Continuum Modeling of Granular Systems

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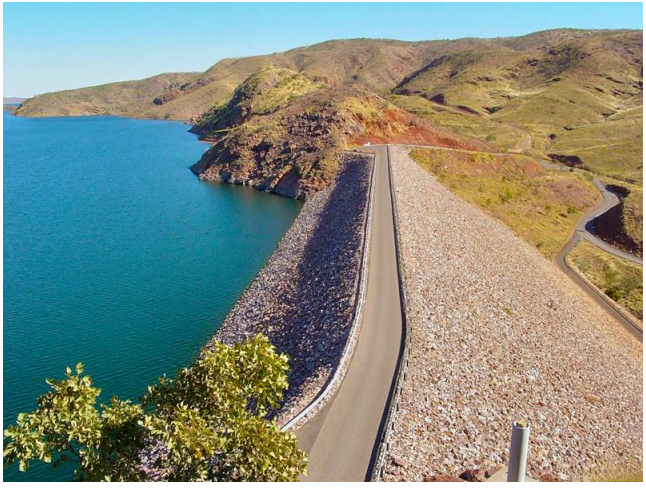
Granular Matter in Human Activity

Nuts

Silos

Quarry

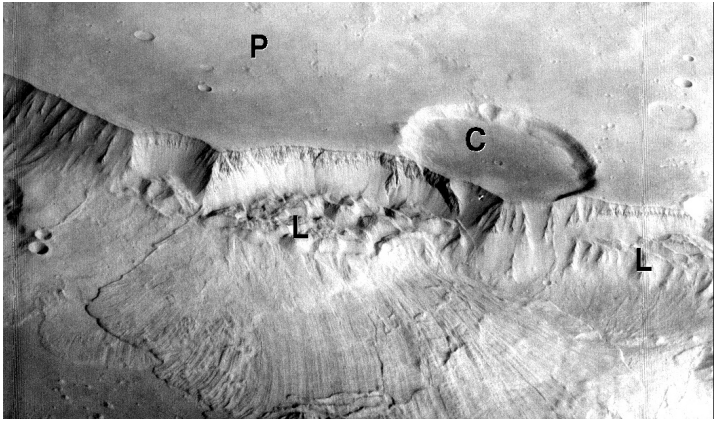


Dam

Granular Matter on Earth and other Planets

Hope slide, British Columbia

Franck Slide, Alberta



Valles Marineris, Mars

Sahara, Tunisie

Why is granular matter so difficult to model?

Definition 1 `Collection of grains for which thermal effects are small compared to variations of potential energy`

Definition 2 `...and such that the forces necessary to deform the system are small compared to those necessary to break the grains`

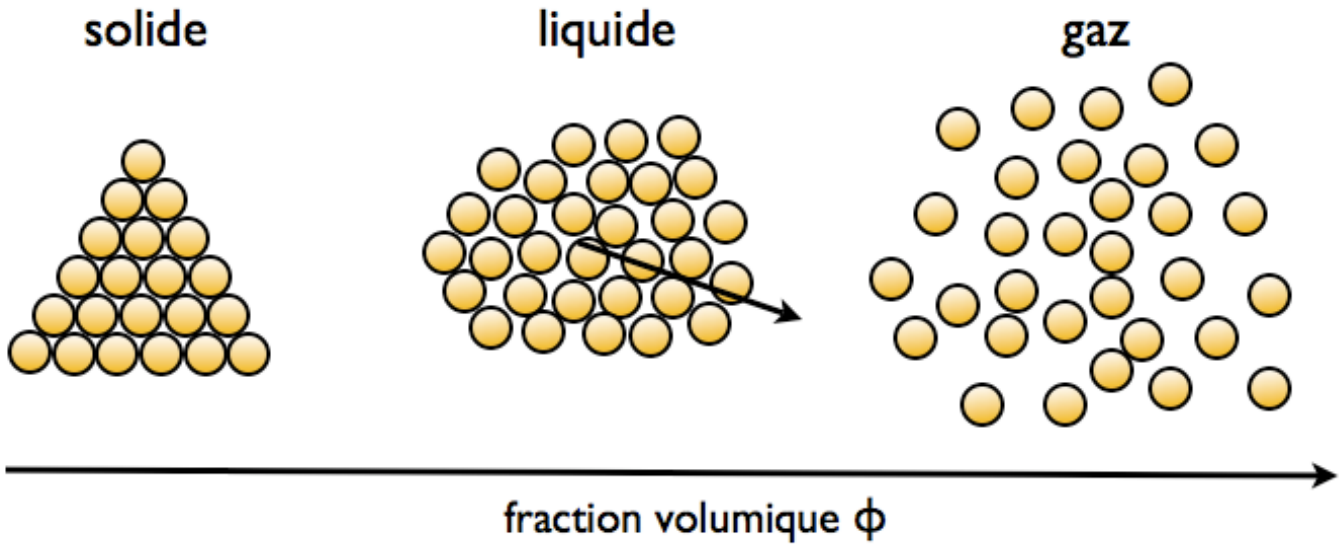
1. The behaviour is dominated by dissipative contact interactions:
   - Inelastic collisions $e << 1$
   - Friction and viscous deformations at enduring contacts

   $\Longrightarrow$ No scale separation between micro, meso and macro description
   $\Longrightarrow$ Thixotropic behaviour
   $\Longrightarrow$ Non-local effect
   $\Longrightarrow$ Segregation patterns

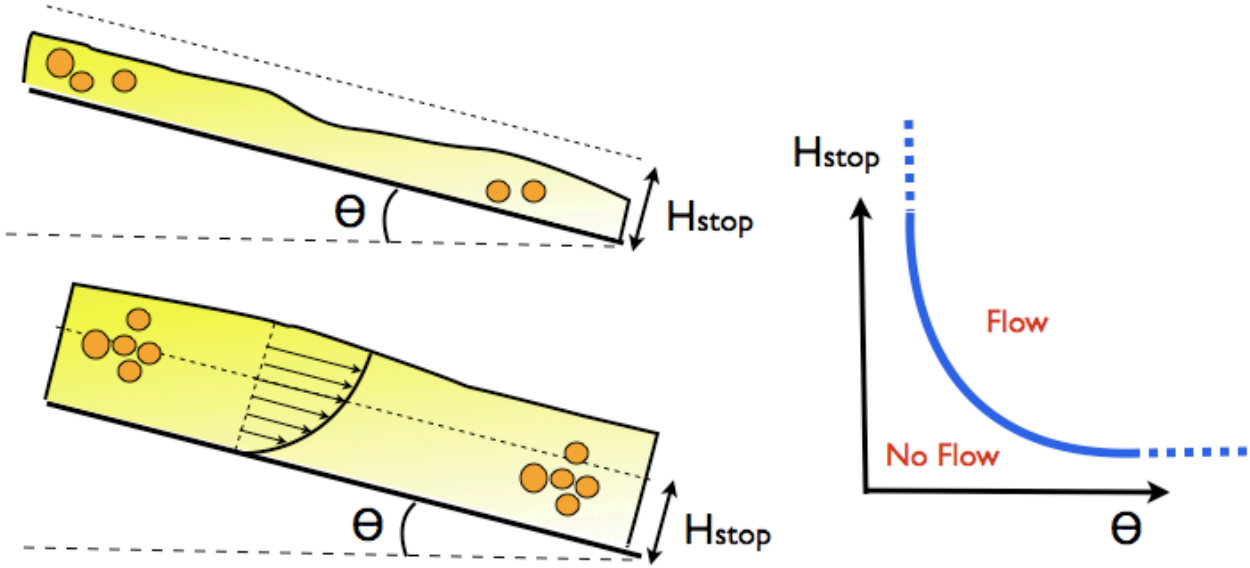
2. Non-linearity of force transmission
   - Contact rigidity

   $\Longrightarrow$ Mesoscopic force chains
   $\Longrightarrow$ No obvious representative elementary volume
   $\Longrightarrow$ Important role of fluctuations

Granular matter is thixotropic

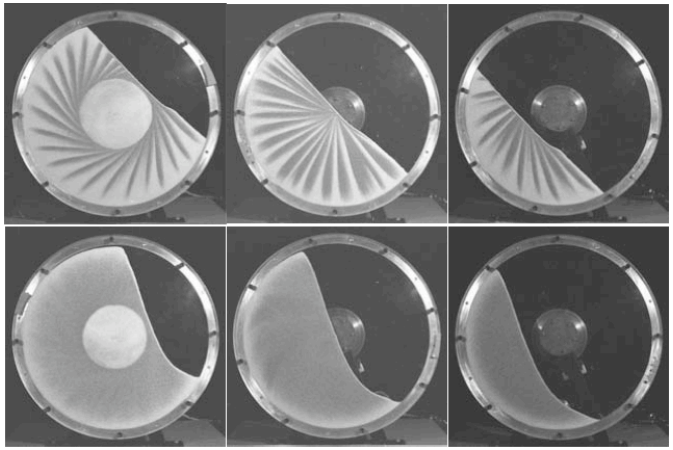


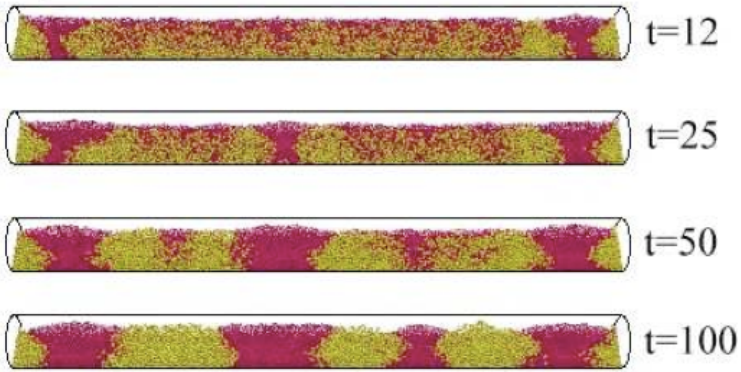
Non-local effects:



⇒ Frictional properties depend on the height of the flow (Pouliquen 1999)

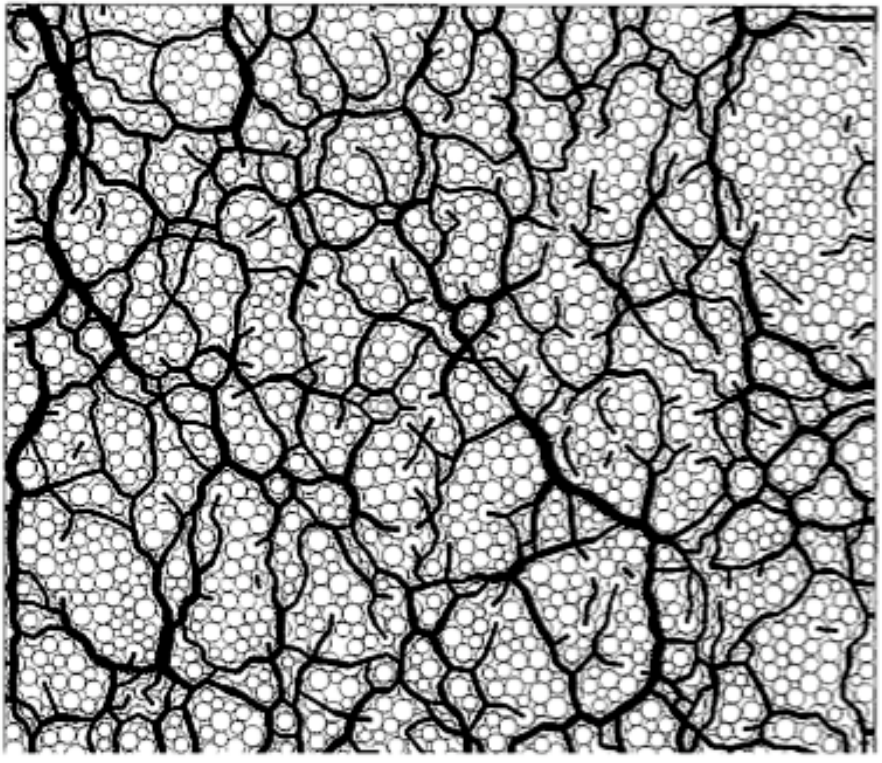
Segregation patterns (size, friction)





⇒ Non-uniformity of granular flows

Gray & Chugunov 2006, Taberlet *et al.* 2006



Radjai et al, 1996

$\Rightarrow$ How to include force chains in mean quantities?

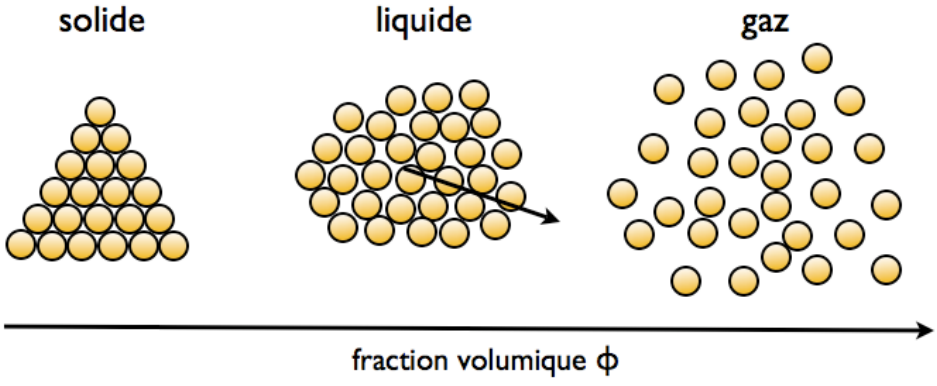
To Summarize

Simulating granular matter

→ Reproducing the multi-contact dynamics
→ Reproducing the contact features

Modeling granular matter

→ Which theoretical frame(s) to model the different "states" ?
→ Which variable(s) (volume fraction)?



In the case of fluid-like behaviour

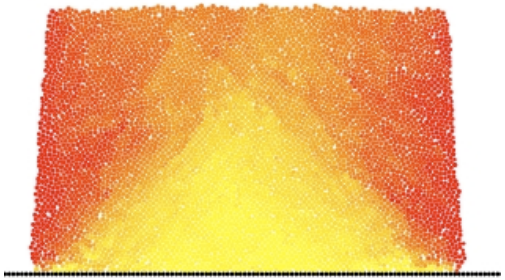
→ How to define an equivalent viscosity?

To Summarize

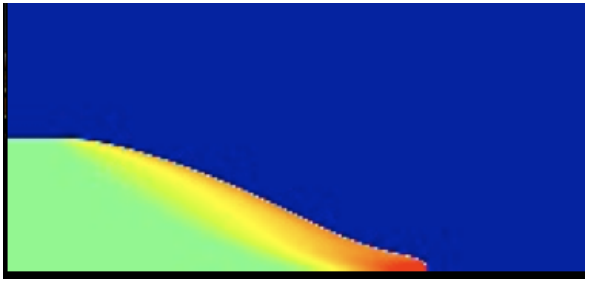
How to simulate granular matter numerically?

How to model granular matter physically?

Discrete



Continuum



Theoretical modeling towards equivalent continuum

Outline

- Discrete Simulation of Granular Media
- Continuum Modeling: Defining an equivalent viscosity
- Continuum vs discrete simulation

Simulating Granular Media

1. Modeling the interactions between the elements of the solid discrete phase
2. Modeling the rheology of the intersticial fluid
3. Modeling the interactions between the solid discrete phase and the intersticial fluid

We consider cases dominated by solid-solid interactions

$\Longrightarrow$ No explicit account of the intersticial fluid

- No hypothesis on the mean behavior
- Hypothesis on the nature of the interactions between the many solid elements

$\Longrightarrow$ Account of the existence of different time and length scales

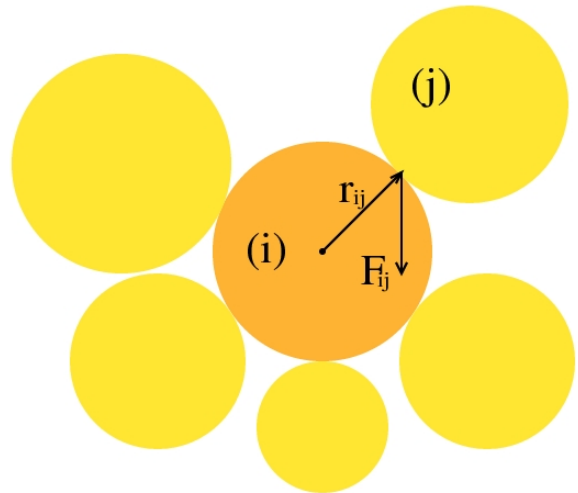
- The system scale L and t
- The grain scale D, ℓ and $(D/g)^{1/2}$
- The contact scale $\lambda << D$ and $\tau << (D/g)^{1/2}$

Newtonian Methods: writing the equations of motion for the grains

N_p grains, N contacts between them
(in 2D)

$$m_i \frac{d\boldsymbol{v}_i}{dt} = \sum_{k \in \mathcal{N}_i} \boldsymbol{F}_{ik} + \boldsymbol{F}_{ext}$$

$$I_i \frac{d\boldsymbol{\omega}_i}{dt} = \sum_{k \in \mathcal{N}_i} \boldsymbol{C}_{ik} + \boldsymbol{C}_{ext}$$



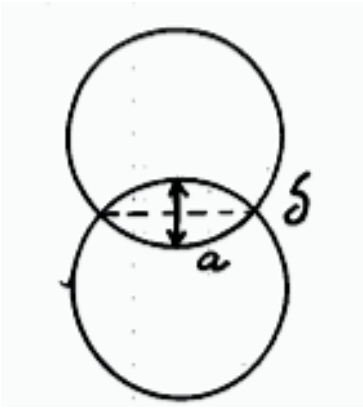
$3 \times N_p$ equations
$3 \times N_p + 2 \times N_c$ unknowns
$\Downarrow$
$2 \times N_c$ relations missing
$=$ **contact laws**

$\Rightarrow$ **Prescribing the shape of the interactions between the grains ...**

Newtonian Methods: modeling contacts

- Deformability/rigidity of the surfaces?
- Dissipation of energy?
- Level of realism required at the contact scale to achieve realism at the system scale?

The soft model: visco-elasto-plastic contact between two surfaces.

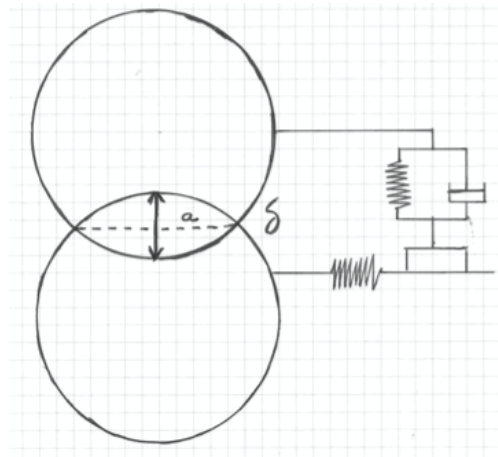


- a: contact surface area
- δ: normal deformation, overlap
- τ: typical time scale of a contact

$$F = \mathcal{F}(\delta, \dot{\delta}, a, \tau)$$

How to handle the small time and length scales related to the contact phenomena?

The Molecular Dynamics (Cundall)

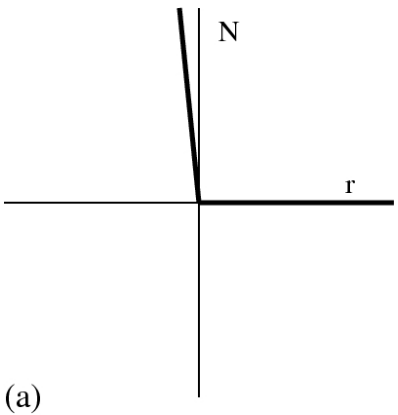


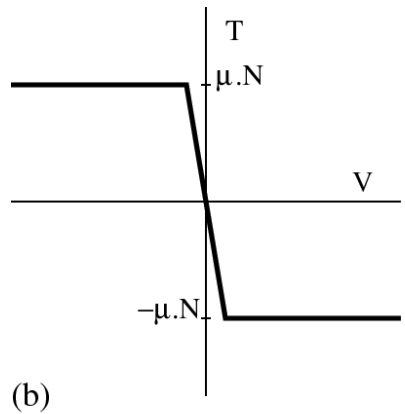
Contact model

Combination of a dashpot and a spring in both normal and tangential directions:

$$N_{ij} = K_n \delta_n + C_n \dot{\delta}_n^{ij}$$

$$T_{ij} = K_t \delta_t + C_t \dot{\delta}_t^{ij}$$





The Contact Dynamics (Moreau, 1988)

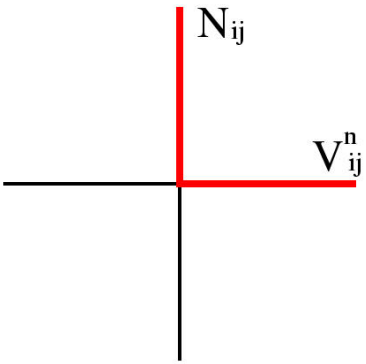
The contact model

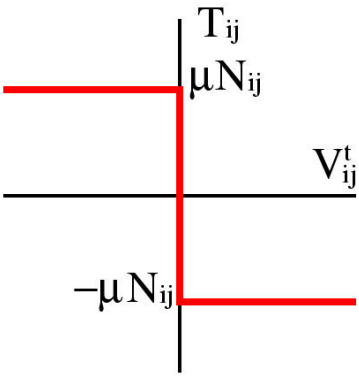
NO explicit model, but two proscriptions instead:

- Hardcore approximation: no overlap
- Coulombic friction: the tangential force is bounded

$\Longrightarrow$ $2N_c$ inequalities:

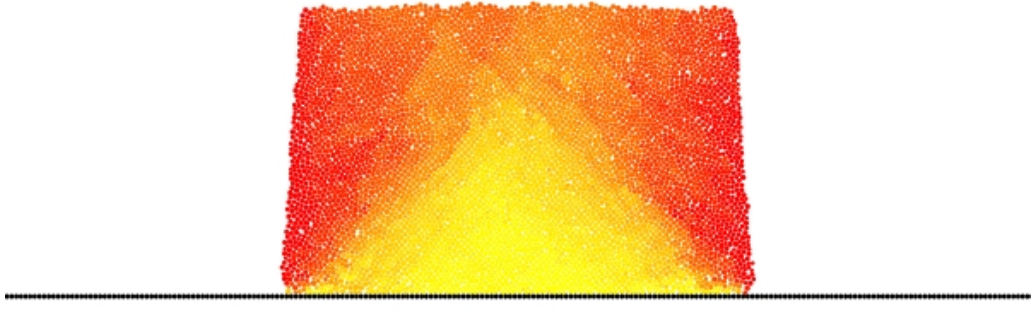
$$\begin{aligned} (d_i + d_j) &< |r_i - r_j| \\ \left|\frac{T_{ij}}{N_{ij}}\right| &< \mu \end{aligned}$$





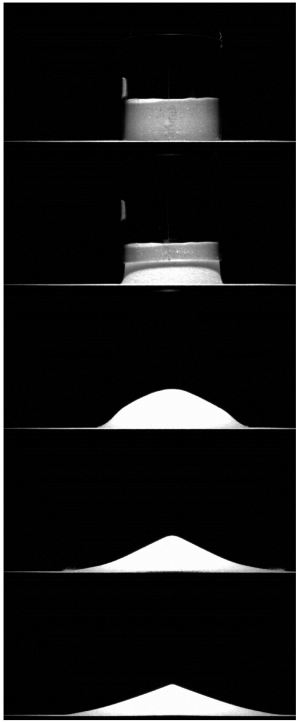
Cannot be included in a set of motion equations...

Numerical collapses

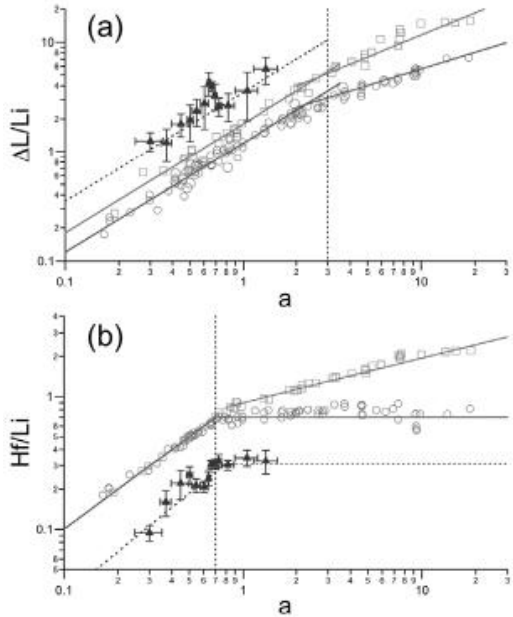


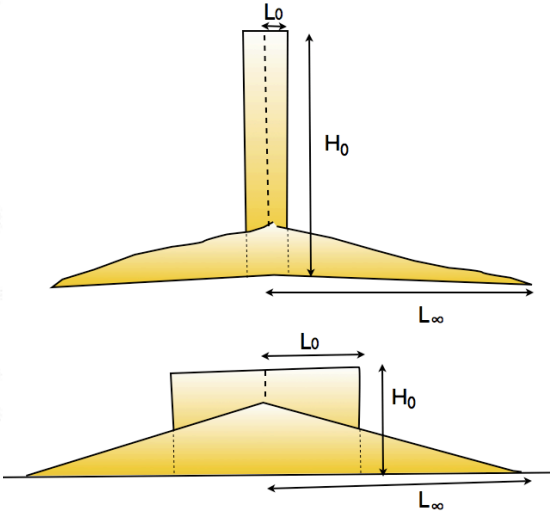
Application to the collapse experiment

Initial geometry: $\boldsymbol{H_0}$, $\boldsymbol{L_0}$, $\boldsymbol{a = H_0/L_0}$ Final geometry: the runout $\boldsymbol{L}$



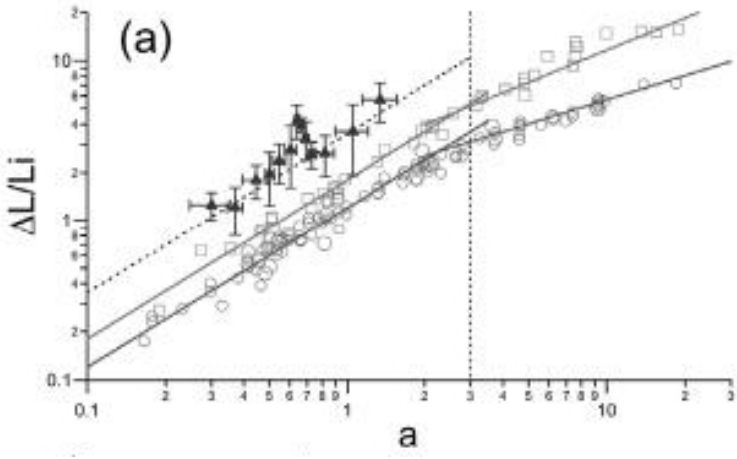
(Lajeunesse et al 2004)

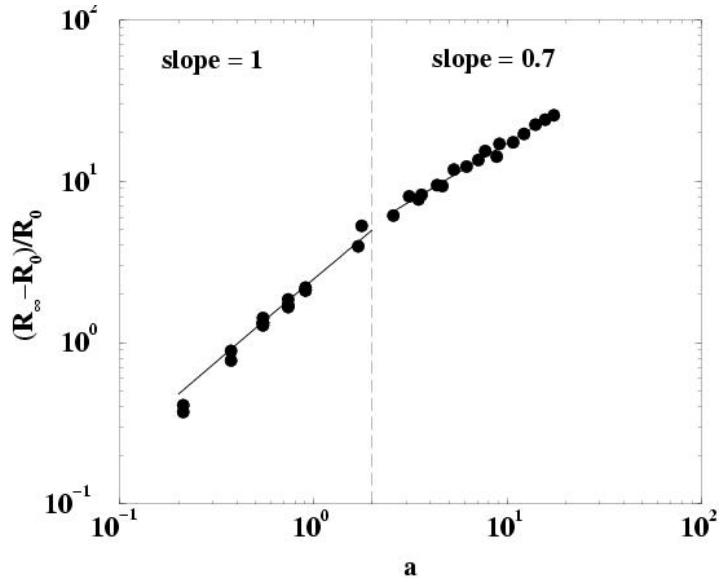




$$\frac{L - L_0}{L_0} \simeq \begin{cases} \lambda_1\, a & a < a_0 \\ \lambda_2\, a^{2/3} & a > a_0 \end{cases}$$

Numerical collapses





We recover the experimental scaling law:

$$\frac{L - L_0}{L_0} = \begin{cases} 2.50\,\boldsymbol{a} & \text{if } a \lesssim 2, \\ 3.25\,\boldsymbol{a}^{\mathbf{0.7}} & \text{if } a \gtrsim 2. \end{cases}$$

Outline

- Discrete Simulation of Granular Media
- Continuum Modeling: Defining an equivalent viscosity
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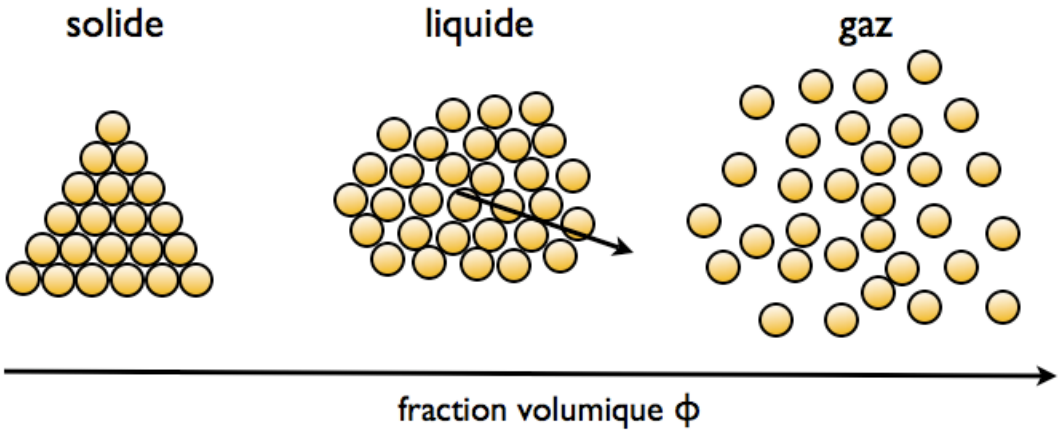
Physical Modeling



- Modeling the fluid-like behavior (equivalent viscosity...)
- Describing the static-flowing transition

The Gas-Like State

For dilute agitated flows: $\phi \leq \phi_m$, temperature $T = < v^2 > - < v >^2$

- Fluctuating velocity $c \simeq T^{1/2}$
- Mean free path ℓ
- Typical inter-collision time $\tau = \ell / c$

Pressure:

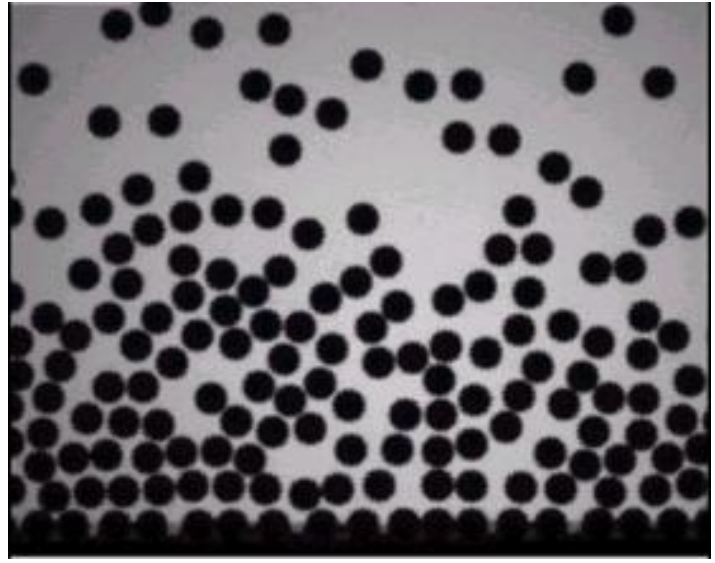
$$(\ell + D)P = m\frac{dv}{dt} = \rho D^2 \frac{c}{\tau}$$

$$P = \rho \frac{D^2}{\ell(\ell + D)} T$$

Viscosity:

$$\sigma = \rho \frac{D^2}{\ell} T^{1/2} \frac{dv}{dy}$$

$$\eta \propto T^{1/2}$$



(Azanza, Chevoir & Moucherond, 1998)

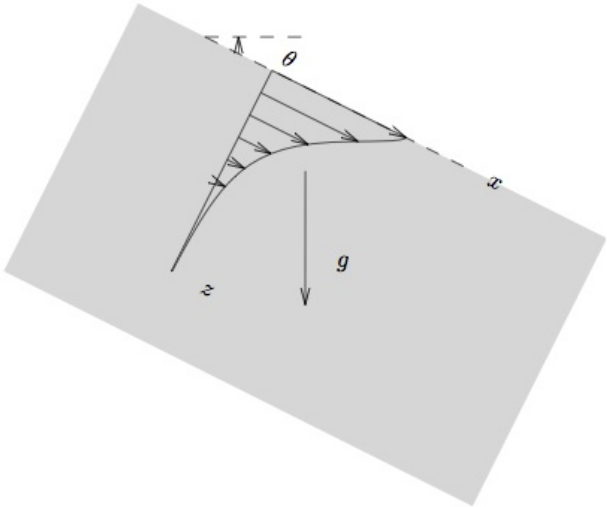
$\Rightarrow$ Application of the kinetic theory of gas to granular flows

The Case of Dense Flows

For dense flows: $\phi_m \leq \phi \leq \phi_M$, $\lambda << D$

Bagnold (1954):

Momentum transfer due to collisions for a dense gravity-driven flow?



Typical time: $\tau = (dv/dz)^{-1}$

$$\sigma_{xz} S = m \times \frac{dv}{dt}$$

$$\sigma_{xz} = \rho D \times \frac{dv}{dz} \times \frac{D}{\tau}$$

$$\boldsymbol{\sigma_{xz} = \rho D^2 \times \left(\frac{dv}{dz}\right)^2}$$

Shear-rate dependent viscosity :

$$\boldsymbol{\eta = \rho D^2 \frac{dv}{dz}}$$

Physical Modeling

- Granular systems can be static: frictional properties
- Granular systems can flow: viscous properties

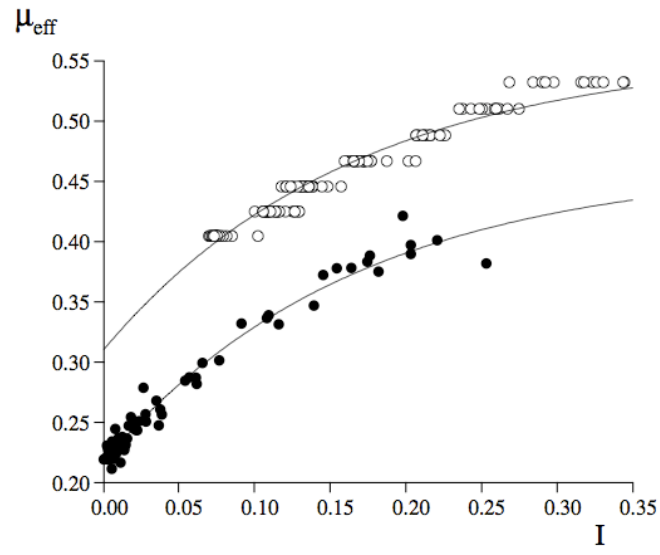
$\Rightarrow$ We use friction to define viscosity:

We form the visco-plastic law:

$$\frac{\tau}{P} = \mu$$
$$\tau_{ij} = \frac{\mu P}{|\dot{\gamma}|}\dot{\gamma}_{ij}$$

using the phenomenological $\mu(I)$ rheology:

$$\mu(I) = \mu_1 + \frac{\mu_2 - \mu_1}{I_0/I + 1}$$
$$I = \frac{D\dot{\gamma}}{\sqrt{P/\rho}}$$



Gdr MIDI 2004, Dacruz et al 2005, Jop et al 2006

- Solving numerically the equations for the motion of viscous fluid
  Open-source solver for incompresible Navier-Stokes using a VOF method
  (Popinet 2003, 2009)
  http://gfs.sourceforge.net

$$
\begin{aligned}
\boldsymbol{\nabla}.\boldsymbol{u} &= 0 \\
\rho\left(\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u}.\boldsymbol{\nabla}\boldsymbol{u}\right) &= -\boldsymbol{\nabla}p + \boldsymbol{\nabla}.(2\eta\boldsymbol{D}) + \rho\boldsymbol{g} \\
\frac{\partial c}{\partial t} + \boldsymbol{\nabla}.(c\boldsymbol{u}) &= 0 \\
\rho = c\rho_1 + (1-c)\rho_2 \quad , \quad \eta &= c\eta_1 + (1-c)\eta_2
\end{aligned}
$$

- Implementing the non-standard granular rheology

$$
\mu(I) = \mu_s + \frac{\Delta\mu}{I_0/I + 1} \quad \text{with} \quad I = d\frac{D_2}{\sqrt{p/\rho}}
$$

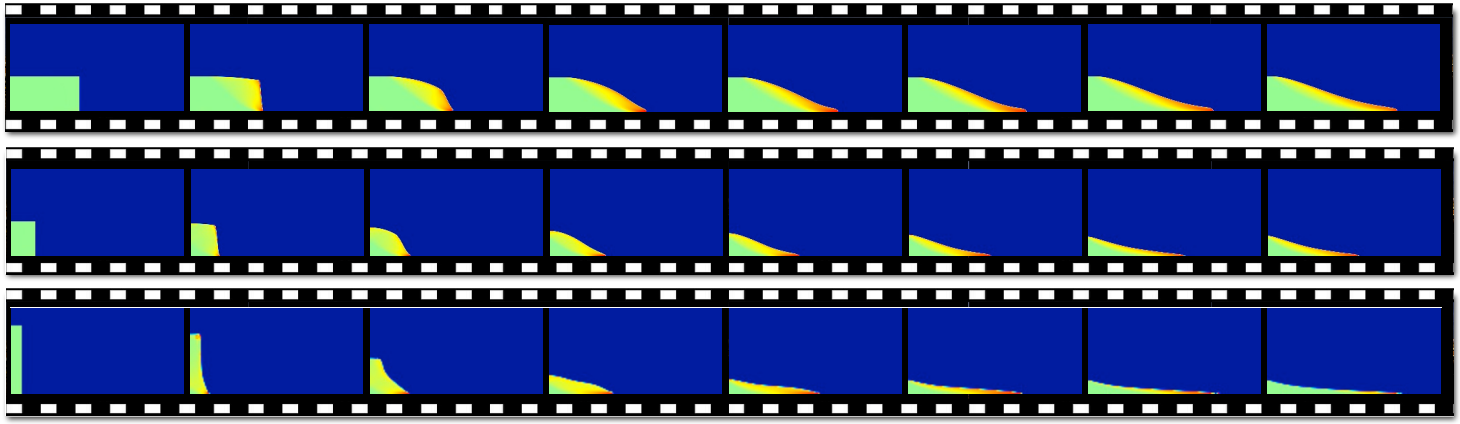
Viscosity:

$$
\eta = \frac{\mu(I)P}{D_2}
$$

The Collapse Experiment by Gerris

Implementation: we consider from small to large aspect ratios
we chose μ_s, $\Delta\mu$ et I_0 maximizing agreement with discrete simulations.

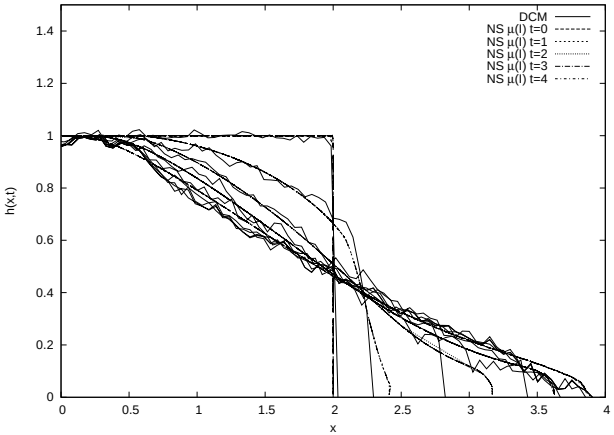
- Low density and viscosity for the surrounding fluid,
- No-slip condition at the bottom



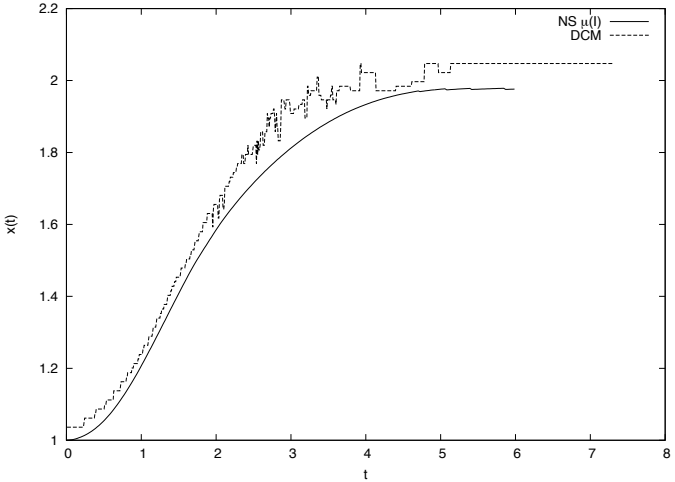
The Collapse Experiment by Gerris

Small aspect ratio:

Évolution de la forme

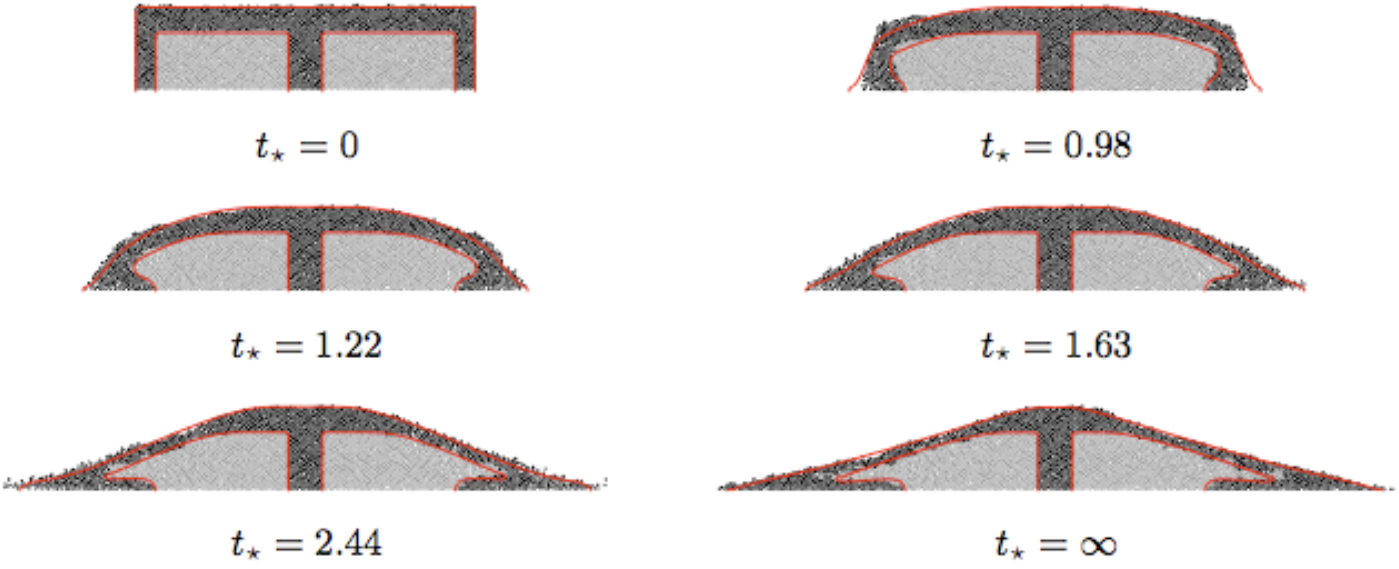


Propagation du front



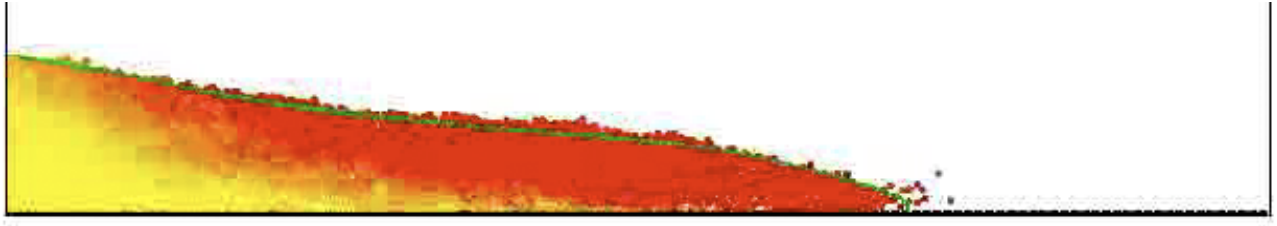
The Collapse Experiment by Gerris

Small aspect ratio: Inner deformation

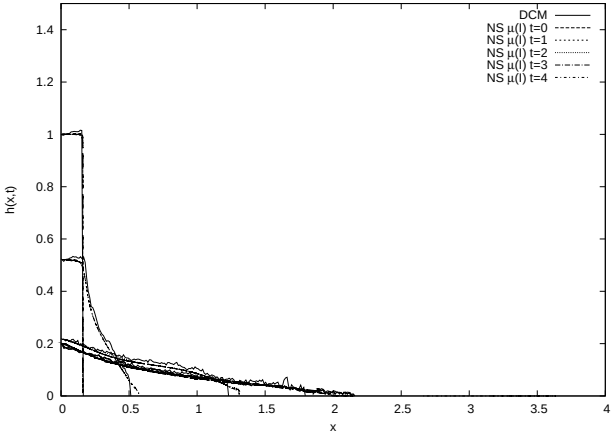


The Collapse Experiment by Gerris

Large aspect ratio:



Évolution de la forme



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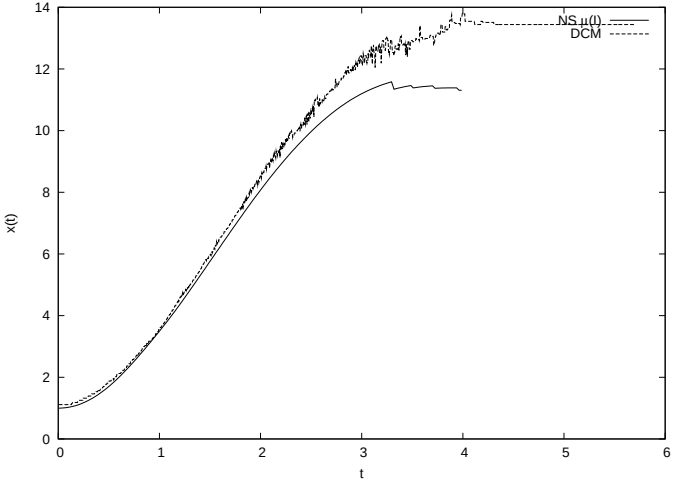
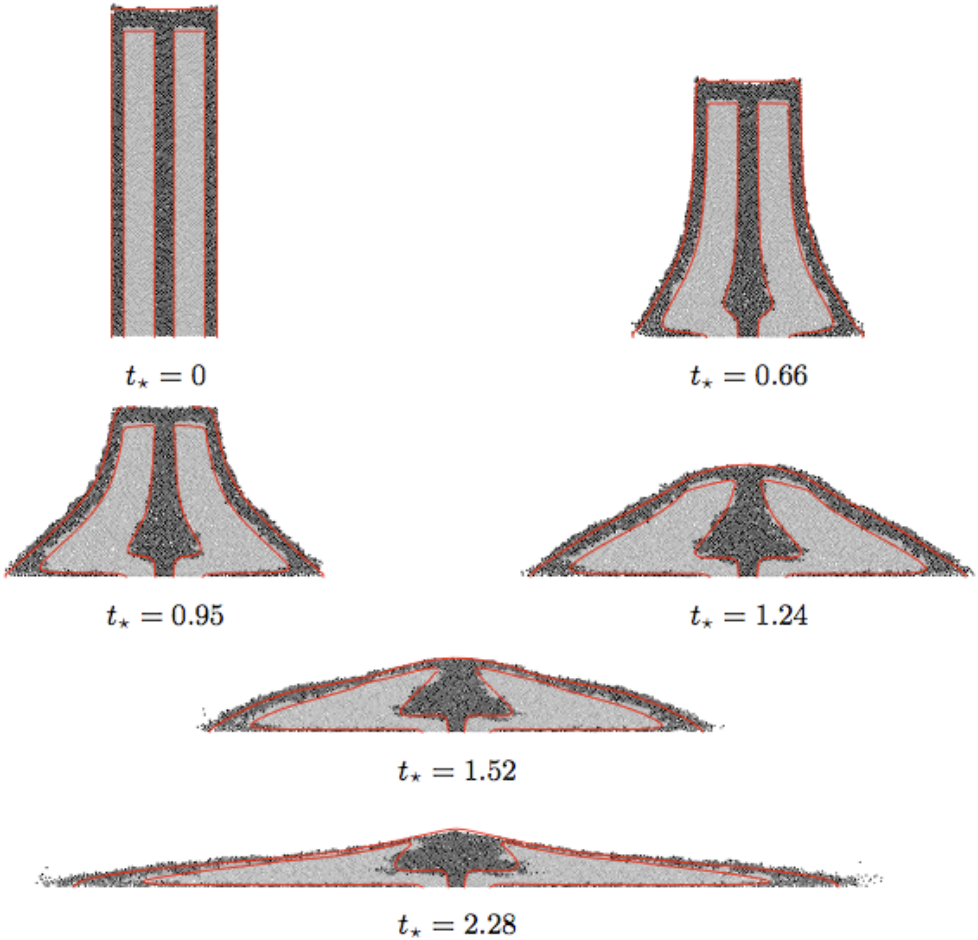
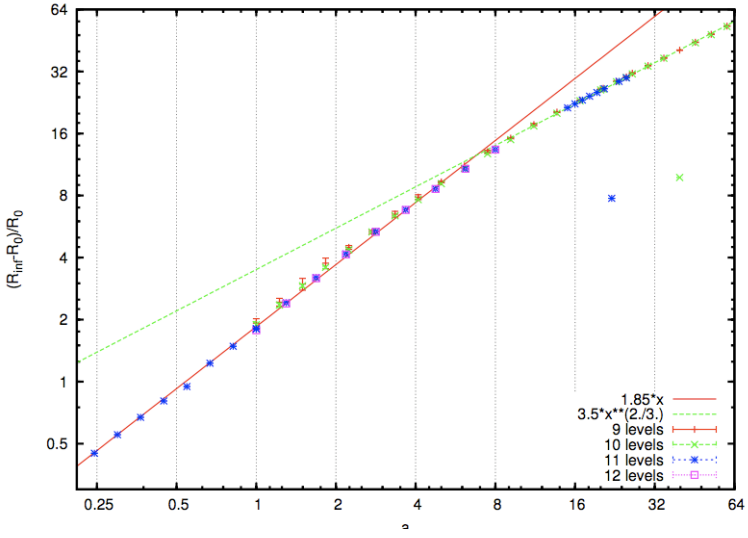


Large aspect ratio: Inner deformation



Scaling law for the runout:

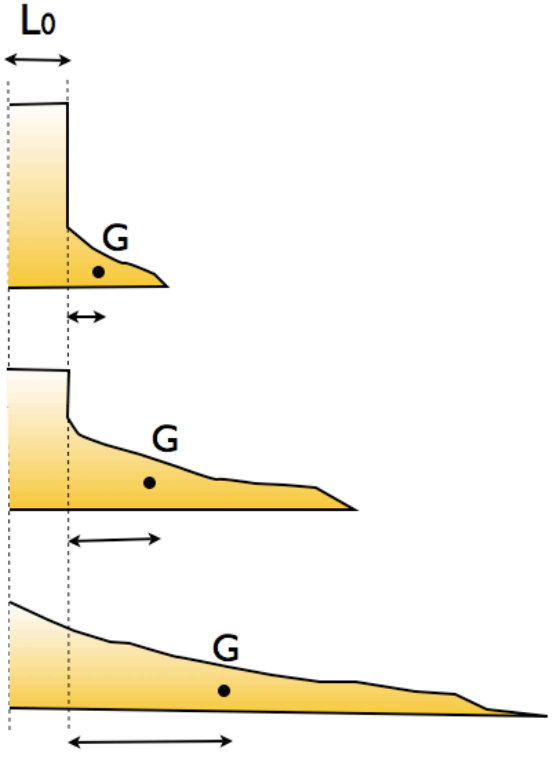


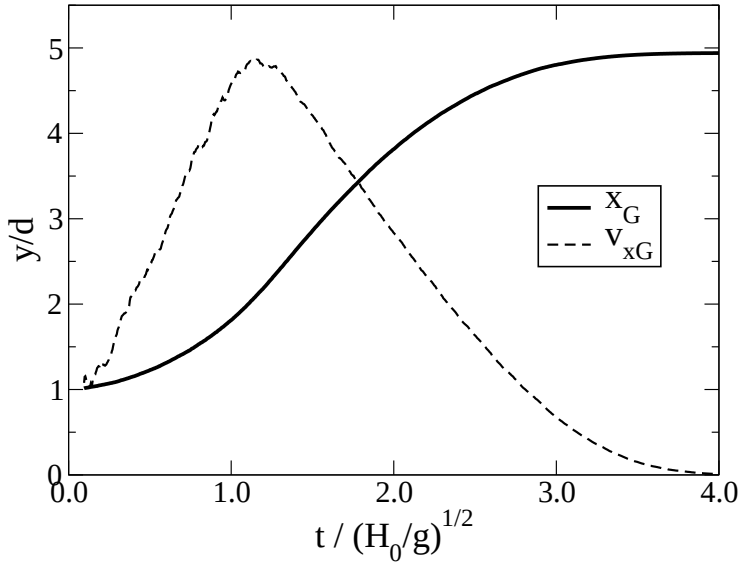
We do recover:

$$\frac{L - L_0}{L_0} \simeq \begin{cases} \lambda_1 \, a & a < a_0 \\ \lambda_2 \, a^{2/3} & a > a_0 \end{cases}$$

Around the Center of Mass

Structure de l'écoulement autour du centre de masse

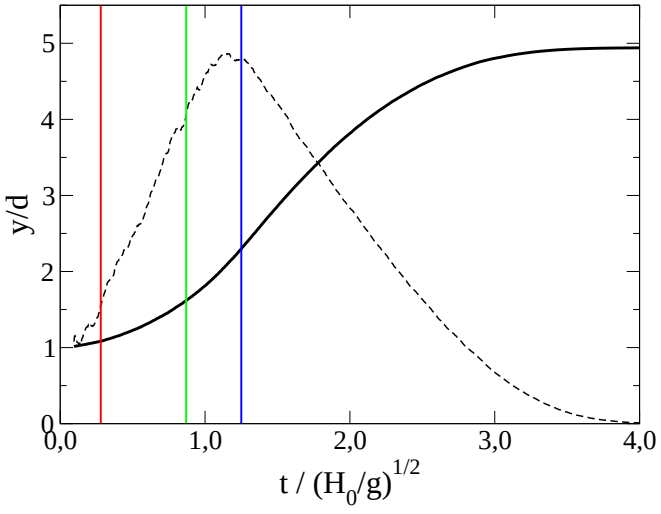


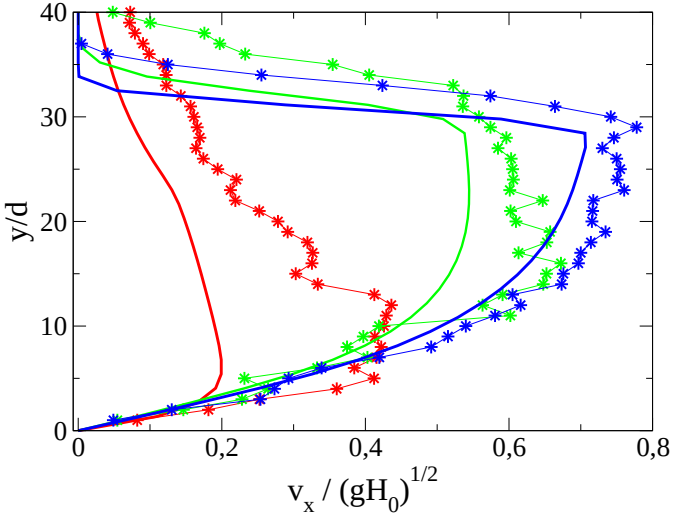


$\Rightarrow$ Velocity profile around the center of mass?

Around the Center of Mass

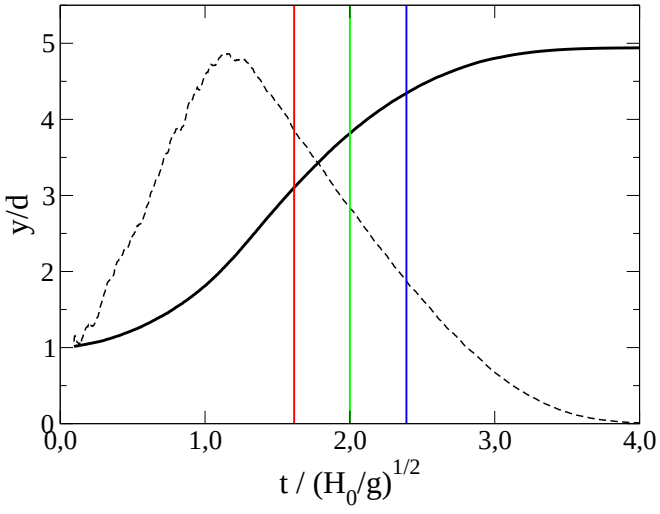
The acceleration phase

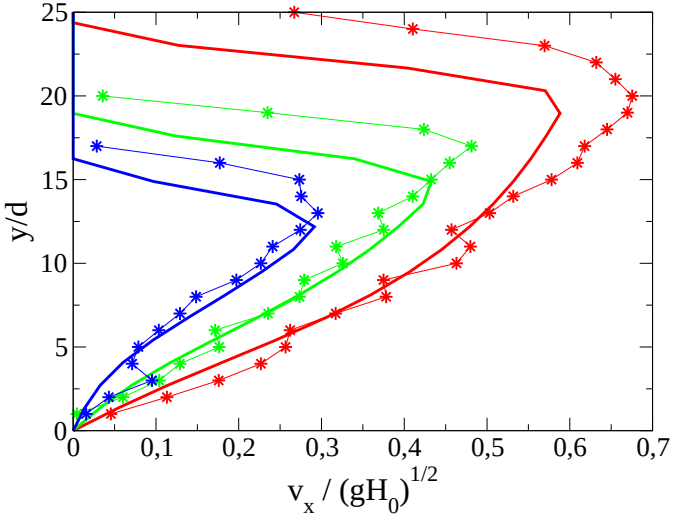




Around the Center of Mass

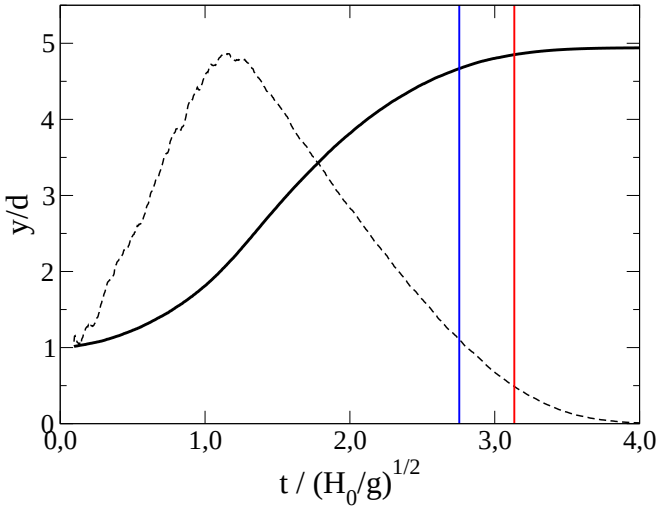
The deceleration phase

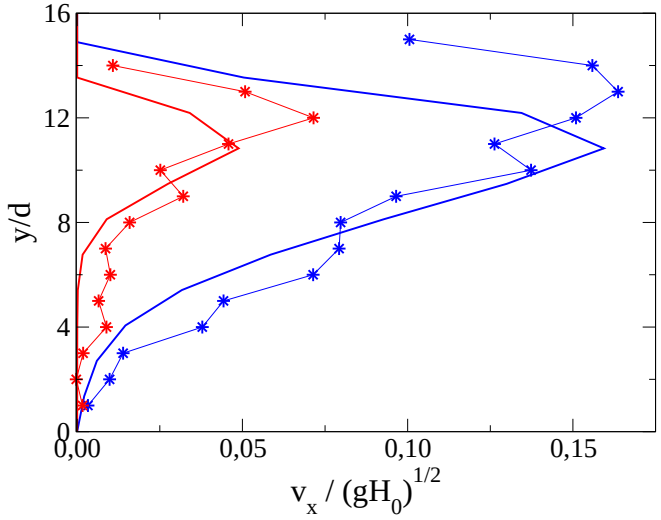




Around the Center of Mass

The stopping phase





The Collapse Experiment by Gerris

Case of a very large aspect ratio

Lagrée, Staron & Popinet, JFM 2011

Conclusions

- Different tools for the simulation of granular matter
  $\rightarrow$ problem of computation cost
- Prospect of reliable continuum simulation
  $\rightarrow$ problem of physical modeling