Granular Materials Rheology (quasi-static regime)

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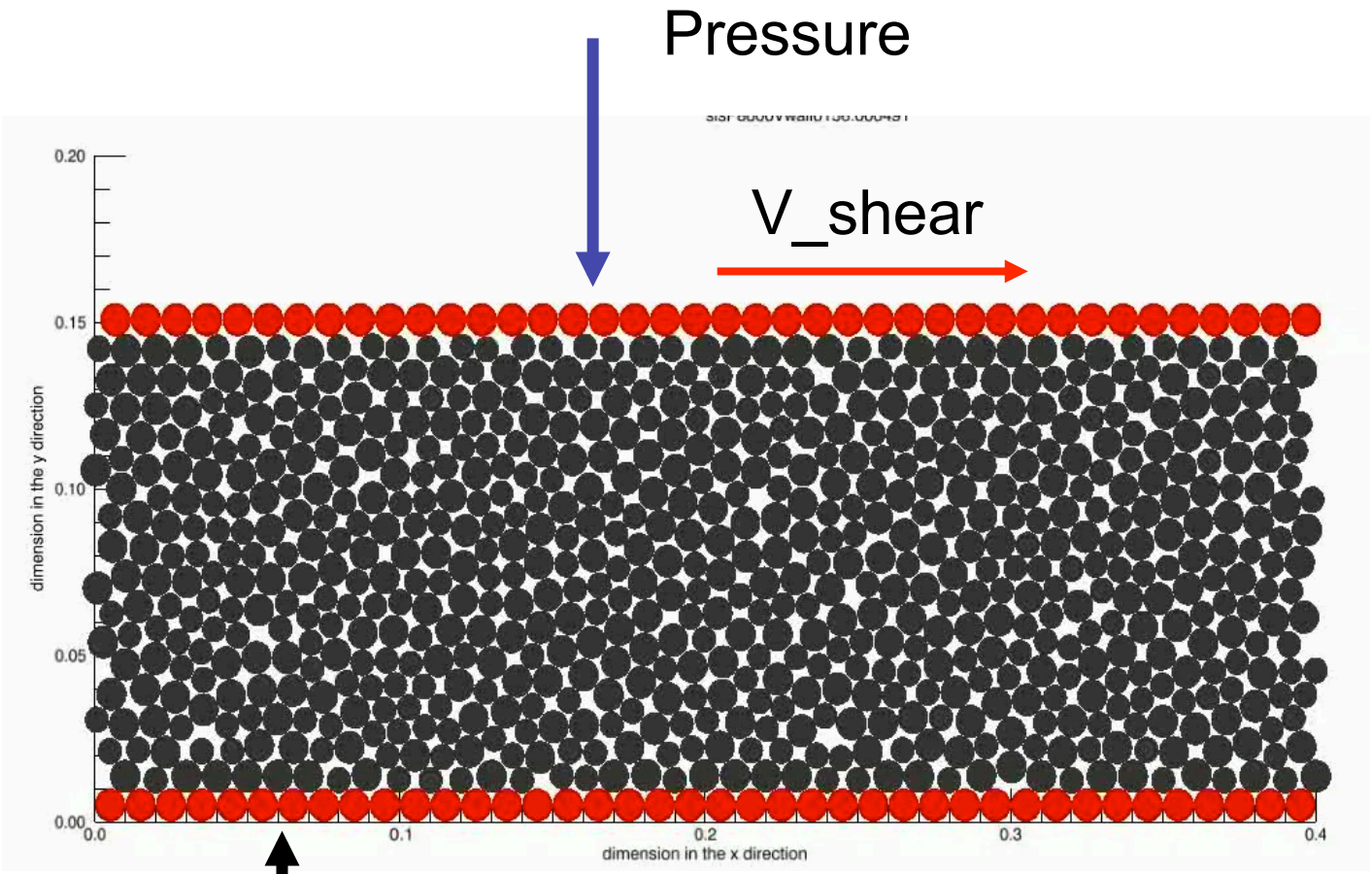
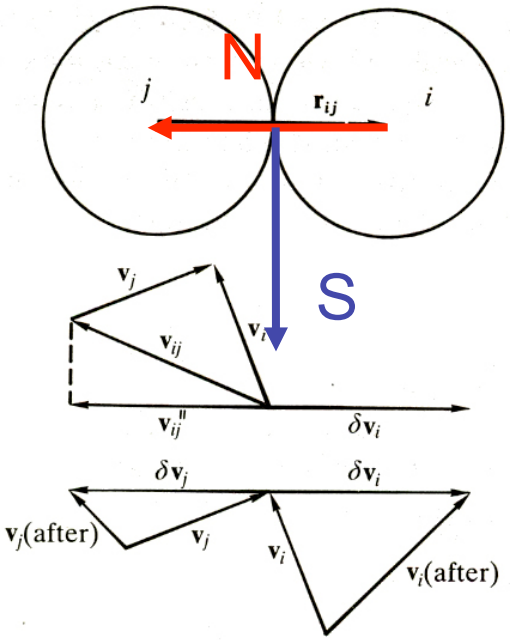
Working session Aout 2011 ESPCI

Goal: Study the rheology of granular assemblies under vibration



Linear Dashpot model



deformation

$$\vec{F}_{ij} = \left\{-K_n * Deformation + \gamma_n * mass * (\vec{v}_{ij} \bullet \vec{n}) / 2.0\right\}\vec{n} + \left\{\min(\gamma_s v_{rel} / 2, \left|\vec{F} \bullet \vec{n}\right|\right\}\vec{s}$$

dissipation

Dissipation by friction

$$F_{ij}(t) = F_{Normal} + F_{\tan gential}$$

$$F_{Normal} = F_{Normal}^{Elastic} + F_{Normal}^{Disipation}$$

$$F_{Normal}^{Elastic} = K_n * (Diam - \left| r_{ij} \right|) * N$$

$$F_{Normal}^{Elastic} = K_n * deformation * N$$

$$F_{Normal}^{Disipation} = \gamma_N * \text{mass} * \text{v}_{\text{ij}} * N$$

Model Constants

Young Modulus

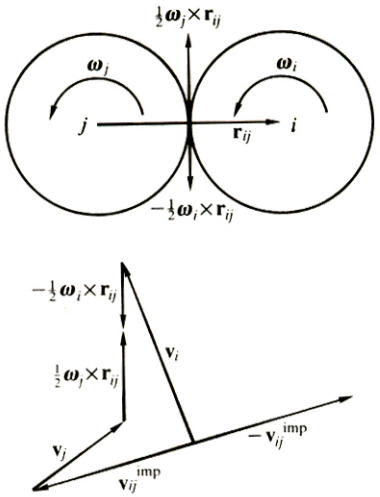
$$K_n = \frac{Y * diam}{3 * (1 - \sigma^2)}$$

Poisson Coeff

$$\gamma_n = 2.0 * \alpha * \sqrt{m_{eff} K_n}$$

$$\alpha = \frac{\log(\varepsilon)}{\sqrt{\pi^2 - \log(\varepsilon)^2}}$$

$$\gamma_t = \frac{\gamma_n}{2.0}$$



$$V_{\mathrm{Re}lTan} = V_{ij} - \frac{1}{2}(w_i r_i + w_j r_j)$$

Thompson & G.S. Grest PRL vol 67 No 13 pp1751

Coulomb's criterion

It is a limit between the normal and tangential forces

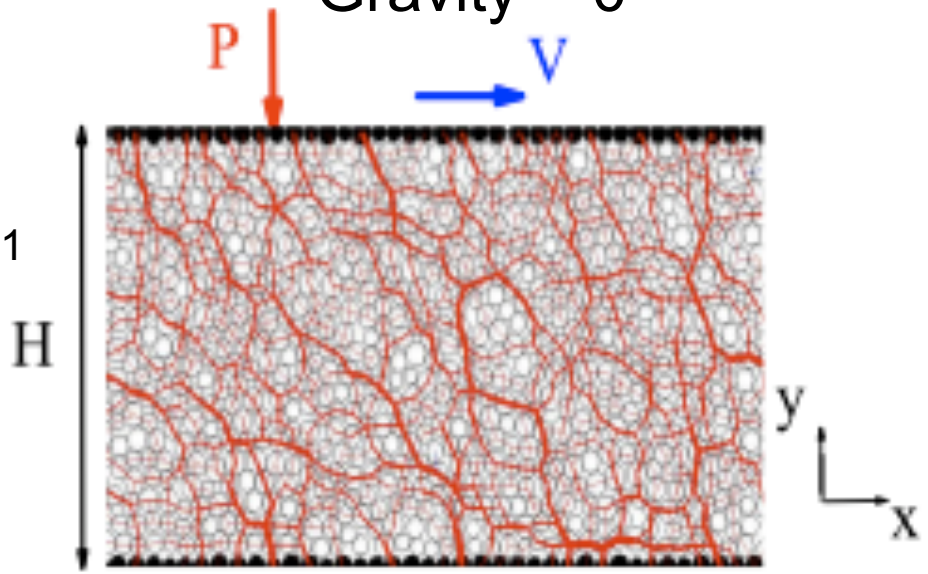
$$\vec{F}^T = -sign(v_{rel}^T) * \min(\left|k_T \zeta\right|, \mu \left|\vec{F}^N\right|$$

$$\zeta(t) = \int_{t_k}^{t} v_{rel}^T(t')dt'$$

$$N = \frac{\vec{r}_{ij}}{\left|r_{ij}\right|} \qquad S = (S_x, S_y) = (-N_y, N_x)$$

2D

Gravity = 0



$L_y=H=20\sigma_1$

$L_x=40\sigma_1$

σ_1 and σ_2

$\sigma_1=1$ $\sigma_2=0.8\sigma_1$

2) Polydispersity 20%

$$\sigma \approx 1$$

$N_{particles}$= 700

Compacity= 0.84

Parameters : V_{shear}, P

Here we use a controlled Pressure

$$V_n = (P_{applied} * l_x - N) / g_p$$

g_p= Wall viscous damping coefficient

Equilibrium appears when P= P_{wall}/l_x

$$friction = \mu = [0.4,05]$$

$$restitution_coeff = \varepsilon = [0.1,0.9]$$

$$mass = m = 1$$

$$diameter = d = 1$$

$$K_n = 1.0D + 4$$

$$\alpha = \frac{-2.0\log(\varepsilon)}{\left(\pi^2 + \log^2(\varepsilon)\right)}$$

$$g_n = \alpha\sqrt{\frac{K_n}{2.0 * m}}$$

$$\gamma_n = 2.0 * \alpha\sqrt{m_{eff} * K_n}$$

$$\gamma_t = \frac{\gamma_n}{2.0}$$

Damping frequency

$$w_o = \sqrt{\frac{K_n}{m_{red}}} = 316,22 \qquad \left[\frac{1}{\text{sec}}\right]$$

Collision time

$$t_c = \sqrt{\frac{m\left(\pi^2 + \ln^2(\varepsilon)\right)}{4.0 * K_n}} = 8.7096E-3$$

density

$$\rho = \frac{4 * m}{\pi d} = 1.27$$

Microscopic time

$$t_{micro} = \frac{d}{\sqrt{\frac{P}{\rho}}} = 9.213E-2$$

$T_{collision}$= 8.7096E-3 $T_{micro=}$ 9.2132E-2

$T_{collision\text{-}correlation}$=1.74E-4 dt (iteration step)=8.70E-5

Natural units: d, m, collision time $T_{collision}$

Inertial number I

Its function is to characterize the shear state

Inertial Number

Macroscopic time scale
Shear rate

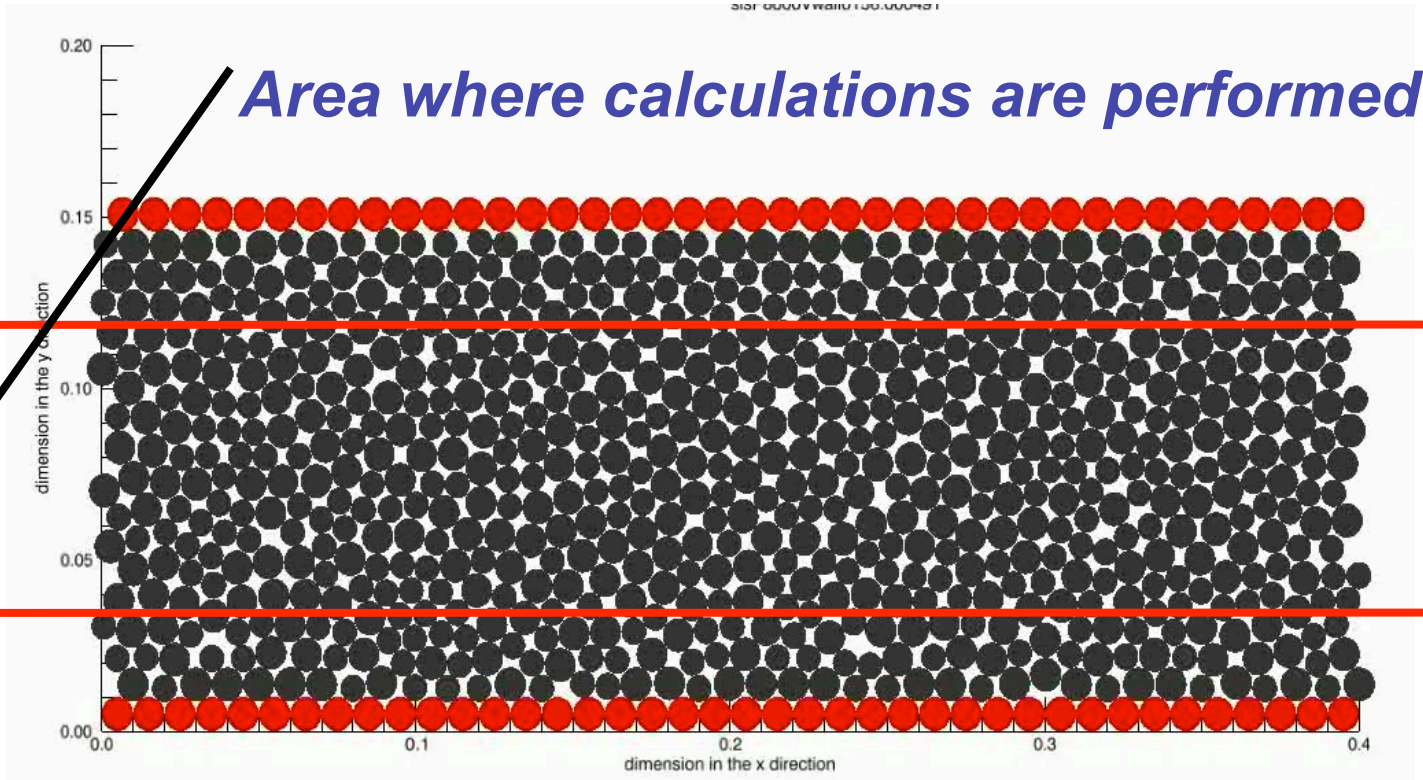
$$I = \frac{V_{shear}}{H}\sqrt{\frac{m}{d * P}} = \dot{\gamma}\sqrt{\frac{m}{d * P}}$$

$$\langle I \rangle = (6.10^{-4}, 0.3)$$

Microscopic time scale (typical time of grain rearrangement)

Effectif friction coefficient

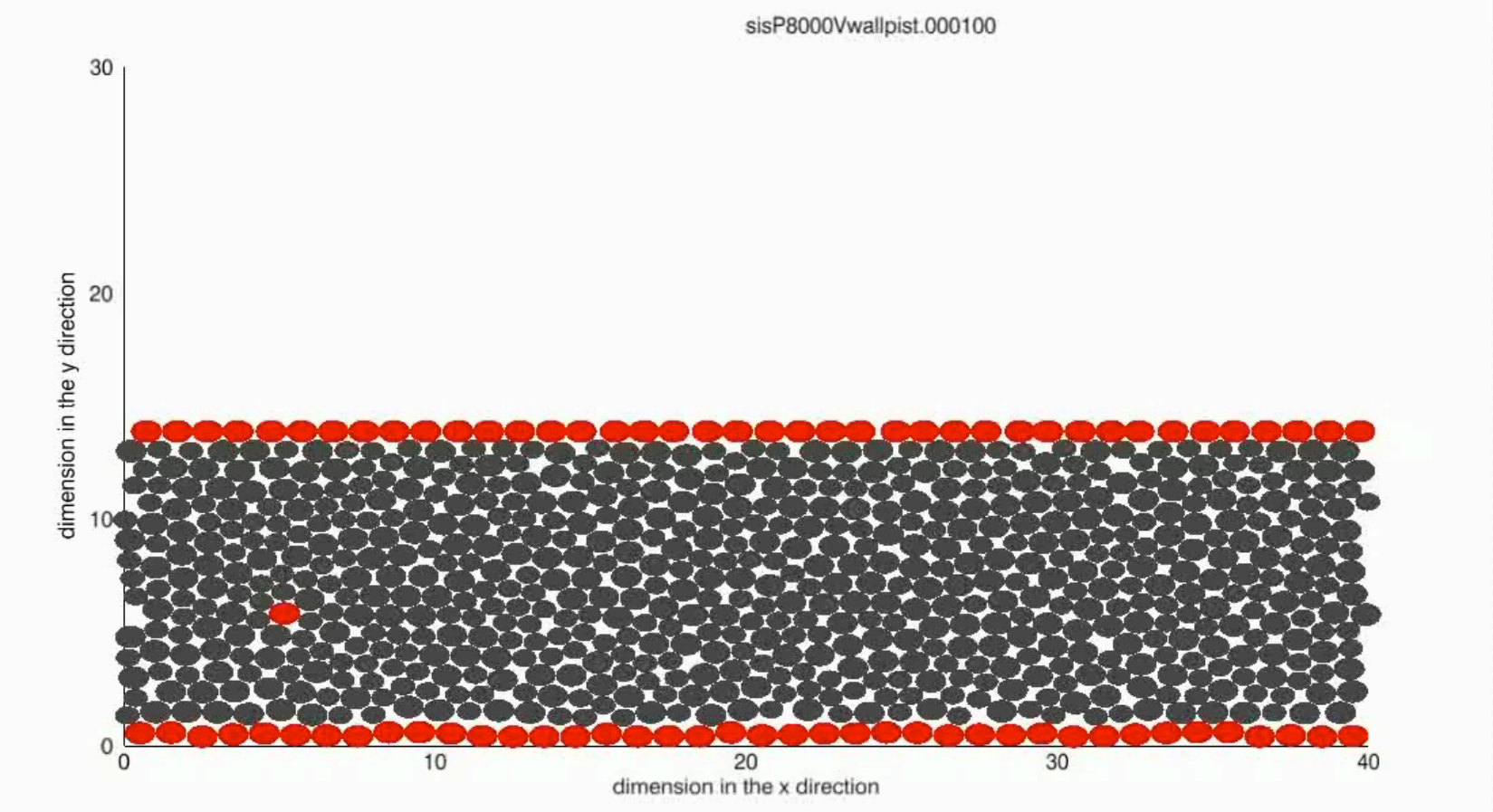
$$\mu^* = \frac{\Sigma_{xy}}{0.5 * \left(\Sigma_{xx} + \Sigma_{yy}\right)}$$



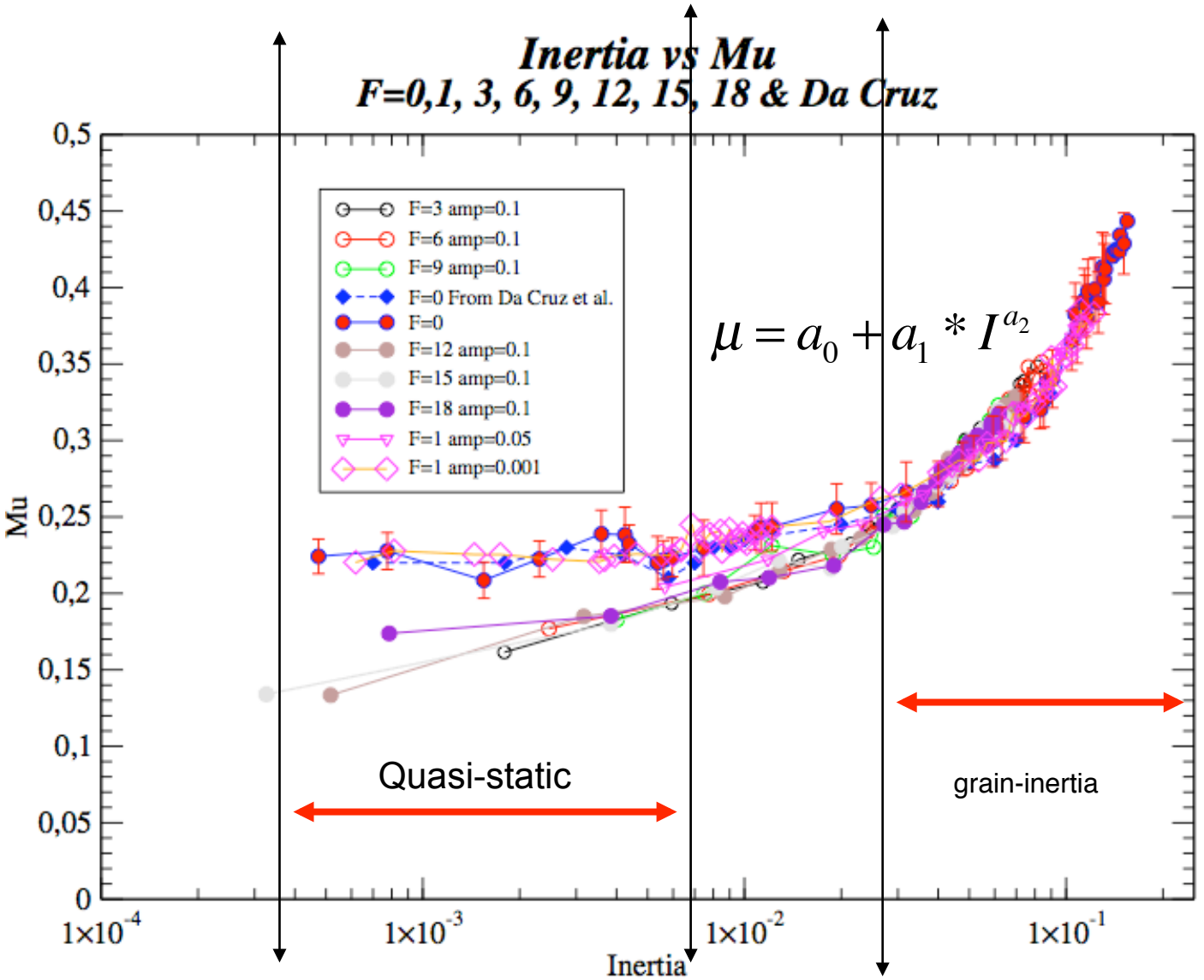
Code validation (no pistons)

$$d_1 = 1.0; d_2 = 0.8; F = 3.0; \mu = 0.5; \varepsilon = 0.1$$

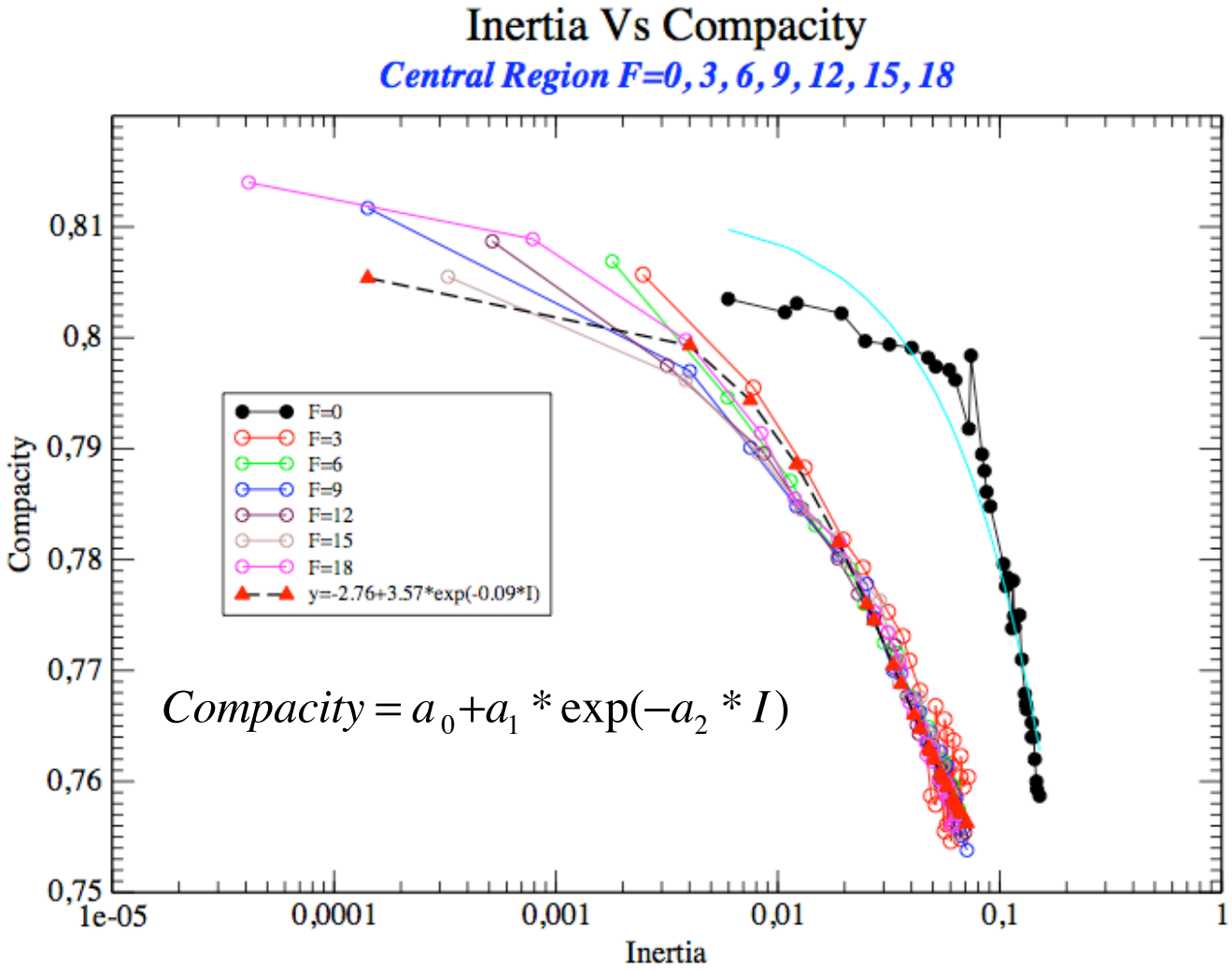
$$\text{Pr}\,essure = 1.0; I = 0.1$$



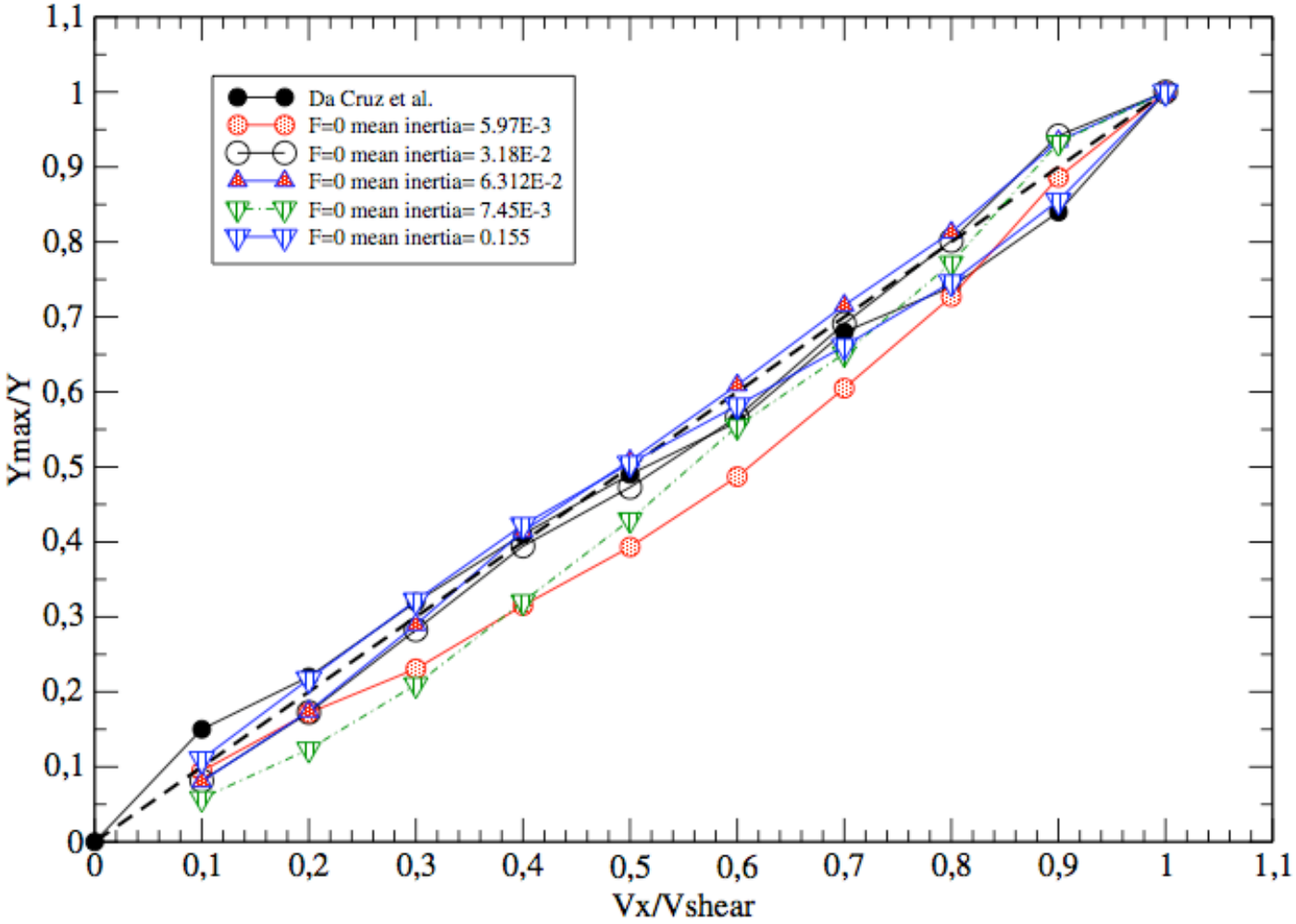
DaCruz et al., PRE 72, 021309 2005



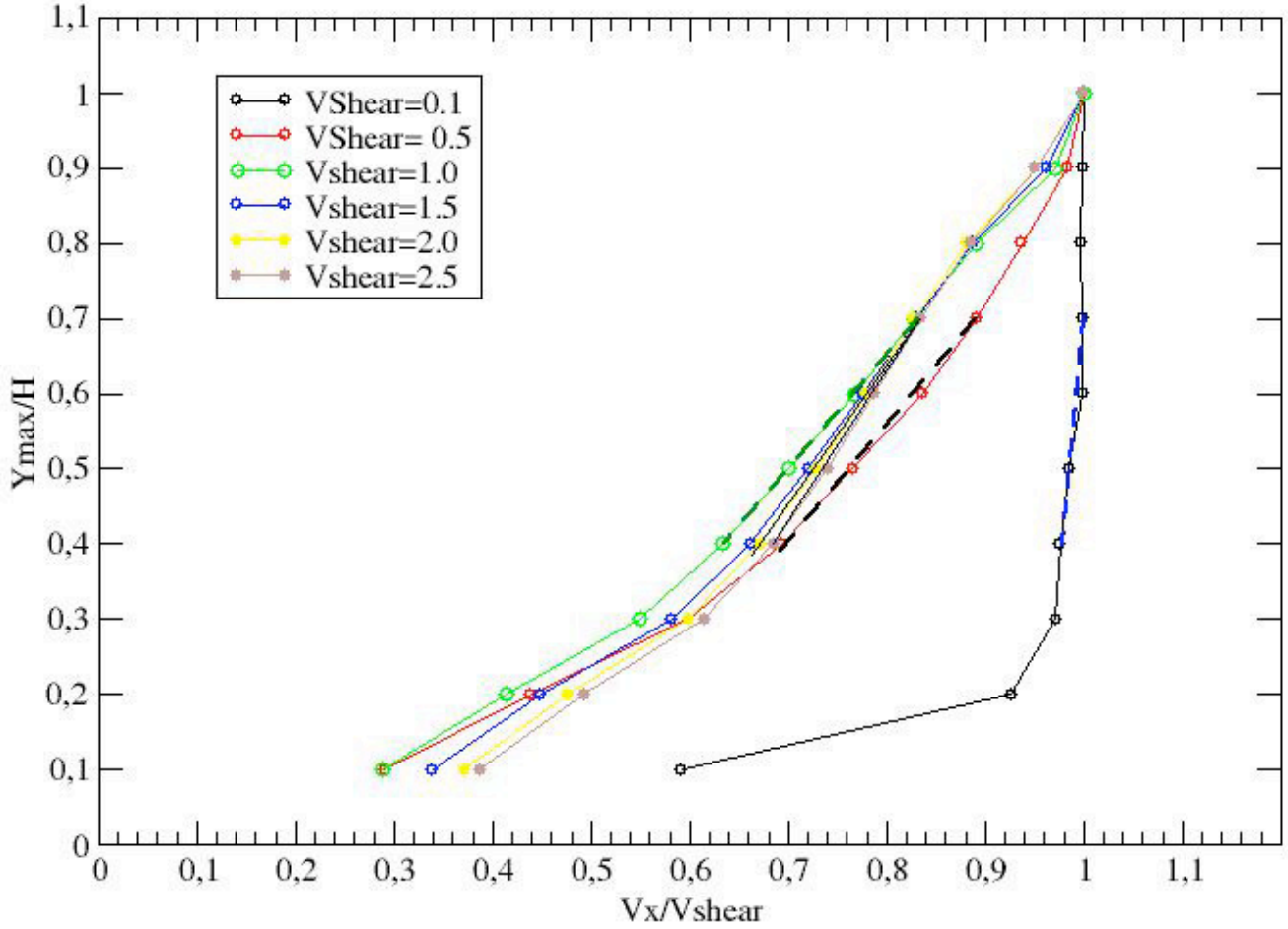
a_2=1/3


Inertia Vs Compacity
Central Region F=0, 3, 6, 9, 12, 15, 18
Compacity
Inertia
0,81
0,8
0,79
0,78
0,77
0,76
0,75
1e-05
0,0001
0,001
0,01
0,1
1
F=0
F=3
F=6
F=9
F=12
F=15
F=18
y=-2.76+3.57*exp(-0.09*I)
Compacity = a0+a1 * exp(−a2 * I)

Velocity Profile Frequency =0



Velocity Profile F=9

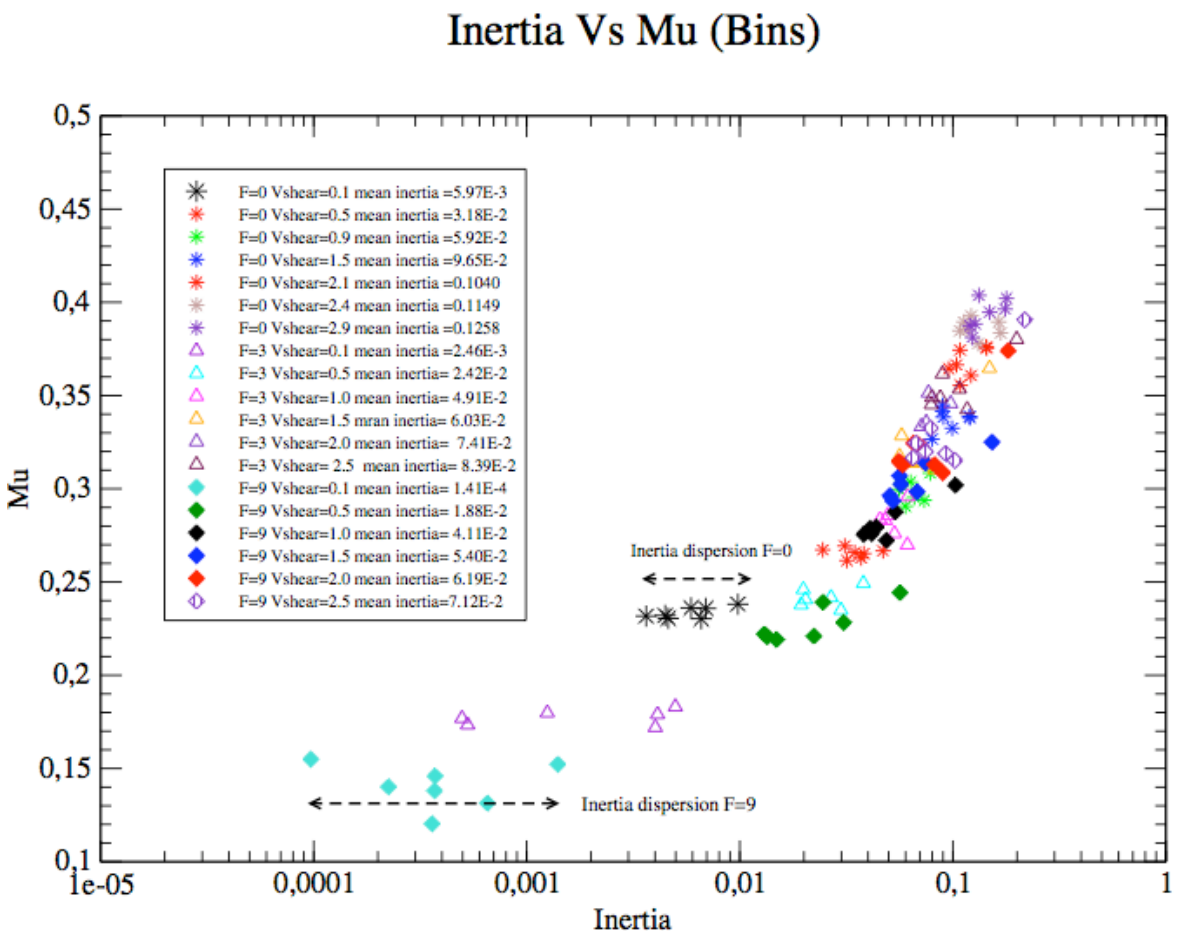


•The Inertia, I(y), has been calculated by considering a central area delimited by 3 layers below and above the top and bottom walls respectively.

$$I(y) = \dot{\gamma}(y)\sqrt{\frac{m}{P(y)}}$$

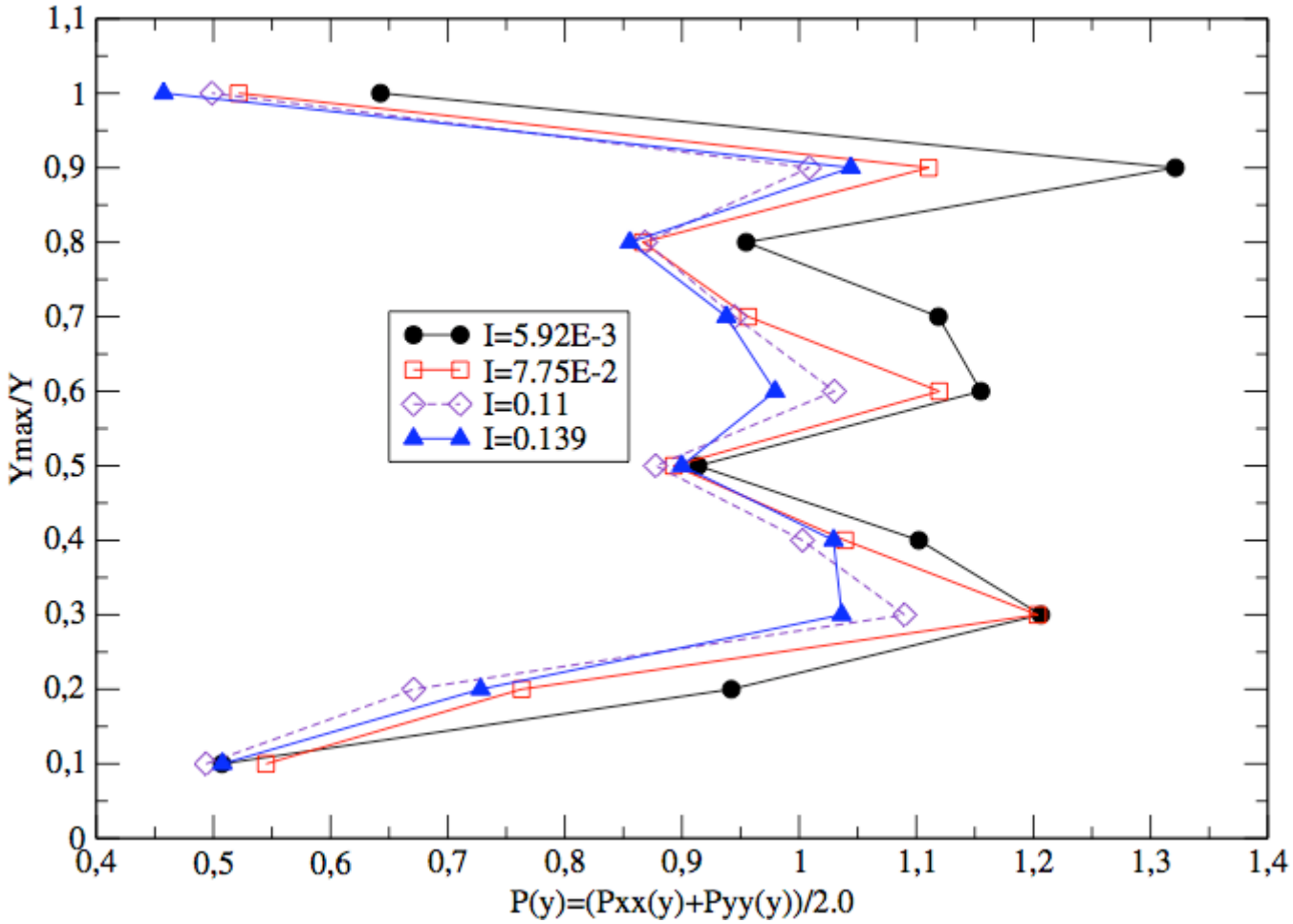
$$I = \langle I(y) \rangle = \frac{1}{N}\sum I(y)$$

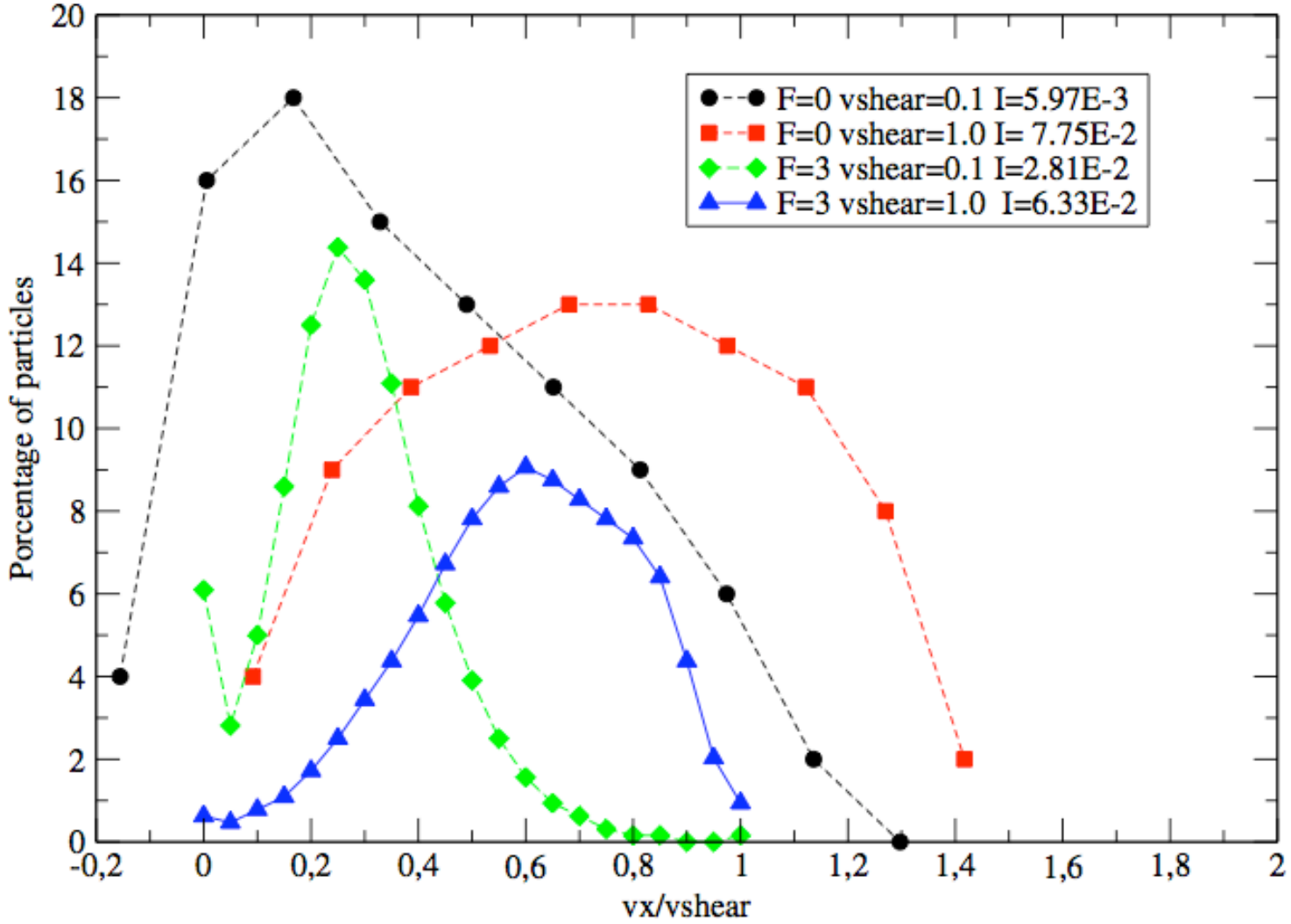
$$I(y) = \dot{\gamma}(y)\sqrt{\frac{m}{P(y)}}$$



F=0

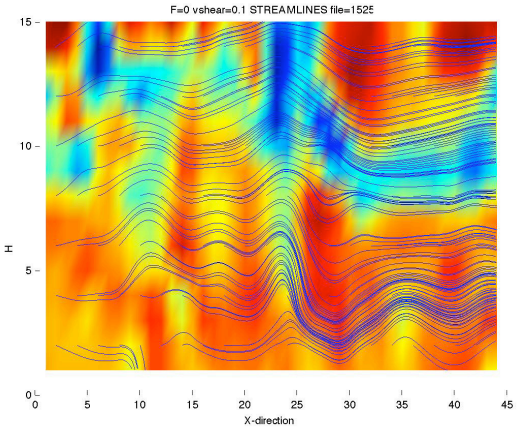
***Charles Cambell & C.E Brennen J.Fluid Mech.* (1985), *vol.* 151, *pp.* 167-188**

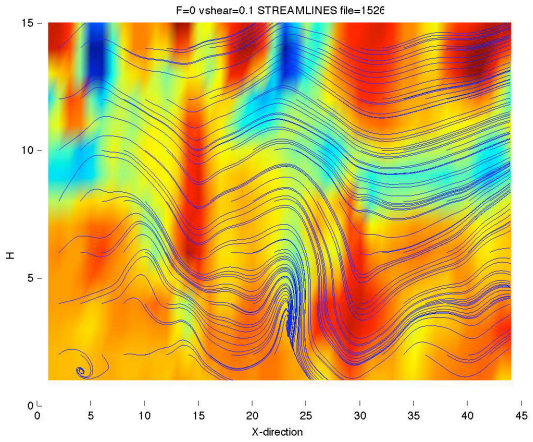


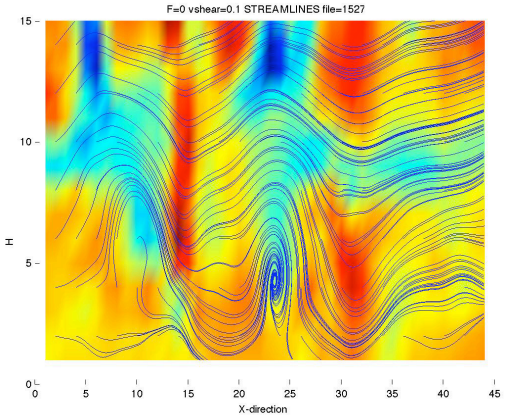

F=0 vshear=0.1 I=5.97E-3
F=0 vshear=1.0 I= 7.75E-2
F=3 vshear=0.1 I=2.81E-2
F=3 vshear=1.0 I=6.33E-2
Porcentage of particles
vx/vshear
20
18
16
14
12
10
8
6
4
2
0
-0,2
0
0,2
0,4
0,6
0,8
1
1,2
1,4
1,6
1,8
2

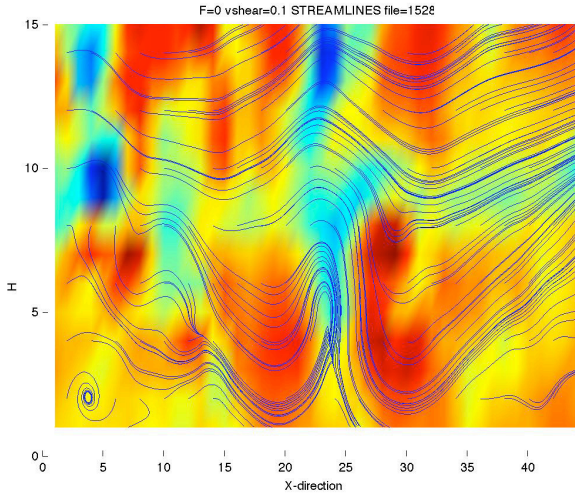
$$\Omega = \nabla \times \vec{v}$$

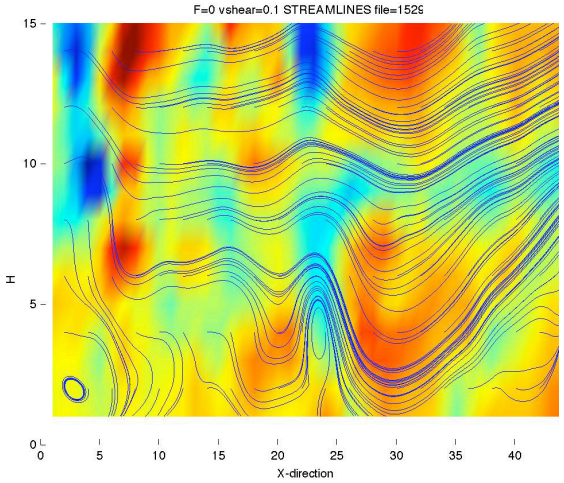
B. J. Glasser & I. Goldhirsch
PHYSICS OF FLUIDSVOLUME 13, N0 2, P407

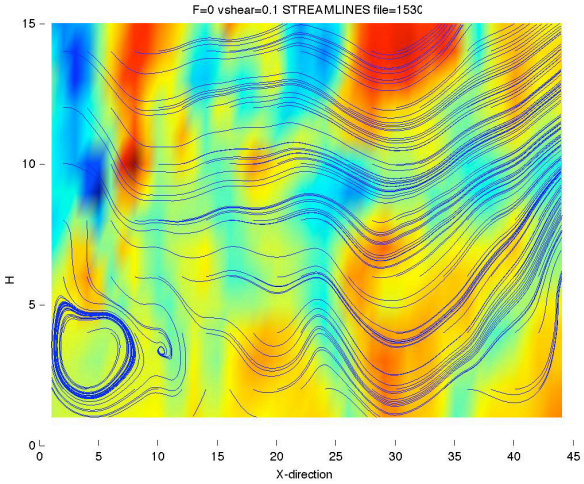


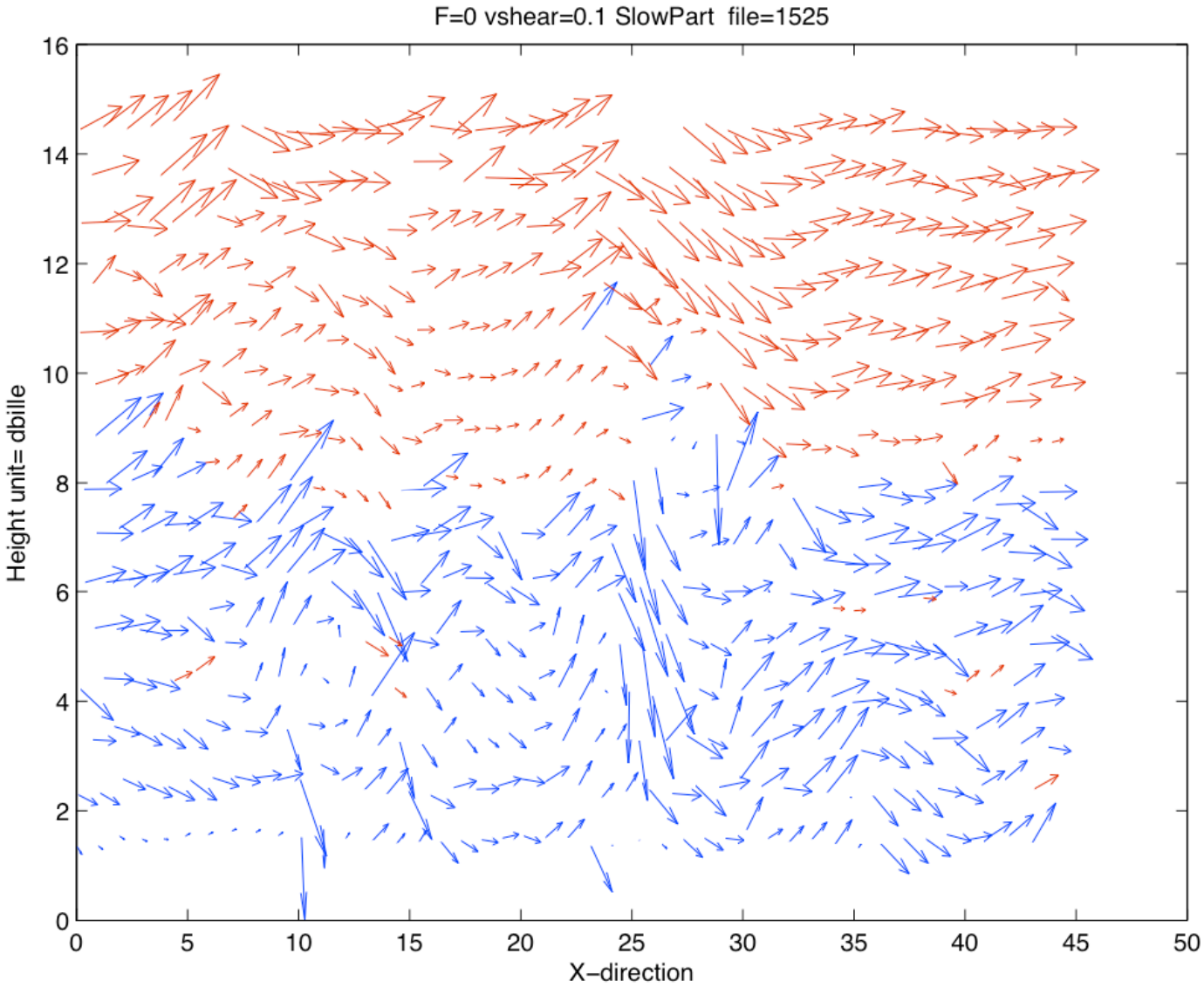


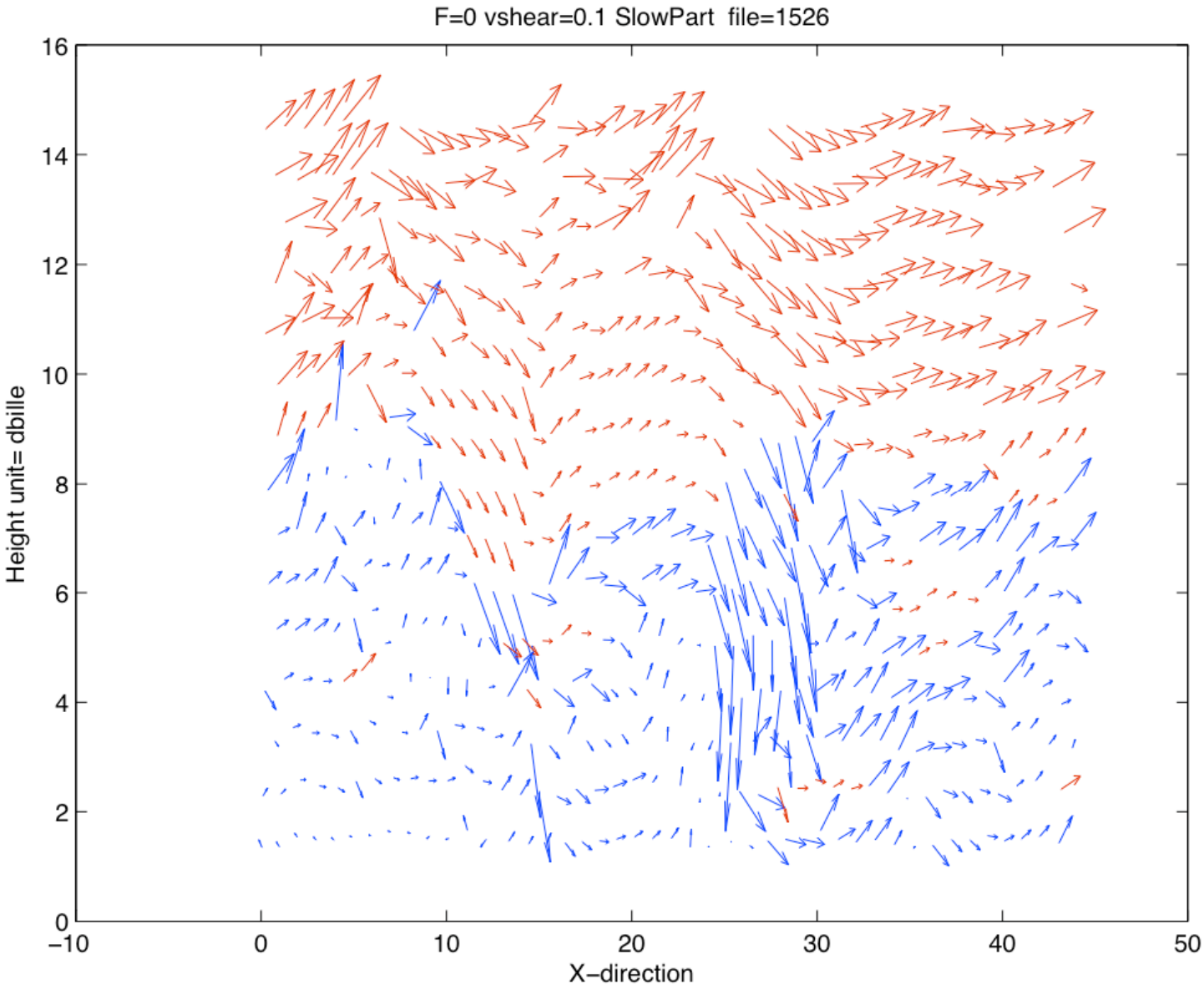


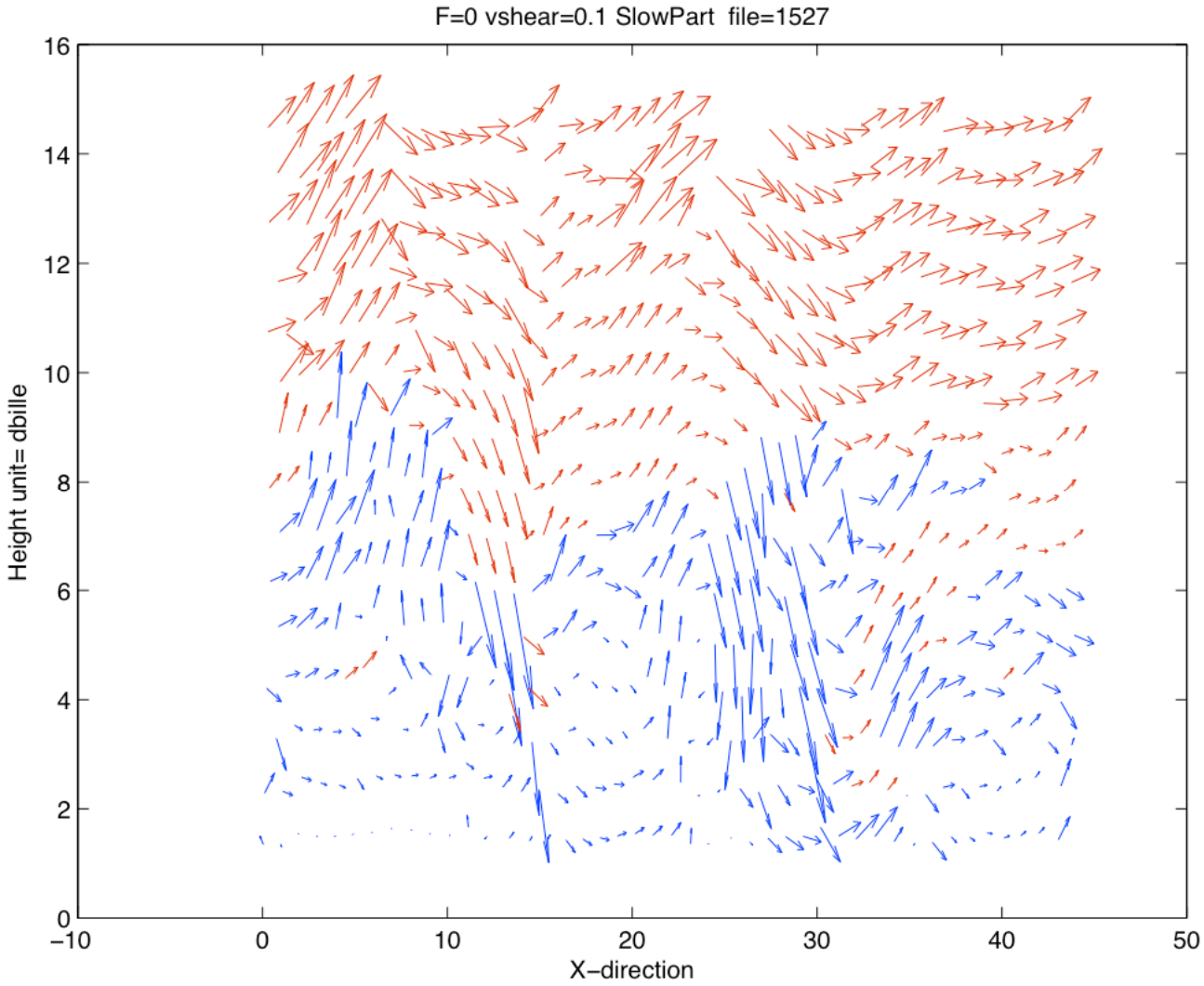







F=0 vshear=0.1 SlowPart file=1525
Height unit= dbille
X−direction
0
2
4
6
8
10
12
14
16
0
5
10
15
20
25
30
35
40
45
50


F=0 vshear=0.1 SlowPart file=1526
Height unit= dbille
X−direction
16
14
12
10
8
6
4
2
0
−10
0
10
20
30
40
50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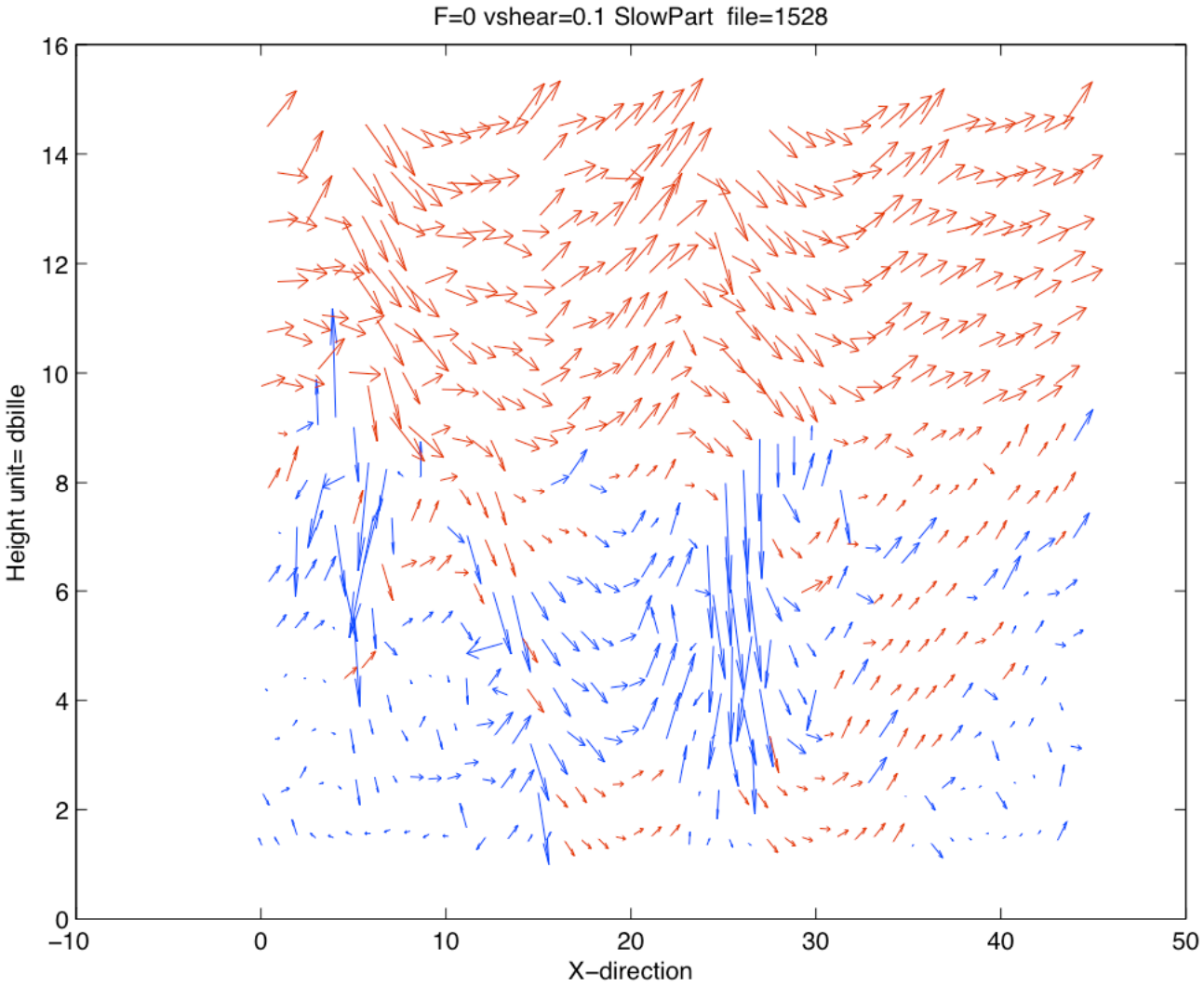
X–direction
16
14
12
10
8
6
4
2
0
–10
0
10
20
30
40
50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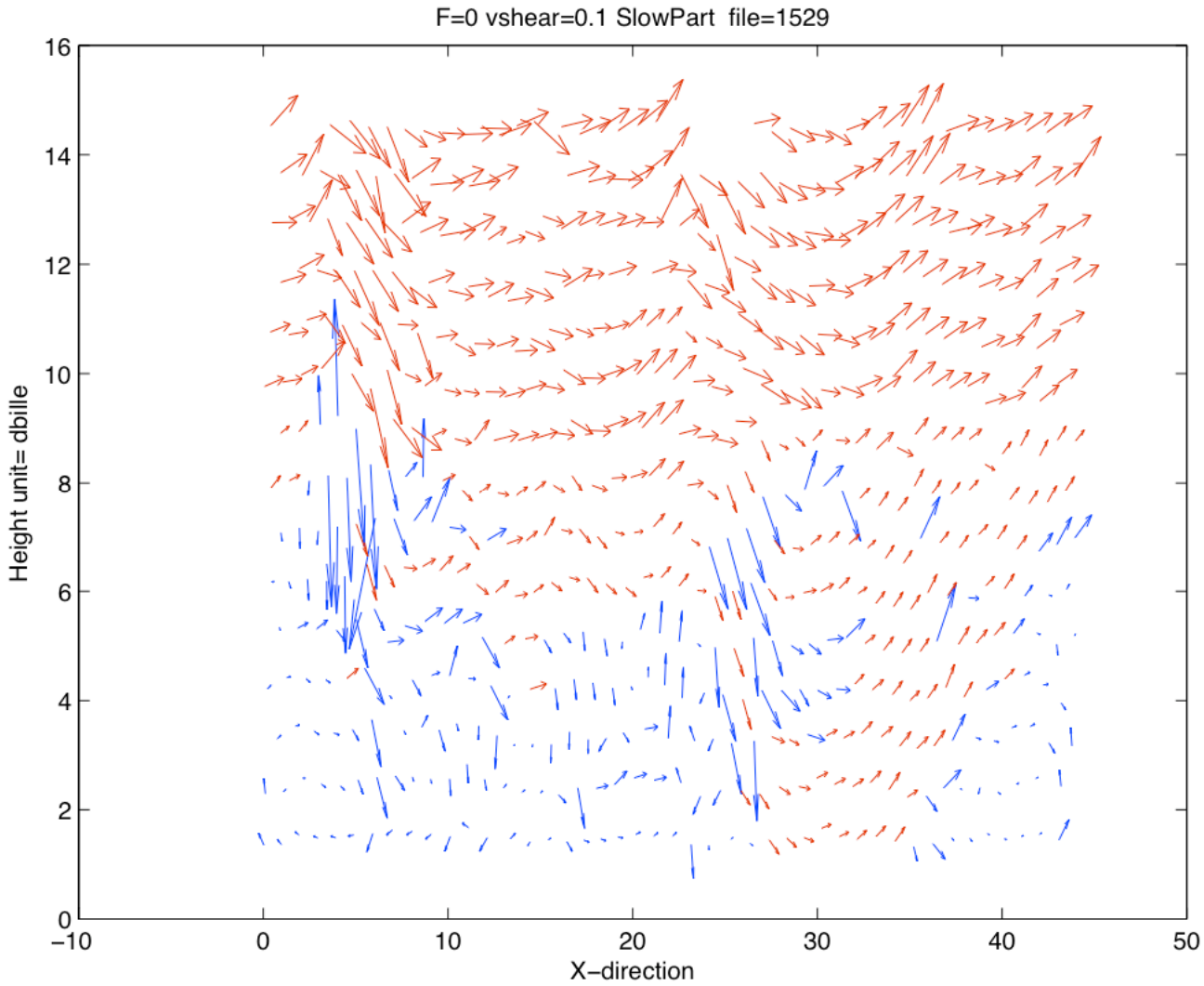
X−direction
16
14
12
10
8
6
4
2
0
−10
0
10
20
30
40
50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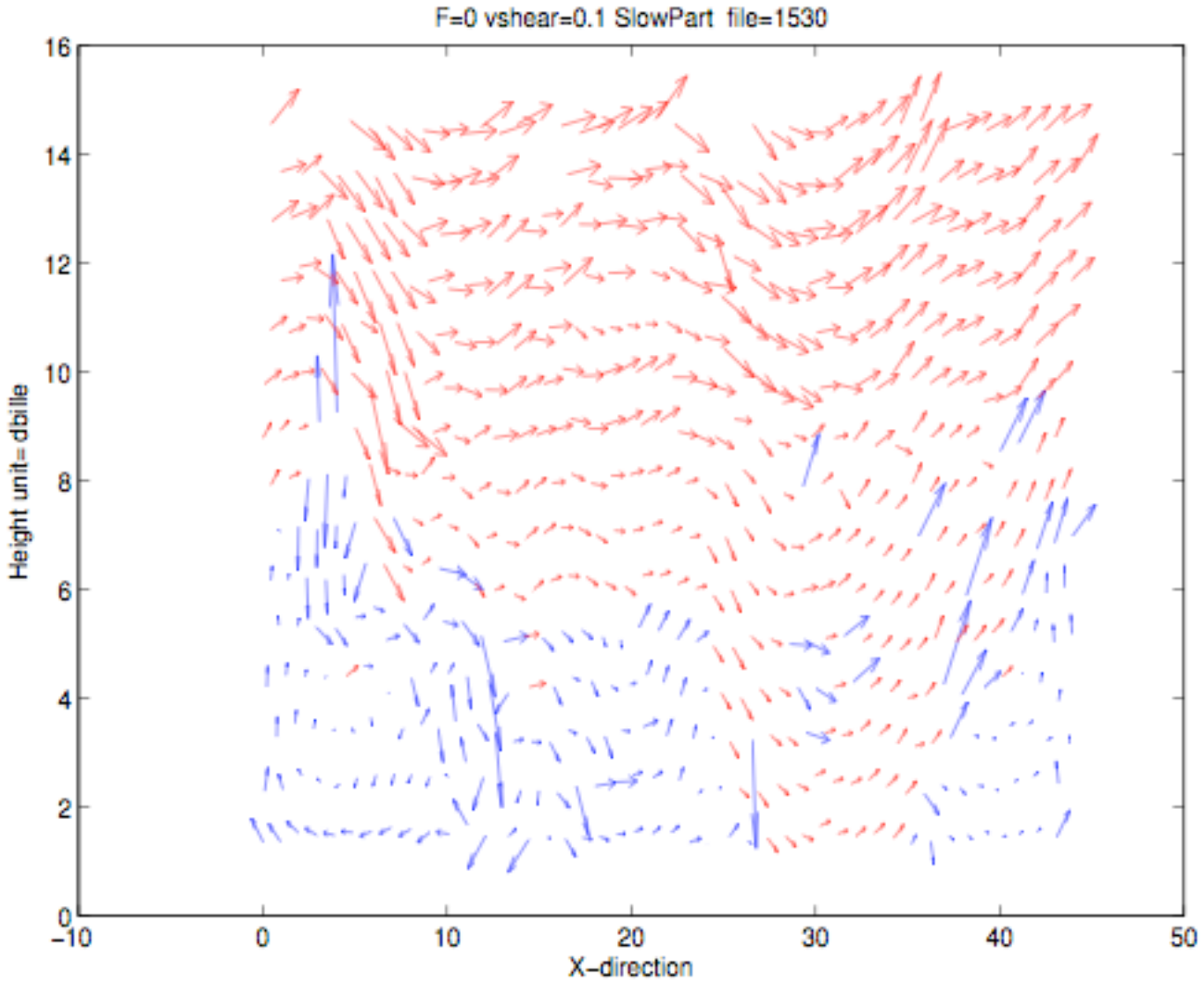
Height unit= dbille
X–direction
16
14
12
10
8
6
4
2
0
–10
0
10
20
30
40
50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Height unit= dbille
X-direction
16
14
12
10
8
6
4
2
0
−10
0
10
20
30
40
50

FUTURE: describe I(y) as a activation process

$$\dot{\gamma}(z) = \sum_{z'} f_{z' \to z} (p^{+}_{z' \to z} - p^{-}_{z' \to z})$$

$f_{z' \to z}$ Is a frequency of jump

$p^{+}_{z' \to z}$ Jumping probability

O.Pouliquen et al., Adv Complex Syst 4, 441-450, 2001