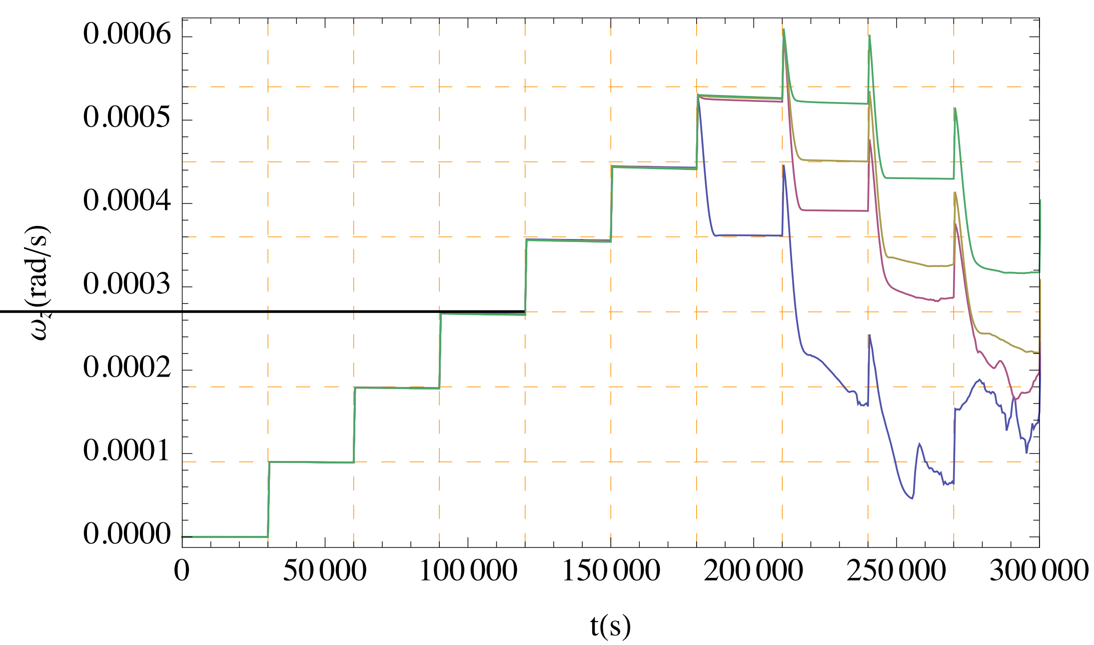
2000, 3000, 4000 and 8000 particles.

No friction



Angle of friction: 12°-14° (Drucker-Prager)
First reshaping event: 3.6×10^{-4} rad/s

Friction



Angle of friction: 24°-34° (Drucker-Prager).
First reshaping event: $>5.3 \times 10^{-4}$ rad/s

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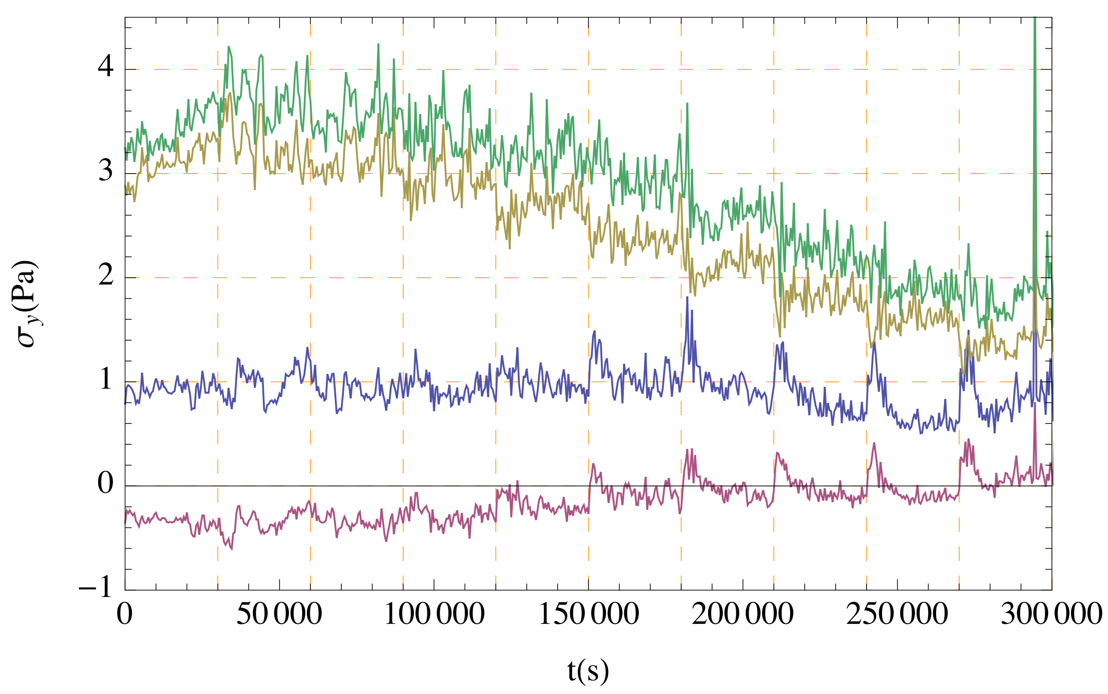
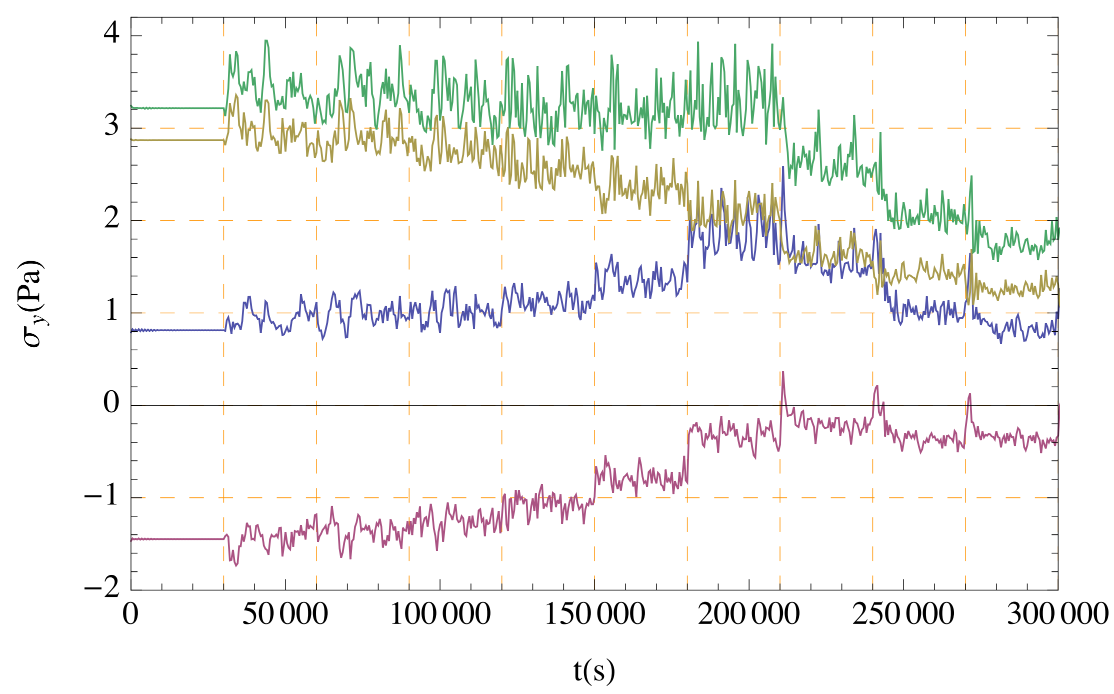
Evaluation of the Yield-Stress Expression

Spherical - no friction



Spherical - friction



Simulations in Numbers

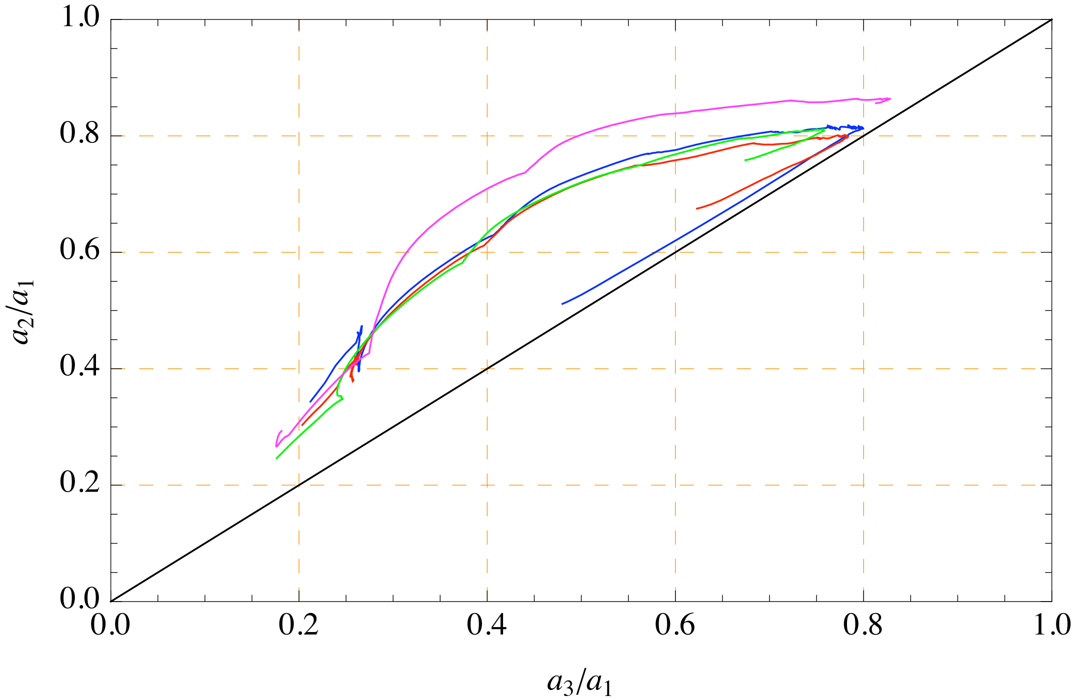
	Sphere-nf	Sphere-fr	Ellipsoid-nf	Ellipsoid-fr
Angle of Friction (θ)	14°-16°	30°-45°	12°-14°	24°-34°
Packing fraction (ϕ)	0.65	0.65→0.61	0.65	0.65→0.61
Critical ang.vel. (ω - rad/s)	3.6×10^{-4}	$>5.3 \times 10^{-4}$	3.6×10^{-4}	$<5.3 \times 10^{-4}$
Period (T - h)	4.85	<3.2	4.85	>3.2→2.9*

- Spherical aggregates seem to be structurally stronger than ellipsoidal ones.
- Angles of friction obtained through the evaluation of the Drucker-Prager criterion are higher for spherical than for ellipsoidal aggregates.
- Only ellipsoidal aggregates formed by 8000 particles could rotate at higher velocities.

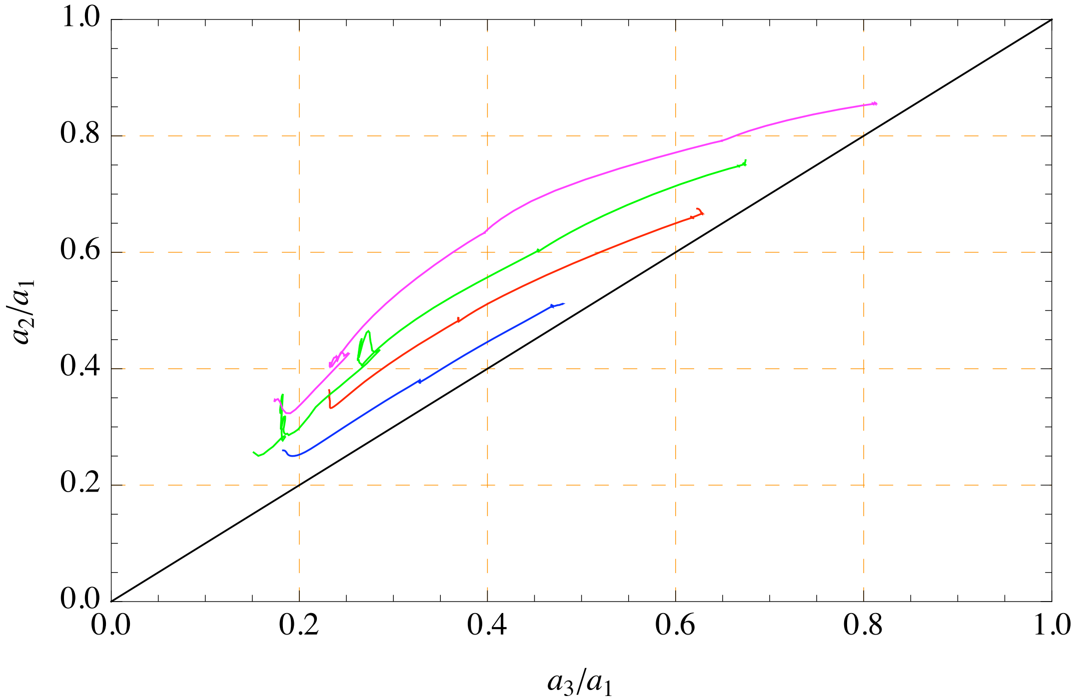
Shape History - Comparison with Theory

2000, 3000, 4000 and 8000 particles.

No friction



Friction

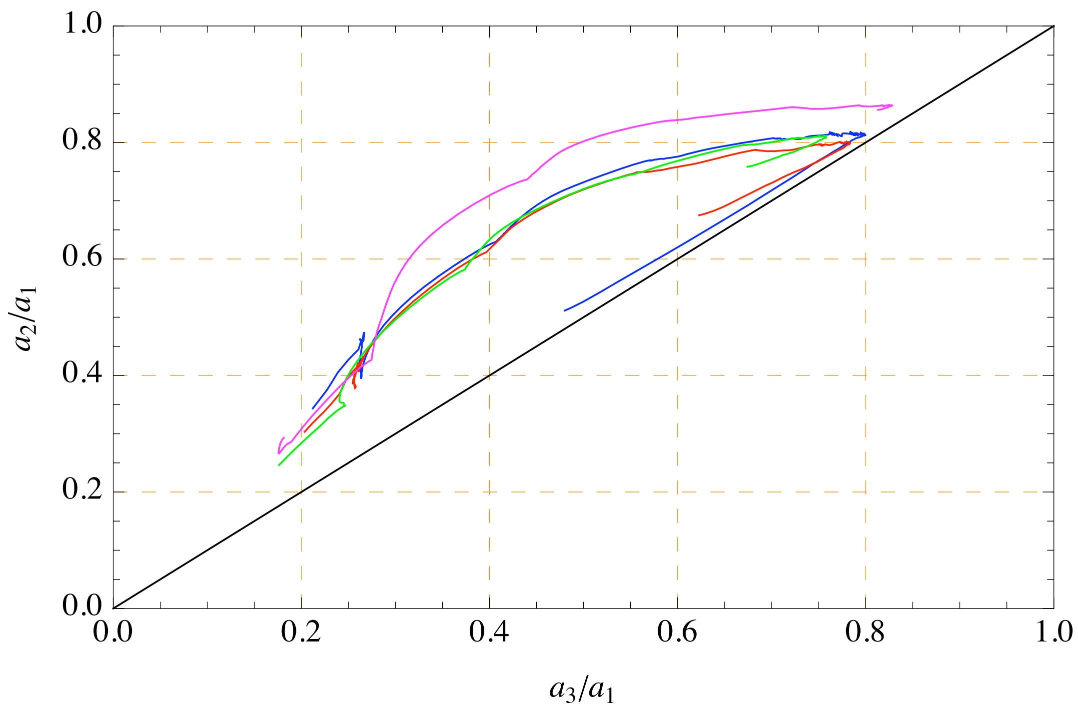


dynamically equivalent equal-volume ellipsoid
(DEEVE)

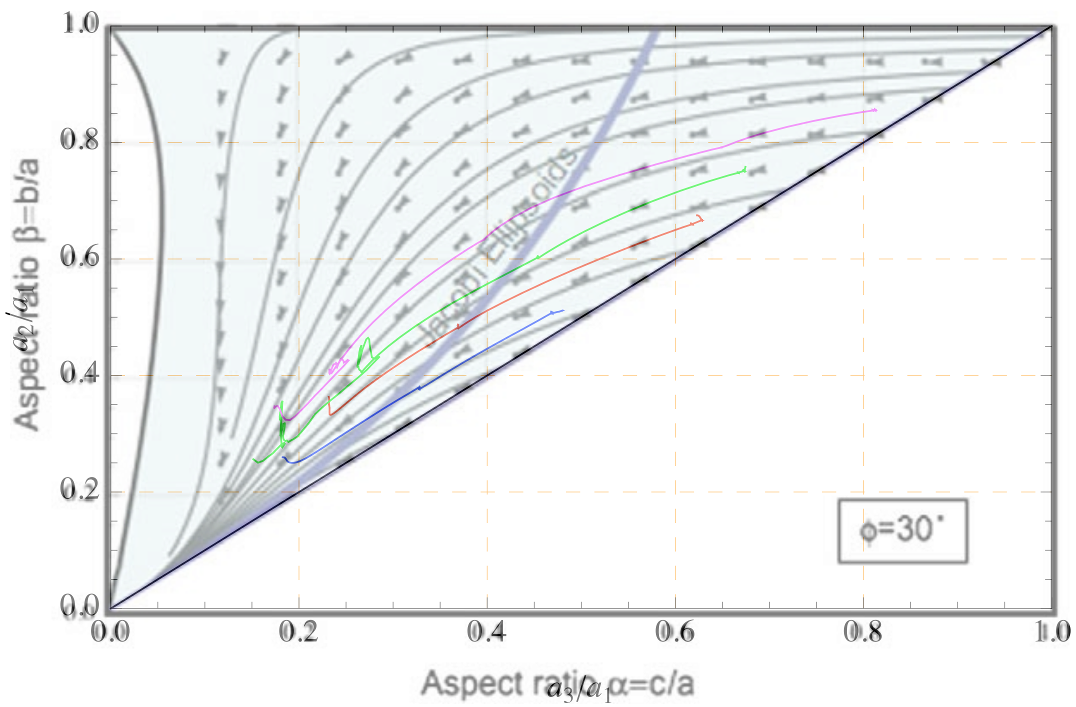
Shape History - Comparison with Theory

2000, 3000, 4000 and 8000 particles.

No friction



Friction

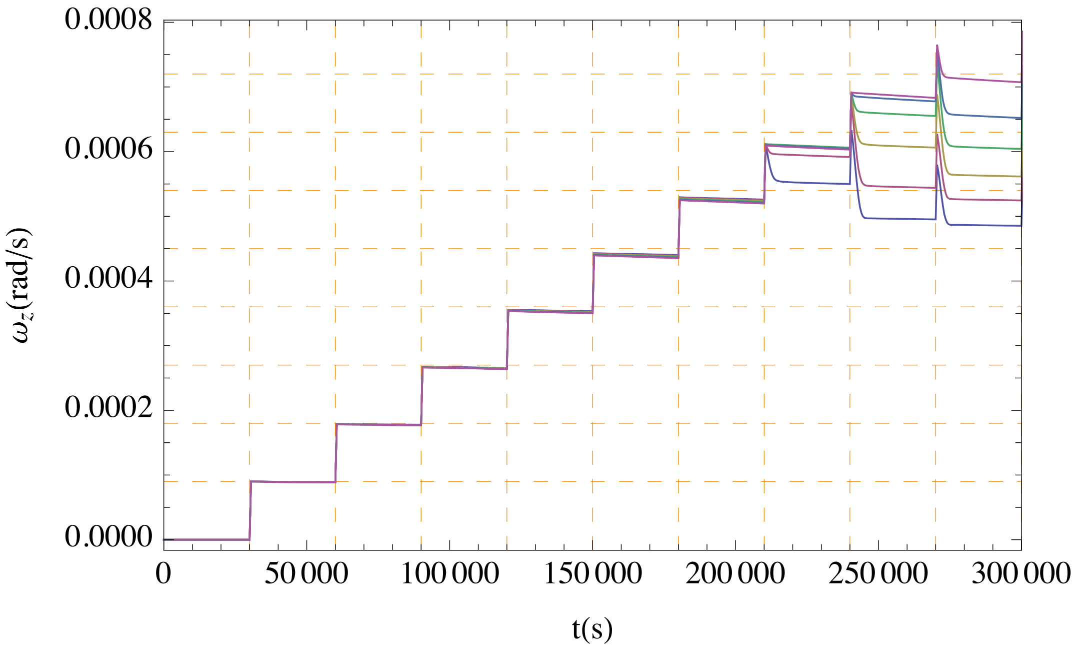


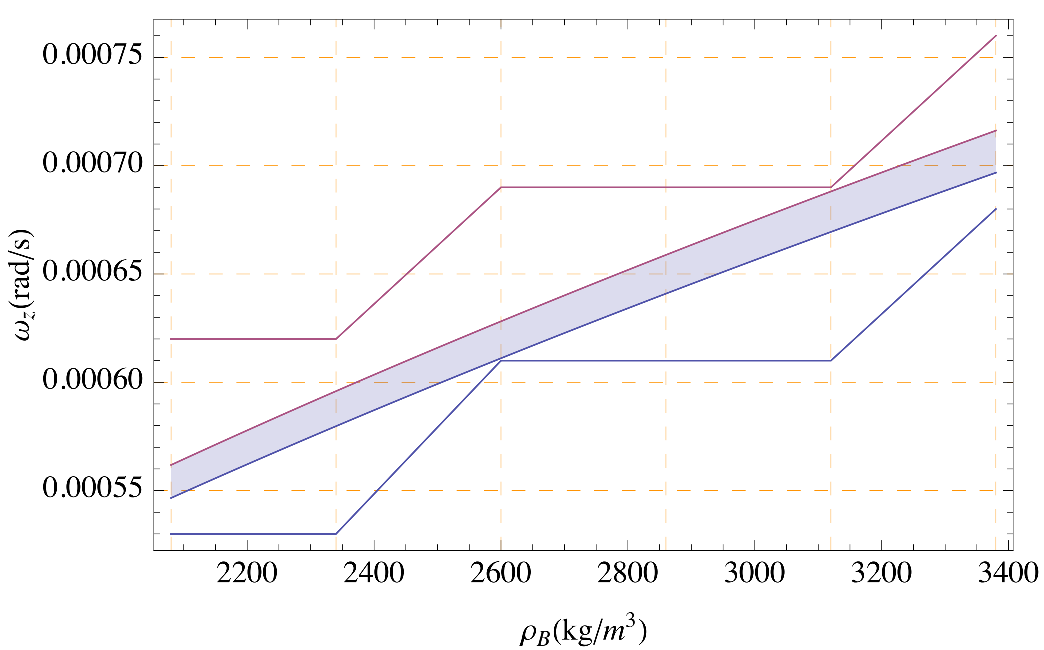
Holsapple, K., On YORP-induced spin deformations of asteroids, Icarus, 2010

dynamically equivalent equal-volume ellipsoid (DEEVE)

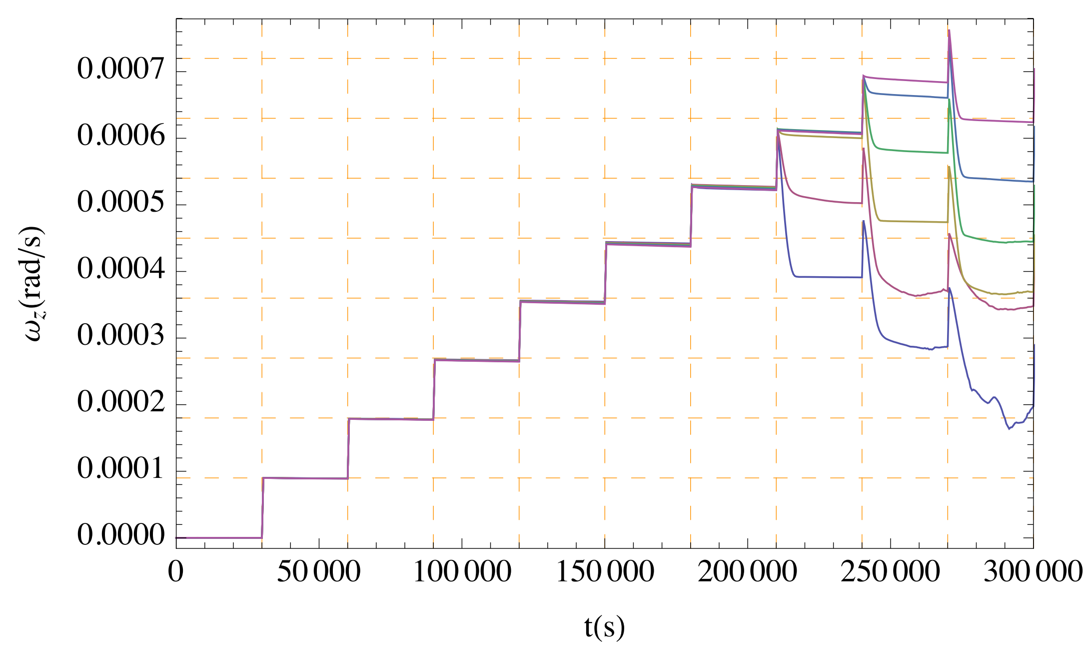
Density Dependance

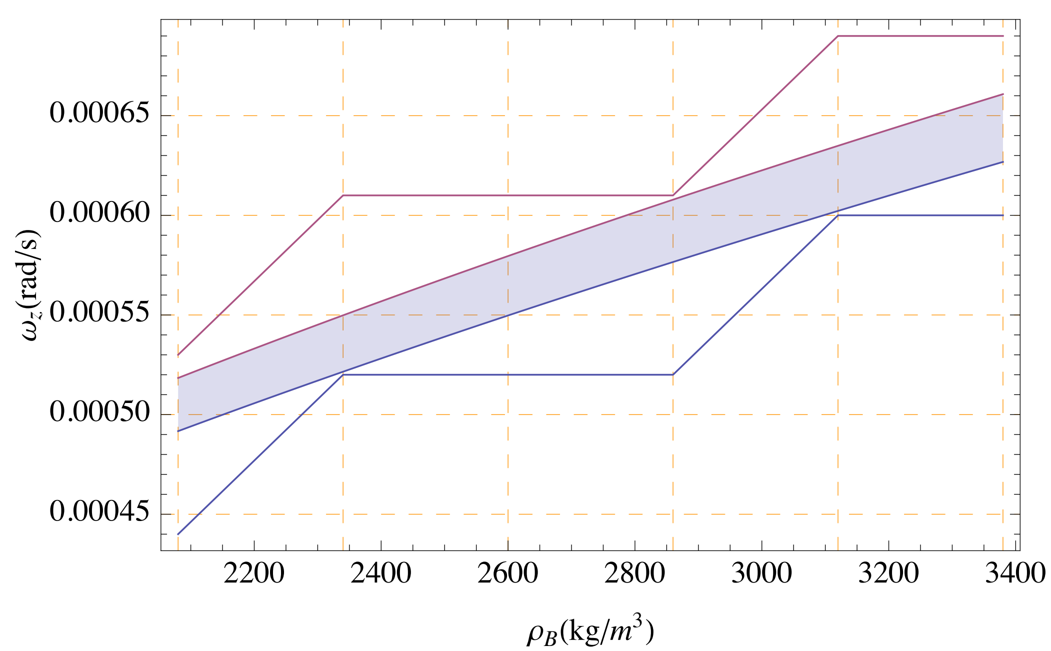
Spherical agg.-friction





Ellipsoidal agg.-friction





Conclusions

- Spherical aggregates are in general stronger than ellipsoidal ones.
- Frictionless aggregates deform at lower spin rates than aggregates with friction.
- Angles of friction are always higher if friction is included in the simulations.
- Regardless of the source of friction it is possible to recover the theoretical results for the DEVEEs.
- The actual shape of the aggregates however, depends on whether of not friction was present.

✤ Open Questions:

  - How are these results affected by size distribution, number of particles used and other material parameters?
  - What determines if disruption is reached by shedding or by fission?
  - How do cohesive forces affect these results?
  - How can cohesive forces be implemented given that they are important for particles in a size range that would take too long to simulate?
  - What about particles of irregular shapes?

Thanks, any questions?